

Branch-Dependent Proper Time

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Abstract

We propose tying the fundamental scale of gravitationally induced space-time blurring to *proper time* rather than to spatial length, and postulate that, under an ensemble reading of the quantum state in which each branch carries its own world line, proper-time fluctuations are statistically independent across branches. Interference is sensitive to the *difference* of branch phase contributions; with independent clocks this difference accumulates at the frequency $\omega_{\text{eff}} = E_{\text{br}}/\hbar$. For a composite system we define this branch-sensitive energy operationally after removing the Hamiltonian sector common to both branches and measuring the remaining split-sector energy from its physical vacuum or ground state: the rest energy of each particle species is weighted by the trace distance between the one-particle reduced density matrices of the two branches; a fully distinguishable which-path carrier gives $E_{\text{br}} = mc^2$, while strictly internal branches with a common spatial-motional state give $E_{\text{br}} = \Delta E$. This prescription is invariant under a global shift of the total Hamiltonian. It yields $V/V_0 = \exp(-\chi)$ with $\chi = (\omega_{\text{eff}})^2 (\xi t_P^2 t)^{2/3}$ for a symmetric motional branch pair (for internal branches with one split-sector ground state an additional factor 1/2 appears); for a centre-of-mass interferometer with fully distinguishable packets $\chi = (mc^2/\hbar)^2 (\xi t_P^2 t)^{2/3}$, where t is the superposition time and $\xi = \mathcal{O}(1)$ is a numerical coefficient. The scaling form $\chi \propto m^2 t^{2/3}$ follows from the postulates; the overall coefficient carries an $\mathcal{O}(1)$ uncertainty. The cubic scaling is borrowed from the Károlyházy relation; the decoherence mechanism differs from Diósi–Penrose collapse. Confronted with sodium-nanoparticle interferometry of the Vienna group [1], this gives an upper bound $\xi \lesssim 1.1$. A decisive test is near: at $m \sim 1$ MDa and $v \approx 25$ m/s the model predicts $\chi \gg 1$ and practically complete loss of interference, whereas the standard quantum model of the setup at the same parameters expects measurable fringes with ordinary contrast.

Keywords: proper time; branch clock independence; Károlyházy uncertainty; decoherence; matter-wave interferometry; Margolus–Levitin theorem; quantum gravity

1 Introduction

The status of the wave function—an ontic object or a computational bookkeeping of an ensemble of possibilities—remains unsettled. We adopt the second, ensemble reading [2, 3]: $|\psi\rangle = \sum_i c_i |i\rangle$ catalogues distinct *variants* of the system, and the squared amplitudes are the relative weights of those variants. This is a deliberately minimal ontology; the substantive postulates on independent branch clocks (§ 2) are added separately, once gravity enters the picture.

In the relational view of time, a clock is a physical subsystem and “time” is a correlation within the total state [4]. If each branch is a genuine variant of the system, then it carries its own world line and hence its own proper time. Penrose reaches a structurally identical statement from the opposite direction: each component of the superposition sources its own space-time, and “since there is no canonical way of relating the time parameters in the two space-times, a fundamental uncertainty in the time-translation operator arises” [5]. Consistently with this, proper time accumulates along a world line and is not fixed by a single common

scale for distinct four-dimensional paths: in an interferometer with different trajectories one cannot, without an additional postulate, impose a single common proper-time parameter on all branches. While the system is in a superposition of paths, interference requires a coherent relative phase between them; branch-independent proper-time fluctuations (postulates P2–P3 below) introduce a random shift of this phase and bound the visibility from above, $V/V_0 \leq e^{-\chi}$ (see (7))—this is a prediction of an interference limit under path-dependent proper time, not a denial of amplitude superposition. After a which-path registration, a single scenario and a single realized world line are fixed; the $|c_i|^2$ remain the weights of the outcomes. The mechanism below is pure dephasing between branches: populations are not redistributed, and no which-path information is recorded. The mixed state arises only after ensemble averaging over $\delta t_a, \delta t_b$, not as the selection of a single realization.

2 Postulates

We introduce the key notation:

- t — the time the system spends in superposition (a proper-time interval along the world line);
- δt — the scale of proper-time uncertainty over the interval t ; δt_a — the realized shift on branch a (P3), $\text{Var}(\delta t_a) = \delta t^2$;
- $t_P = \sqrt{\hbar G/c^5}$ — the Planck time;
- E_{br} — the characteristic energy of the *split* subsystem in the interfering branch pair;
- $\varepsilon_{a|b}$ — the energy carried on branch a by degrees of freedom that distinguish it from branch b , measured from the physical vacuum or ground state of that split sector after removing the sector common to both branches;
- $\omega_{\text{eff}} = E_{\text{br}}/\hbar$ — the corresponding effective phase-noise frequency;
- m — the carrier mass in the which-path case (then $E_{\text{br}} = mc^2$, $\omega_{\text{eff}} = \omega_C = mc^2/\hbar$);
- $\xi = \mathcal{O}(1)$ — a dimensionless numerical coefficient.

2.1 (P1) Branch structure.

A branch is a state component differing in a *physical degree of freedom* that participates in interference: a spatial-motional state (which-path or current), or an internal eigenstate of H_{int} (spin, a hyperfine or optical level, a qubit). One connected and indistinguishable wave packet carries one clock; branching occurs when evolution splits the state into distinguishable components, and ceases upon their recombination or which-path registration.

To prevent the choice of the “split subsystem” from depending on an arbitrary grouping of a composite system, we fix an operational rule. For particles of species s , let $\gamma_{a,s}^{(1)}$ and $\gamma_{b,s}^{(1)}$ be the one-particle reduced density matrices of the branches, normalized by $\text{Tr} \gamma_{a,s}^{(1)} = \text{Tr} \gamma_{b,s}^{(1)} = N_s$. The number of one-particle states distinguishable between the branches and the corresponding branch-sensitive energy are

$$\mathcal{D}_s(a, b) = \frac{1}{2} \left\| \gamma_{a,s}^{(1)} - \gamma_{b,s}^{(1)} \right\|_1, \quad E_{\text{br}}^{\text{mot}} = c^2 \sum_s m_s \mathcal{D}_s(a, b). \quad (1)$$

Here $0 \leq \mathcal{D}_s \leq N_s$; the definition is basis-invariant and applies to identical particles [6]. For fully distinguishable spatial packets of every constituent, $\mathcal{D}_s = N_s$, and hence $E_{\text{br}}^{\text{mot}} = Mc^2$.

