 "The simplest solution is usually the best solution"---Albert Einstein

To Options

Proving & Teaching Collatz Conjecture

A. A. Frempong

Abstract

Collatz conjecture states that beginning with a positive integer, if one repeatedly performs the following operations to form a sequence of integers, the sequence will eventually reach the integer 1; the operations being that if the integer is even, divide it by 2, but if the integer is odd, multiply it by 3 and add 1; and also, use the result of each step as the input for the next step.
To prove Collatz conjecture, one would apply a systematic observation of the sequences produced by the Collatz process, the $(3n+1)\frac{1}{2}$ process, and note especially, the patterns of the sequence terms as the process reaches the equivalent powers, 2^{2k} $(k = 2,\ 3,...)$ and the sequences reach the integer 1, by repeated division by 2, i.e., $2^{2k-1},\ 2^{2k-2},\ 2^{2k-3},\ \ ,...,2^{2k-2k}$. Two main cases are covered. In Case 1, the integer can be written as a power of 2 as 2^k $(\mathrm{k} = 1,2,3,...)$. In this case, the sequence will reach the integer 1 by repeated division by 2, i.e., $2^{k-1},2^{k-2},2^{k-3},...2^{k-k}$.
In Case 2, the integer cannot be written as a power of 2, but the sequence terms of the integer reach the equivalent power, 2^{2k} $(k = 2,\ 3,...)$, and by repeated division by 2, the sequence will reach the integer 1, i.e., $2^{2k-1},2^{2k-2},2^{2k-3},,...,2^{2k-2k}$. Thus, if an integer is of the form, 2^k $(\mathrm{k} = 1,2,3,...)$ or the sequence reaches 2^{2k} $(k = 2,\ 3,...)$, the sequence will definitely continue to 1. In Case 2, when the sequence terms reach some particular integers such as 5, 21 and 85, the application of 3n +1 to these integers will result in the powers, 2^{2k} $(k = 2,\ 3,...)$. One would call these integers, the 2k-power converters. There are infinitely many power converters as there are 2^{2k} powers. If the sequence reaches a 2^{2k} $(k = 2,\ 3,...)$ power, the terms will continue as $2^{2k-1},\ 2^{2k-2},\ 2^{2k-3},...,.$ $,...,2^{2k-2k}$. A term of the sequence must be converted to an integer equivalent to 2^{2k} $(k = 2,\ 3,...)$. There are infinitely many paths for converting integers to 2^{2k} $(k = 2,\ 3,...)$ powers. Of these conversion paths, the integer 5-path, is the nearest 2^{2k} $(k = 2)$ converter path to the integer 1 on the 2^{2k}–route. For the 5-path, when a sequence terms reach the integer, 5, the next term would be $3(5)+1 = 16$ or 2^4. Other integers can follow the integer 5-path to the 2^4 power as follows: Let n be an integer whose sequence terms would reach 16 or 2^4 in the $(3n+1)\frac{1}{2}$ process, and let $n \pm r = 5$, where r is the net change in the sequence terms before the integer 5; and one uses the positive sign if $n < 5$, but the negative sign if $n > 5$. One will call the following, the 5-path 2k-converter formula: $\boxed{3(n \pm r)+1 = 16 \text{ or } 2^4}$. By the substitution axiom, using this formula, the sequence of every positive integer that cannot be written as a power of 2, would reach the integer, 16, as in Case 2; and once the sequence reaches 16, by repeated division by 2, the sequence would reach the integer 1. ((16,8,4,2,1). In Case 1**,** the integer can be written as a power of 2 as 2^k $(\mathrm{k} = 1,2,3,...)$; and the sequence will reach the integer 1, by repeated division by 2, i.e., $2^{k-1},2^{k-2},2^{k-3},...2^{k-k}$. Therefore, the sequence of every positive integer would reach the integer 1. It is worth mentioning that the integers, 1 quadril1ion, 1 trillion, 1 billion, 1 million, and the integer, 123,456,789,012,345, all, use the 5-path to reach the 2k-power forms.