States common to both branches contribute zero. If only an internal level differs while the spatial-motional carrier $|C\rangle$ is common, we use the transition energy $E_{\text{br}}^{\text{int}} = |\Delta E|$, not the rest energy of the common carrier.

The energies that multiply the independent clock shifts are not absolute eigenvalues of the total Hamiltonian. For each ordered branch pair we denote by $\varepsilon_{a|b}$ the energy carried on branch a by the branch-distinguishing degrees of freedom, measured relative to the physical vacuum or ground state of that split sector. The Hamiltonian sector represented identically in both branches is common and is removed before coupling to $\delta t_a, \delta t_b$. Thus, for a motional pair,

$$\varepsilon_{a|b}^{\text{mot}} = \varepsilon_{b|a}^{\text{mot}} = E_{\text{br}}^{\text{mot}} = c^2 \sum_s m_s \mathcal{D}_s(a, b), \quad (2)$$

where the rest energy is the energy of occupied branch-exclusive matter modes relative to the vacuum. For internal states $|0\rangle, |1\rangle$ of a common carrier, energies are measured from the ground state of the split internal sector: $\varepsilon_{0|1}^{\text{int}} = 0$ and $\varepsilon_{1|0}^{\text{int}} = \Delta E$. If motional and internal differences coexist, the corresponding exclusive contributions are added separately on each branch, $\varepsilon_{a|b} = E_{\text{br}}^{\text{mot}} + \varepsilon_a^{\text{int}}$ and $\varepsilon_{b|a} = E_{\text{br}}^{\text{mot}} + \varepsilon_b^{\text{int}}$.

This prescription is invariant under a global shift $H \mapsto H + C\mathbf{1}$. Such a shift changes the reference energy of the common or ground-state sector by the same amount and leaves $\varepsilon_{a|b}$, $\varepsilon_{b|a}$, and hence all predicted visibilities unchanged. The occurrence of Mc^2 for resolved motional branches is therefore not the use of an arbitrary additive constant in a fixed-particle Hamiltonian; it is an additional premise that proper-time noise couples to the vacuum-referenced energy of the branch-exclusive massive modes.

2.2 (P2) Magnitude of the proper-time fluctuation.

The accumulated uncertainty of the proper-time interval t measured along a world line is

$$\boxed{\delta t^3 = \xi t_P^2 t}, \quad (3)$$

with $\xi = \mathcal{O}(1)$. Equation (3) is the temporal form of the Károlyházy uncertainty relation [7]; the same cubic scaling is consistent with the clock arguments of Salecker–Wigner [8], Ng–van Dam [9, 10], and the holographic bound on space-time uncertainty [11]. We do not fix the numerical coefficient ξ in the postulate: in these works it is likewise never pinned down more precisely than to $\mathcal{O}(1)$.

We stress that gravity is essential: without the “not a black hole” constraint the effect vanishes ($\delta t \rightarrow 0$ as $m \rightarrow \infty$). The rest mass is equally essential: the uncertainty δt is defined along a world line with nonzero proper time; without rest mass there is no proper time. For a photon $m = 0$, $d\tau = 0$, and the entire mechanism does not apply to a light-like carrier. The exponent $1/3$ and the Planck scale t_P^2 are gravitational in origin; the postulate is formulated along a world line, without explicitly invoking spatial length.

2.3 (P3) Branch clock independence.

For two branches a, b (in the sense of P1) over the superposition interval t (the same as in (3)), the realized shifts $\delta t_a, \delta t_b$ are independent, centered Gaussian random variables of accumulated proper time with $\text{Var}(\delta t_a) = \text{Var}(\delta t_b) = \delta t^2$ and $\text{Cov}(\delta t_a, \delta t_b) = 0$. This is a substantive postulate, motivated by the ensemble ontology (§ 1): distinct variants do not share a common clock; there is no strict derivation from it, and what remains irreducible is the premise that the holographic information budget is allocated *per branch* rather than as a single shared register. Within one branch, all degrees of freedom *not* participating in the split share that branch’s clock.

Crucially, δt_a is not a fluctuation of an external metric field and not environmental noise common to nearby trajectories. It is an *ontic* random shift of proper time; its scale over the

interval t is set by δt from (3)—the holographic scale of such uncertainty [11]. An environmental source—for example, a difference of gravitational potentials on the interferometer arms—would give a *correlated* shift δt_a , the closer to common the nearer the branches, and it would cancel in the relative phase; but that is a systematic shift, not noise. The holographic uncertainty attaches to the split subsystem of a branch, not to spatial points, so spatial proximity of branches does not by itself correlate their clocks. Attachment to discrete branches rather than positions is essential: noise independent for arbitrarily close *positions* would mean a random phase field with unbounded gradient—hence a force and momentum diffusion; noise independent for discrete *branches* is pure dephasing without force. Experimental consequences are discussed in § 4.1.

3 Decoherence functional

For a composite system we use the decomposition $H = mc^2 + P^2/(2m) + H_{\text{int}}$. For which-path ($\varepsilon_{a|b} = \varepsilon_{b|a} = E_{\text{br}} = mc^2$ by (P1)) the phase contribution of the split subsystem gives the relative phase

$$\Delta\varphi = \frac{\varepsilon_{a|b}}{\hbar} \delta t_a - \frac{\varepsilon_{b|a}}{\hbar} \delta t_b \approx \frac{mc^2}{\hbar} (\delta t_a - \delta t_b), \quad (4)$$

since $\varepsilon_{a|b} = \varepsilon_{b|a} = mc^2$ and interference involves the *difference* of branch-exclusive contributions, not an absolute eigenvalue of the total Hamiltonian.

For a single superposition interval, postulates (P2)–(P3) fully specify the distribution of $\delta t_a, \delta t_b$ used in the derivation below; the multi-time structure of the noise is discussed in § 6.1. The general formula follows from (4) and postulates (P2)–(P3). By (P3), with $\text{Var}(\delta t_a) = \text{Var}(\delta t_b) = \delta t^2$:

$$\text{Var}(\Delta\varphi) = \frac{\varepsilon_{a|b}^2 + \varepsilon_{b|a}^2}{\hbar^2} \delta t^2. \quad (5)$$

If the centre of mass branches (which-path), (5) gives

$$\text{Var}(\Delta\varphi) = 2\omega_{\text{eff}}^2 \delta t^2, \quad \omega_{\text{eff}} = \frac{mc^2}{\hbar}. \quad (6)$$

With *shared* clocks the global phase $mc^2\tau/\hbar$ would cancel in $\Delta\varphi$; with independent clocks (P3) the vacuum-referenced branch-exclusive accumulations enter the difference. A constant common term in the total Hamiltonian is removed by (P1) and cannot generate dephasing. For internal branches $|0\rangle, |1\rangle$ with a common carrier $|C\rangle$ we take $E_{\text{br}} = \Delta E$ and $\omega_{\text{eff}} = \Delta E/\hbar$; since here $\varepsilon_{0|1} = 0$ and $\varepsilon_{1|0} = \Delta E$, dephasing is twice as weak as in the symmetric case: $\chi = \omega_{\text{eff}}^2 \delta t^2/2$.

Since $\delta t_a, \delta t_b$ are Gaussian (P3), the relative phase $\Delta\varphi$ is also Gaussian and $\langle e^{i\Delta\varphi} \rangle = \exp[-\text{Var}(\Delta\varphi)/2]$; the factor 2 from the two branches and the factor 1/2 from Gaussian averaging cancel. In the which-path case, for fringe visibility (V relative to the reference V_0):

$$\boxed{\frac{V}{V_0} = e^{-\chi}, \quad \chi = \omega_{\text{eff}}^2 \delta t^2 = \left(\frac{mc^2}{\hbar}\right)^2 (\xi t_P^2 t)^{2/3}.} \quad (7)$$

The general expression for an arbitrary branch pair is

$$\boxed{\chi_{ab} = \frac{\varepsilon_{a|b}^2 + \varepsilon_{b|a}^2}{2\hbar^2} \delta t^2.} \quad (8)$$

For $\varepsilon_{a|b} = \varepsilon_{b|a} = mc^2$ it reduces to (7). Because the ε 's are vacuum- or ground-state-referenced energies of the split sector, (8) is unchanged by $H \mapsto H + C\mathbf{1}$. The scaling form $\chi \propto m^2 t^{2/3}$ in a centre-of-mass interferometer follows from the postulates and constitutes the central testable

prediction; the overall coefficient carries an $\mathcal{O}(1)$ uncertainty (through $\xi^{2/3}$, the interpretation of δt , and the choice of the effective t ; see § 4). The mass-quadratic dependence follows from rest-phase accumulation ($\omega_{\text{eff}} \propto mc^2$) and matches the typical dephasing scaling in models of metric and conformal space-time fluctuations [12, 13]; in the high-mass interferometry programme, this mass growth of the signal is highlighted as a practical motivation for searching for hypothetical effects [14].

Relation to clock interference and the Compton phase. The conceptual framework of “proper time along each branch” is already used in quantum-clock interference. In Zych et al., an internal state acts as a clock: a deterministic proper-time difference along the two paths records which-path information in the internal degree of freedom and reduces visibility [15]. Piovski et al. developed this mechanism for a composite object in a gravitational field: an externally imposed potential difference unitarily entangles the centre of mass with the internal-energy distribution, and dephasing appears after tracing out the internal degrees of freedom [16]. In contrast, the present model does not rely on an external potential difference or on a dedicated internal degree of freedom that unitarily encodes which-path information; it introduces the stochastic Planckian scale P2 and branch clock independence (P3), and therefore predicts irreversible ensemble dephasing even for an object in its internal ground state.

Using $\omega_C = mc^2/\hbar$ requires a caveat: in standard quantum mechanics the Compton phase cancels in a closed interferometer [17], whereas atom interferometry has been interpreted as providing access to the mass frequency [18]. The present model adds the independent premise that the vacuum-referenced rest energy of branch-exclusive massive modes couples to the independent shifts $\delta t_a, \delta t_b$ and thereby becomes a random *relative* contribution. It does not couple an arbitrary identity term in the total Hamiltonian: P1 first removes the common sector and its reference energy. Without this premise the central prediction $\chi \propto m^2$ does not arise (see also § 6).

Planck mass boundary. The decoherence time t_{dec} is defined by the condition $\chi = 1$. From (7):

$$t_{\text{dec}} = \frac{t_C^3}{\xi t_P^2}, \quad t_C = \frac{\hbar}{mc^2}. \quad (9)$$

Setting $t_{\text{dec}} = t_C$, we obtain $t_C = \sqrt{\xi} t_P$, i.e. the characteristic scale

$$m_* = \frac{m_P}{\sqrt{\xi}}, \quad m_P = \sqrt{\hbar c/G} \approx 2.2 \times 10^{-8} \text{ kg}. \quad (10)$$

For $\xi = \mathcal{O}(1)$ this is of the order of the Planck mass. For $m \gg m_*$ we have $t_{\text{dec}} \ll t_C$: inter-branch coherence vanishes over an interval below the uncertainty δt from (3) and in principle inaccessible to direct measurement. The mechanism remains pure dephasing without population redistribution or spontaneous emission [19]. The ontological consequences of the macroscopic limit—an effective ensemble mixture in the branch basis, objective loss of inter-branch interference before readout, and distinction from dynamical collapse models—are discussed in § 6.3.

4 Confrontation with experiment

4.1 Matter-wave interferometry: the primary probe

By construction (P1)/(P3) the mechanism is pure rest-phase dephasing without force or momentum diffusion (see (P3) and § 6). The experimental consequence is that non-interferometric bounds on Diósi–Penrose (the underground search for spontaneous radiation [19], the Majorana Demonstrator limit [20], and X-ray tests [21]) do not apply; single atomic clocks are likewise unaffected—the effect exists only *between* branches of a superposition. Matter-wave interferometry of the centre of mass is therefore the most direct laboratory probe of the mechanism with $E_{\text{br}} = mc^2$.

For the same reason, purely optical interferometers provide no direct constraint ($d\tau = 0$ for a photon); LIGO and the Holometer constrain external metric and spatial models [22], but not branch-dependent proper time (BDPT; see § 6). Massive mirrors would be subject to the mechanism only in their own centre-of-mass superposition; in the usual classical optical regime they are not, and existing optical null results therefore place no direct bound on ξ .

4.2 Sodium nanoparticles: the Vienna group (2026)

Setup parameters. We confront Equation (7) with the sodium-nanoparticle interferometer of the Vienna group [1, 23]:

- clusters of 5000–10000 Na atoms with a mass-distribution centre $m \approx 172$ kDa ($m \approx 2.86 \times 10^{-22}$ kg);
- velocity $v \approx 160$ m/s;
- three gratings of period $d = 133$ nm spaced by $L = 0.983$ m;
- a measured fringe visibility $V = 0.10 \pm 0.01$ (second run 0.08 ± 0.01) at 172 kDa, described by the quantum model with a single global factor 0.78 for conventional contrast losses (alignment, gravitational and rotational phase averaging, vibrations, thermal and collisional decoherence);
- a record macroscopicity $\mu = 15.5$ [24, 1].

The relevant superposition time is the Talbot flight of one section $t = L/v \approx 6.1$ ms (justification of the choice—in § 4.3).

Under the sharpened P1 this is not a separate counting rule: the two resolved branches of a rigid cluster are centre-of-mass shifts of the whole body (without change of internal configuration). When their one-atom spatial supports have negligible overlap, $\mathcal{D}_{\text{Na}} = N$ and Equation (1) gives $E_{\text{br}} = Nm_{\text{at}}c^2 = Mc^2$, whence $\chi \propto M^2$. If the overlap is incomplete, M in the formulas below must be replaced by $m_{\text{at}}\mathcal{D}_{\text{Na}}$; the assumption of full branch distinguishability is thus explicit and can be checked against the packet geometry.

4.3 Numerical estimate and constraint on ξ

We first fix the effective superposition time. The postulates do not uniquely define the moment at which independent branch clocks “switch on” (§ 6.2): delocalization of the wave function grows already after the first grating (full flight $2L/v \approx 12.3$ ms), but distinguishable diffraction orders—branches in the strict sense of (P1)—form only at the second grating, leaving one section until recombination at the third, $t = L/v \approx 6.1$ ms. For an upper bound we take the *minimal* interpretation $t = L/v$: the smaller t , the smaller the predicted dephasing and the *weaker* (more conservative) the resulting bound on ξ ; any broader interpretation of the superposition time would only tighten the limit. With $\omega_C = mc^2/\hbar = 2.43 \times 10^{29} \text{ s}^{-1}$ (at $m = 172$ kDa) and $t_P^2 = 2.91 \times 10^{-87} \text{ s}^2$, Equation (7) gives

$$\chi \approx 0.41 \xi^{2/3} \quad (t = L/v \approx 6.1 \text{ ms}). \quad (11)$$

At $\xi = 1$ this is $\chi \approx 0.41$, i.e. a visibility deficit of $\approx 34\%$.

We use the published raw data and analysis code [25]. In the bin $P_2 = 15.252$ mW three independent scans give

$$V_i = 0.0732 \pm 0.0081, \quad 0.0975 \pm 0.0070, \quad 0.0780 \pm 0.0074, \quad (12)$$

where the errors are 1σ from the covariance matrix of a nonlinear sinusoidal fit. For the displayed rounded values, the inverse-variance mean is $\bar{V} = 0.0840$ with a formal error of 0.0043. For

these three scans $\chi_{\text{red}}^2 = 3.08$; following the experiment's procedure for overdispersed bins, we inflate the error by $\sqrt{\chi_{\text{red}}^2}$ and obtain $\sigma_{\bar{V}} = 0.0076$.

The official quantum curve at this power gives $V_{\text{QM}} = 0.1108$ (without conventional losses). The one-sided lower bound on the observed visibility at 95% is

$$\bar{V} - 1.645 \sigma_{\bar{V}} = 0.0716. \quad (13)$$

We construct the upper bound on ξ strictly. The observed visibility decomposes as $V_{\text{obs}} = R_{\text{ord}} V_{\text{QM}} e^{-\chi}$, where $R_{\text{ord}} \leq 1$ is the total conventional contrast-loss factor (alignment, vibrations, phase averaging, gas, radiation). Since R_{ord} cannot exceed unity by its physical meaning, regardless of its true value we have $e^{-\chi} \geq V_{\text{obs}}/V_{\text{QM}}$. Attributing *the entire* visibility deficit to our mechanism ($R_{\text{ord}} = 1$), we obtain

$$e^{-\chi} \geq \frac{0.0716}{0.1108} = 0.646, \quad \chi \leq 0.44, \quad (14)$$

and from $\chi = 0.41 \xi^{2/3}$ —a strict conservative bound

$$\boxed{\xi \lesssim 1.1 \quad (95\% \text{ CL, one-sided}).} \quad (15)$$

This bound does not depend on the experimental calibration: any real conventional losses ($R_{\text{ord}} < 1$) could only tighten it. The exclusion threshold corresponds to a visibility deficit $1 - 0.646 \approx 35\%$: values of ξ predicting a larger deficit are incompatible with the data at 95%.

If one trusts the published estimate of conventional losses $R_{\text{ord}} = 0.78$, the limit tightens: $e^{-\chi} \geq 0.0716/(0.78 \cdot 0.1108) = 0.828$, $\chi \leq 0.188$, whence $\xi < 0.31$. But this is a conditional result: at a single mass the product $R_{\text{ord}} e^{-\chi}$ is degenerate, and the data do not allow separating the new mechanism from underestimated conventional losses; the interpretation of δt , the choice of effective t , and the overall postulate coefficient also remain model systematics.

Summary for the data: at $\xi = 1$ the model predicts ($R_{\text{ord}} = 1$) visibility $0.1108 e^{-0.41} \approx 0.074$ against the observed 0.0840 ± 0.0076 —a deviation of only 1.4σ , so $\xi = \mathcal{O}(1)$ is *not excluded* by the data; the strict bound presses the coefficient to $\xi \lesssim 1.1$, the conditional one (with the 0.78 calibration) to 0.31. A statistically rigorous limit requires a joint likelihood analysis of the raw scans, the quantum-curve parameters, and all conventional contrast-loss channels.

4.4 The decisive near-term test

The already-measured high visibility $V = 0.66 \pm 0.09$ at 0.4–1 MDa [1] does *not* test the mechanism as distinguishable quantum coherence: it was taken in the regime $\lambda_{dB} \lesssim 3$ fm, $L \leq L_T$ ($L_T \sim d^2/\lambda_{dB}$ —the *Talbot length*, the scale of grating self-imaging in the near field of a Talbot–Lau interferometer [1, 14]), where the quantum and classical predictions coincide (geometric optics). A discriminating test requires the quantum regime at high mass. The same programme targets $m \sim 1$ MDa with $v \approx 25$ m/s [1]. Then even with the conservative $t = L/v \approx 39$ ms and $\omega_{\text{eff}} \approx 1.4 \times 10^{30} \text{ s}^{-1}$, Equation (7) at $\xi = 1$ gives

$$\chi \approx 47, \quad V/V_0 = e^{-\chi} \lesssim 10^{-20}. \quad (16)$$

Practically complete loss of interference is robust against $\mathcal{O}(1)$ uncertainty in the coefficient. The standard quantum model of the setup at the same parameters expects measurable fringes with ordinary contrast. Observing fringes at ~ 1 MDa and $v \approx 25$ m/s would falsify the model.

4.5 Distinguishing signature

A visibility deficit alone is not diagnostic, since thermal radiation, collisions with residual gas, and velocity spread also reduce the contrast. The signature of the present mechanism is its scaling $\chi \propto m^2 t^{2/3}$ together with *immunity to shielding*: it cannot be reduced by

improving the vacuum or cooling the particle. The experimental discriminator is whether a residual deficit persists after the suppression of all known environmental channels and whether it follows the m^2 law. Disappearance of fringes at ~ 1 MDa and $v \approx 25$ m/s with scaling $V \propto \exp[-(mc^2/\hbar)^2(\xi t_P^2 t)^{2/3}]$ after excluding environmental losses would be evidence in favour of the model.

An additional discriminator is the dependence on the maximum spatial branch separation Δx . For momentum-transfer decoherence (collisions and photon emission or scattering), the localization exponent contains factors of the form $1 - \text{sinc}(q\Delta x/\hbar)$: it vanishes for unresolved paths, grows as the environment starts to resolve them, and saturates at the event rate once Δx exceeds the environment's spatial resolution [26]. It would therefore be incorrect to say that *all* environmental channels grow without bound with Δx . A gravitational gradient usually produces a deterministic geometry-dependent phase; contrast is lost only after averaging over fluctuations of the gradient or trajectories.

The present model is likewise not strictly independent of Δx over the entire range. Under the sharpened P1,

$$\chi_{\text{BDPT}}(\Delta x) = \left[\frac{c^2 \sum_s m_s \mathcal{D}_s(\Delta x)}{\hbar} \right]^2 (\xi t_P^2 t)^{2/3}. \quad (17)$$

While the packets overlap, $\mathcal{D}_s(\Delta x)$ grows from zero. The specific signature emerges once the branches are resolved: as $\mathcal{D}_s \rightarrow N_s$, increasing Δx further does not change χ_{BDPT} , producing a plateau that depends only on m and t . This is the direct consequence of attaching the noise to distinguishable branches rather than to spatial points.

The test can be performed in one apparatus: at fixed m , v , superposition time, pressure, and temperature, vary the transverse momentum transfer (preferably the diffraction order; changing the grating period also requires compensating the change in Talbot length) and thereby scan Δx . The quantity to compare is not raw visibility but the residual exponent

$$\chi_{\text{res}}(\Delta x) = -\ln \left[\frac{V_{\text{obs}}(\Delta x)}{V_{\text{QM}}(\Delta x)} \right] - \chi_{\text{env}}^{\text{calc}}(\Delta x), \quad (18)$$

where V_{QM} includes coherent dynamics and grating interactions, while $\chi_{\text{env}}^{\text{calc}}$ contains the calculated gas and thermal contributions. The model predicts a constant addition to χ_{res} on the distinguishability plateau, whereas unsaturated environmental channels retain a characteristic Δx dependence. A flat residual alone is not unique: saturated environmental decoherence and Δx -independent technical losses can mimic it. The Δx scan should therefore be combined with changes in vacuum or temperature and with a test of the $m^2 t^{2/3}$ scaling.

5 Limits on quantum computation

5.1 Composite superpositions: flux qubits and atomic NOON states

The unqualified phrase “energy of the split subsystem” is particularly hazardous for a superconducting flux qubit. Naively counting all 10^6 – 10^{10} electrons carrying superconductivity would give, at the upper end, $E_{\text{br}} \simeq 8.2 \times 10^{-4}$ J and, for $t = 100$ μs and $\xi = 1$, $\chi \simeq 26.5$; the observed coherence of current states would then exclude the model. A microscopic analysis of the BCS states, however, shows that the near-unit overlap of the one-electron states in the two current branches leaves only a small fraction of occupations changed. An estimate equivalent to the one-particle distinguishability in (1) gives $\mathcal{D}_e \simeq 42$ – 5750 for the devices considered, rather than the total number of condensate electrons [27, 28]. At $t = 100$ μs this gives $\chi \lesssim 9 \times 10^{-12}$: existing flux qubits are safe for the model. The difference between naive and microscopic counts is not an $\mathcal{O}(1)$ uncertainty; this is why the distinguishability measure is part of P1.

The same rule separates two often conflated regimes of atom interferometry. A product state

$$\left[(|L\rangle + |R\rangle)/\sqrt{2} \right]^{\otimes N} \quad (19)$$

contains independent one-atom superpositions; observed first-order coherence compares components with $\mathcal{D} = 1$ and acquires the atomic, not collective, energy mc^2 . In contrast, a NOON state

$$(|N, 0\rangle + |0, N\rangle)/\sqrt{2} \quad (20)$$

has two collective branches with $\mathcal{D} = N$ and $E_{\text{br}} = Nmc^2$. For $N = 10^5$ ^{87}Rb atoms and $t = 10$ ms, Equation (7) predicts $\chi \simeq 1.4 \times 10^3$ at $\xi = 1$, i.e. practically complete suppression of interference for a massive motional NOON state. This additional falsifiable prediction does not yet conflict with experiment: demonstrated atomic NOON states remain few-particle; for example, Ref. [29] realized a second-order state.

5.2 Margolus–Levitin: a heuristic comparison

The decoherence time of the model is the moment when $\chi = 1$ and the visibility falls to e^{-1} ; it is given by Equation (9).

The Margolus–Levitin theorem [30] limits the speed of orthogonal evolution by the energy *above the ground level* $E_{\text{av}} = \langle H \rangle - E_0$: $\tau_{\perp} \geq \pi\hbar/(2E_{\text{av}})$, whence $\nu_{\text{max}} = 2E_{\text{av}}/(\pi\hbar)$. This is a question *distinct* from branch decoherence: the Margolus–Levitin theorem concerns the unitary speed of state change, not the noise of independent clocks.

At fixed particle number, the term mc^2 is proportional to the identity and does not generate orthogonal centre-of-mass evolution; it cancels from $E_{\text{av}} = \langle H \rangle - E_0$. Thus the identification $E_{\text{av}} = E_{\text{br}} = mc^2$ follows neither from the theorem nor from the model’s postulates. Only as a formal dimensional illustration, under the additional and nonstandard assumption of that identification, one may write the number of “ticks” over the window t_{dec} as

$$N_{\text{max}} = \nu_{\text{max}} t_{\text{dec}} = \frac{2\omega_{\text{eff}}}{\pi} \cdot \frac{t_C^3}{\xi t_P^2} = \frac{2}{\pi\xi} \left(\frac{t_C}{t_P} \right)^2. \quad (21)$$

Since $t_C/t_P = m_P/m$, for which-path

$$N_{\text{max}}^{\text{formal}} = \frac{2}{\pi\xi} \left(\frac{m_P}{m} \right)^2. \quad (22)$$

At $m = m_* = m_P/\sqrt{\xi}$ this formal quantity is $2/\pi \approx 0.64$. It is not a computational-power bound derived from the model. For internal branches, where the transition energy actually generates evolution, the theorem should instead be applied directly using the physical E_{av} , without importing the formula based on mc^2 .

5.3 Formal coincidence with holographic scaling

The same additional identification gives the formal rate $N = (2/\pi)t/t_C$. Combining it with the coherence condition $t_{\text{dec}} \geq t$, i.e. $t_C \geq (\xi t_P^2 t)^{1/3}$, and taking equality at the boundary gives

$$N_{\text{formal}}(t) = \frac{2}{\pi\xi^{1/3}} \left(\frac{t}{t_P} \right)^{2/3}. \quad (23)$$

Over one second this is $\approx 4.5 \times 10^{28}$ at $\xi = 1$. The exponent $2/3$ coincides with the scaling $N \propto (t/t_P)^{2/3}$ obtained by Ng for a “space-time-foam computer” and a black-hole quantum computer [11], because both estimates use the cubic law $\delta t^3 \sim t_P^2 t$. This is a meaningful structural analogy, but not an independent derivation of an operation-count bound: without additional physical justification for $E_{\text{av}} = mc^2$, matching exponents does not turn N_{formal} into a computational limit of the model.

5.4 Phase noise and the qubit register

Postulates (P1)–(P3) act on any superposition, including a computational one. For strictly internal levels $|0\rangle, |1\rangle$ with a common one-particle spatial-motional density matrix, P1 gives $E_{\text{br}} = \hbar\omega_q = \Delta E$; the mass of the chip and resonator enters the common $|C\rangle$ and does not enter ω_{eff} . This statement cannot automatically be transferred to a superposition of opposite persistent currents, for which Equation (1) applies as discussed in § 5.1. Superconducting resonators storing a single photon for ~ 34 ms with $\gtrsim 10^{23}$ electrons in their walls [31] illustrate the internal case: with $E_{\text{br}} = \Delta E$ of the photon, not the rest energy of the common walls, long-lived coherence is consistent with the model. The random relative phase between a pair of internal branches over time t :

$$\sigma(t) = \frac{\sqrt{\varepsilon_{a|b}^2 + \varepsilon_{b|a}^2}}{\hbar} \delta t = \omega_q (\xi t_P^2 t)^{1/3} \quad (\varepsilon_{0|1} = 0, \varepsilon_{1|0} = \hbar\omega_q). \quad (24)$$

For a superconducting qubit ($\omega_q \approx 2\pi \times 5$ GHz) and $t = 1$ s: $\delta t \approx 1.4 \times 10^{-29}$ s, $\sigma \approx 4.5 \times 10^{-19}$ rad (the corresponding dephasing parameter $\chi = \sigma^2/2$). Crucially, the variance (24) is *pairwise* and independent of the number of branches: interference involves only differences of independent δt_a , and the extremal phase spread over 2^n branches grows only as $\sigma\sqrt{2 \ln 2^n} \propto \sigma\sqrt{n}$. For a branch pair differing in the state of d qubits, $\chi \propto d^2$; in the worst case, when all n qubits differ, the formal limit $n\omega_q \delta t \lesssim 1$ gives $n \lesssim 10^{18}$ —far beyond practical horizons.

5.5 Phase distinguishability floor: exact and approximate QFT

The quantity σ sets a *floor* for algorithmic phase rotations: a gate at angle $\theta \lesssim \sigma$ is indistinguishable from the model’s intrinsic noise and cannot carry information. An exact quantum Fourier transform on n qubits contains controlled rotations down to $2\pi/2^n$, and already at $n \gtrsim 64$ these angles fall below the floor $\sim 10^{-19}$ rad: exact QFT on large registers is forbidden by the model. However, exact QFT is not used in implementations either: by Coppersmith [32] all rotations with $k > \mathcal{O}(\log(n/\varepsilon))$ can be *omitted from the circuit* entirely, losing only ε success probability; the minimum remaining angle $\sim 2\pi\varepsilon/n$ is polynomially, not exponentially, small. For Shor’s algorithm [33] at $n = 10^4$, $\varepsilon = 10^{-2}$ this is $\sim 10^{-5}$ rad—thirteen orders of magnitude above the floor (24). Phase-estimation algorithms that read the answer from the position of an interference peak are therefore not limited by the mechanism: it forbids precisely those exponentially small rotations that are excluded from circuits anyway.

5.6 Grover’s algorithm limit

The case of amplitude amplification [34] is contrasting. Here there is no QFT and nothing to truncate: each iteration rotates the state vector by a fixed angle $2\theta \approx 2/\sqrt{N}$, set by the geometry of the problem, and all $\sim (\pi/4)\sqrt{N}$ iterations form one continuous coherent accumulation without intermediate readouts. The small rotation per step is irremovable: it *is* the algorithm. Comparability with the floor (24) arises when $2/\sqrt{N} \sim \sigma$, i.e. at $N \sim (2/\sigma)^2 \sim 2 \times 10^{37} \approx 2^{124}$: there the signal step is $2^{-61} \approx 4 \times 10^{-19}$ rad. At smaller N (e.g. $N = 2^{80}$, step $\sim 10^{-12}$ rad) the rotation is orders of magnitude above the floor and the mechanism is imperceptible on a single iteration; at larger N the step falls below the floor, and dephasing accumulated over $\sim \sqrt{N}$ iterations becomes critical—although in practice the computation time kills it first. This is consistent with the known structural fragility of the algorithm: with constant noise speed on the oracle, Grover’s quadratic speedup is destroyed completely [35].

6 Discussion

6.1 Temporal structure of proper-time noise

Postulates P2–P3 specify only the single-interval variance δt ; multi-time covariance, the noise spectrum, and the spin-echo response are not fixed by them. The law $\text{Var}[\delta t(t)] \propto t^{2/3}$ follows from (3), but does *not* by itself determine the full stochastic process. If Gaussianity of the full process, self-similarity, and stationary increments are additionally imposed, the natural completion is fractional Brownian motion with Hurst exponent $H = 1/3$ [36]. In that completion, disjoint increments are anticorrelated, the low-frequency spectrum of fractional Gaussian noise behaves as $S(f) \propto |f|^{1/3}$, and echo requires an explicit calculation with the corresponding filter function rather than an automatic cancellation of a quasistatic phase. These spectral properties follow from an additional temporal postulate, not from P2–P3 as presently stated.

6.2 Limitations and model systematics

The central prediction $\chi \propto m^2 t^{2/3}$ follows from the postulates, but the overall coefficient remains incompletely fixed. The number ξ in (3) is specified only as $\mathcal{O}(1)$; interpreting δt as a single-interval variance and choosing the effective superposition time t (e.g. L/v versus $2L/v$ in a Talbot–Lau interferometer; see § 4.3) enter the $\mathcal{O}(1)$ uncertainty of the predicted visibility on equal footing with ξ . When confronting a single mass, the observed visibility decomposes as $V_{\text{obs}} = R_{\text{ord}} V_{\text{QM}} e^{-\chi}$; the product $R_{\text{ord}} e^{-\chi}$ is degenerate, and without varying m or t one cannot uniquely separate the BDPT contribution from underestimated conventional contrast losses. The conditional bound at fixed $R_{\text{ord}} = 0.78$ is therefore weaker than the strict one-sided estimate that attributes the entire deficit to the mechanism. A statistically robust limit on ξ will require a joint likelihood analysis of the raw scans, the quantum-curve parameters, and all conventional loss channels, not aggregated visibility values alone.

6.3 Relation to objective collapse

A natural question is how BDPT relates to objective collapse if it extinguishes the relative phase between branches. Two levels must be kept apart. For *fixed* realizations $\delta t_a, \delta t_b$ the state

$$|\psi\rangle = c_a|a\rangle + c_b e^{i\Delta\varphi}|b\rangle \quad (25)$$

remains *pure*; $\Delta\varphi$ is given by (4) and depends on the particular realization of the branch clocks. A mixed state appears only after ensemble averaging over independent $\delta t_a, \delta t_b$:

$$\bar{\rho}_{ab} = \rho_{ab}(0) e^{-\chi}. \quad (26)$$

At $\chi \gg 1$ this yields the effective state

$$\rho \approx |c_a|^2 |a\rangle\langle a| + |c_b|^2 |b\rangle\langle b|, \quad (27)$$

without coherent terms $c_a c_b^* |a\rangle\langle b| + \text{h.c.}$ The loss of phase and passage to a mixed state is not reducible to the observer’s incomplete knowledge of the phase: by (P3) the noise is an *ontic* independent shift of proper time per branch, not the recording of which-path information in an environment.

In the ensemble reading (§ 1) each branch is a genuine variant of the system and $|c_i|^2$ are the variant weights. For $\chi \gg 1$ there is no observable interference between variants; the ensemble mixture with the same weights is operationally indistinguishable from a statistical description of outcomes with probabilities $|c_i|^2$. The definite outcome remains unknown until registration, but coherent superposition of alternatives as a physical resource disappears *objectively*—before and independently of readout—over a time $\lesssim t_{\text{dec}}$. In this sense the macroscopic limit of BDPT

yields objective loss of inter-branch interference and effective branch superselection at $\chi \gg 1$, but does *not* select a single realization and does not replace dynamical collapse.

The mechanism differs from dynamical collapse models (Diósi–Penrose and others) in its channel: the populations $|c_i|^2$ are not redistributed by a jump and there is no spontaneous emission [19]; it is pure rest-phase dephasing. The destruction of inter-branch coherence and the passage to an ensemble mixture is the link that *resembles* objectivist programmes, but formally follows only from the ensemble ontology, independent branch clocks, and the P2 scale, without an additional nonlinear term in the state equation. Schrödinger’s cat at $m \gg m_*$ illustrates the limit: the amplitudinal record “alive/dead” remains, but interference between branches vanishes over $t_{\text{dec}} \ll t_C$; what remains is a statistical mixture of variants with weights $|c_i|^2$, not the selection of a single outcome before measurement.

6.4 Distinction from other decoherence mechanisms

The mechanism must be distinguished from nearby sources of interference loss. In quantum-clock interference, a deterministic or potential-induced proper-time difference records which-path information through internal degrees of freedom or an external gravitational gradient [15, 16]; BDPT is not reducible to such unitary readout and does not require a potential difference, but introduces a stochastic Planckian scale with independent $\delta t_a, \delta t_b$. An environment with momentum transfer (gas, thermal radiation, photon scattering) creates position-dependent noise, force, and momentum diffusion and is usually suppressed by improving the vacuum or cooling; the characteristic dependence on Δx is described by localization factors [26] (see § 4.5). Dynamical Diósi–Penrose collapse models predict population redistribution and spontaneous emission [19], to which the present model is *transparent*: it is pure rest-phase dephasing without additional channels. Spatially attached implementations of Planckian uncertainty (correlated shear noise, external metric fluctuations) are constrained by devices such as the Holometer and LIGO-like detectors [22]; BDPT attaches to discrete branches, not to points, and does not act on a massless carrier. Finally, in standard quantum mechanics the Compton phase cancels in a closed interferometer [17]; the present model does not settle that debate but postulates that, with independent branch clocks, the rest phase mc^2 becomes a random *relative* contribution (see § 3)—a separate and falsifiable premise.

7 Conclusion

Tying the Károlyházy fluctuation to proper time and postulating independent branch clocks yields a decoherence law (for which-path, $\xi = \mathcal{O}(1)$)

$$\boxed{V/V_0 = \exp \left[- \left(\frac{mc^2}{\hbar} \right)^2 (\xi t_P^2 t)^{2/3} \right]}, \quad (28)$$

where interference involves the *difference* of branch phase contributions and mc^2 is the energy of the split carrier. The scaling $m^2 t^{2/3}$ is immune to electromagnetic shielding; sodium interferometry [1] gives a strict one-sided 95% bound $\xi \lesssim 1.1$ (the entire contrast deficit attributed to the mechanism; conditionally, with the conventional-loss normalization 0.78, $\xi \lesssim 0.31$). For $\xi = \mathcal{O}(1)$ the characteristic scale $m_* = m_P/\sqrt{\xi}$ is of the order of the Planck mass; for $m \gg m_*$ decoherence occurs over $t_{\text{dec}} \ll t_C$, below the Károlyházy distinguishability limit for proper time. The model is falsifiable by a test at ~ 1 MDa and $v \approx 25$ m/s: either fringes of the standard quantum model, or suppression with the $m^2 t^{2/3}$ scaling.

Declarations

Funding. The author received no specific grant from any funding agency.

Data availability. Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

Competing interests. The author declares no competing interests.

Author contributions. OP conceived the model, performed the calculations, and wrote the manuscript.

References

- [1] S. Pedalino, B. E. Ramírez-Galindo, R. Ferstl, K. Hornberger, M. Arndt, and S. Gerlich. Probing quantum mechanics with nanoparticle matter-wave interferometry. *Nature*, 649:866–870, 2026.
- [2] L. E. Ballentine. The statistical interpretation of quantum mechanics. *Reviews of Modern Physics*, 42(4):358–381, 1970.
- [3] D. Home and M. A. B. Whitaker. Ensemble interpretations of quantum mechanics. A modern perspective. *Physics Reports*, 210(4):223–317, 1992.
- [4] D. N. Page and W. K. Wootters. Evolution without evolution: Dynamics described by stationary observables. *Physical Review D*, 27:2885–2892, 1983.
- [5] R. Penrose. On gravity’s role in quantum state reduction. *General Relativity and Gravitation*, 28(5):581–600, 1996.
- [6] J. I. Korsbakken, K. B. Whaley, J. DuBois, and J. I. Cirac. Measurement-based measure of the size of macroscopic quantum superpositions. *Physical Review A*, 75:042106, 2007.
- [7] F. Károlyházy. Gravitation and quantum mechanics of macroscopic objects. *Il Nuovo Cimento A*, 42:390–402, 1966.
- [8] H. Salecker and E. P. Wigner. Quantum limitations of the measurement of space-time distances. *Physical Review*, 109:571–577, 1958.
- [9] Y. J. Ng and H. van Dam. Limitation to quantum measurements of space-time distances. *Annals of the New York Academy of Sciences*, 755:579–584, 1995.
- [10] Y. J. Ng and H. van Dam. Limit to space-time measurement. *Modern Physics Letters A*, 9:335–340, 1994.
- [11] Y. J. Ng. Spacetime foam, holographic principle, and black hole quantum computers. *International Journal of Modern Physics A*, 20:1328–1335, 2005.
- [12] Paolo M. Bonifacio, Charles H.-T. Wang, J. Tito Mendonça, and Robert Bingham. De-phasing of a non-relativistic quantum particle due to a conformally fluctuating spacetime. *Classical and Quantum Gravity*, 26:145013, 2009.
- [13] Heinz-Peter Breuer, Ertan Göklü, and Claus Lämmerzahl. Metric fluctuations and decoherence. *Classical and Quantum Gravity*, 26:105012, 2009.
- [14] F. Kiałka, Y. Y. Fein, S. Pedalino, S. Gerlich, and M. Arndt. A roadmap for universal high-mass matter-wave interferometry. *AVS Quantum Science*, 4:020502, 2022.
- [15] M. Zych, F. Costa, I. Pikovski, and Č. Brukner. Quantum interferometric visibility as a witness of general relativistic proper time. *Nature Communications*, 2:505, 2011.

- [16] I. Pikovski, M. Zych, F. Costa, and Č. Brukner. Universal decoherence due to gravitational time dilation. *Nature Physics*, 11:668–672, 2015.
- [17] P. Wolf, L. Blanchet, C. J. Bordé, S. Reynaud, C. Salomon, and C. Cohen-Tannoudji. Does an atom interferometer test the gravitational redshift at the Compton frequency? *Classical and Quantum Gravity*, 28:145017, 2011.
- [18] S.-Y. Lan, P.-C. Kuan, B. Estey, D. English, J. M. Brown, M. A. Hohensee, and H. Müller. A clock directly linking time to a particle’s mass. *Science*, 339(6119):554–557, 2013.
- [19] S. Donadi et al. Underground test of gravity-related wave-function collapse. *Nature Physics*, 17:74–78, 2021.
- [20] I. J. Arnquist et al. Search for spontaneous radiation from wave-function collapse in the Majorana Demonstrator. *Physical Review Letters*, 129:080401, 2022.
- [21] K. Piscicchia et al. X-ray emission from atomic systems can distinguish between prevailing dynamical wave-function collapse models. *Physical Review Letters*, 132:250203, 2024.
- [22] A. S. Chou et al. Interferometric constraints on quantum geometrical shear noise correlations. *Classical and Quantum Gravity*, 34:165005, 2017.
- [23] S. Pedalino, B. E. Ramírez-Galindo, R. Ferstl, K. Hornberger, M. Arndt, and S. Gerlich. Probing quantum mechanics using nanoparticle Schrödinger cats, 2025.
- [24] S. Nimmrichter and K. Hornberger. Macroscopicity of mechanical quantum superposition states. *Physical Review Letters*, 110:160403, 2013.
- [25] Sebastian Pedalino, Bruno Eduardo Ramírez Galindo, Richard Ferstl, Klaus Hornberger, Markus Arndt, and Stefan Gerlich. Probing quantum mechanics with nanoparticle matter-wave interferometry - dataset and code, 2025.
- [26] K. Hornberger, J. E. Sipe, and M. Arndt. Theory of decoherence in a matter wave Talbot–Lau interferometer. *Physical Review A*, 70:053608, 2004.
- [27] J. I. Korsbakken, F. K. Wilhelm, and K. B. Whaley. Electronic structure of superposition states in flux qubits. *Physica Scripta*, T137:014022, 2009.
- [28] J. I. Korsbakken, F. K. Wilhelm, and K. B. Whaley. The size of macroscopic superposition states in flux qubits. *EPL*, 89:30003, 2010.
- [29] Y.-A. Chen, X.-H. Bao, Z.-S. Yuan, S. Chen, B. Zhao, and J.-W. Pan. Heralded generation of an atomic NOON state. *Physical Review Letters*, 104:043601, 2010.
- [30] N. Margolus and L. B. Levitin. The maximum speed of dynamical evolution. *Physica D*, 120:188–195, 1998.
- [31] O. Milul, B. Guttel, U. Goldblatt, S. Hazanov, L. M. Joshi, D. Chausovsky, N. Kahn, E. Çiftyürek, F. Lafont, and S. Rosenblum. Superconducting cavity qubit with tens of milliseconds single-photon coherence time. *PRX Quantum*, 4:030336, 2023.
- [32] D. Coppersmith. An approximate Fourier transform useful in quantum factoring, 1994. IBM Research Report RC 19642.
- [33] P. W. Shor. Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. *SIAM Journal on Computing*, 26(5):1484–1509, 1997.

- [34] L. K. Grover. A fast quantum mechanical algorithm for database search. In *Proceedings of the 28th Annual ACM Symposium on Theory of Computing (STOC)*, pages 212–219, 1996.
- [35] O. Regev and L. Schiff. Impossibility of a quantum speed-up with a faulty oracle. In *Automata, Languages and Programming (ICALP 2008), Lecture Notes in Computer Science*, volume 5125, pages 773–781, 2008.
- [36] B. B. Mandelbrot and J. W. Van Ness. Fractional Brownian motions, fractional noises and applications. *SIAM Review*, 10(4):422–437, 1968.