 "The simplest solution is usually the best solution"---Albert Einstein

To First Page

Options

 "The simplest solution is usually the best solution"---Albert Einstein

Back to Options

Option 1
Preliminaries

The following is from the first page of the author's high school practical physics note book: Science is the systematic observation of what happens in nature and the building up of body of laws and theories to describe the natural world. Scientific knowledge is being extended and applied to everyday life. The basis of this growing knowledge is experimental work. To prove Collatz conjecture, one would be guided by a systematic observation of the sequences produced by the Collatz process, $(3n+1)\frac{1}{2}$ process, One will observe that there are two main cases of the patterns of sequence terms in the columns, In Case 1, the positive integer whose sequence would reach the integer 1, can be written as a power of 2. In Case 2, the positive integer whose sequence would reach the integer 1, **cannot** be written as a power of 2.

In **Case 1,** one can make the following observations: In **Table 1**, one observes that if the first element in a column is 2,4,8, or 16, the integer can be written as a power of 2, and by repeated division by 2 the sequence reaches the integer 1, Similarly, in **Table 2** , one observes the integer, 32, can be written as a power of 2 and by repeated division by 2 the sequence reaches the integer 1, In **Case 2**, one observes the first elements in the columns are 3,5, 6, 7, 9,10, 11, 12, 13, 14,15, 17, 18, 19, 20., Each of these integers cannot be written as a power 2. One can observe that in each sequence, the term after the integer 5, is 16 which is a power of 2,. $(16, 8, 4, 2, 1 \text{ or } 2^4, 2^3, 2^2, 2^1, 2^0)$, One can also note that except for 5, the values of the terms increase or decrease before the integer 16., but after the integer 16, the values only decrease, and continue to reach the integer 1.

Tables of Sequences of Positive Integers
Table 1

1	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
2	4	1	10	2	16	3	22	4	28	5	34	6	40	7	46	8	52	9	58	10
3	2		5	1	8	10	11	2	14	16	17	3	20	22	23	4	26	28	29	5
4	1		16		4	5	34	1	7	8	52	10	10	11	70	2	13	14	88	16
5			8		2	16	17		22	4	26	5	5	34	35	1	40	7	44	8
6			4		1	8	52		11	2	13	16	16	17	106		20	22	22	4
7			2			4	26		34	1	40	8	8	52	53		10	11	11	2
8			1			2	13		17		20	4	4	26	160		5	34	34	1
9						1	40		52		10	2	2	13	80		16	17	17	
10							20		26		5	1	1	40	40		8	52	52	
11							10		13		16			20	20		4	26	26	
12							5		40		8			10	10		2	13	13	
13							16		20		4			5	5		1	40	40	
14							8		10		2			16	16			20	20	
15							4		5		1			8	8			10	10	
16							2		16					4	4			5	5	
17							1		8					2	2			16	16	
18									4					1	1			8	8	
19									2									4	4	
20									1									2	2	
21																		1	1	

Note in Table 1 above: 1. If the first element cannot be written as a power 2, the sequence terms reach the integer, 5 whose next term is 16, a power of 2.

Back to Options

Table 2

1	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
2	64	11	70	12	76	13	82	14	88	15	94	16	100	17	106	18	112	19	118	20
3	32	34	35	6	38	40	41	7	44	46	47	8	50	52	53	9	56	58	59	10
4	16	17	106	3	19	20	124	22	22	23	142	4	25	26	160	28	28	29	178	5
5	8	52	53	10	58	10	62	11	11	70	71	2	76	13	80	14	14	88	89	16
6	4	26	160	5	29	5	31	34	34	35	↓	1	38	40	40	7	7	44	268	8
7	2	13	80	16	88	16	94	17	17	106			19	20	20	22	22	22	134	4
8	1	40	40	8	44	8	47	52	52	53			58	10	10	11	11	11	67	2
9		20	20	4	22	4	142	26	26	160			29	5	5	34	34	34	202	1
10		10	10	2	11	2	71	13	13	80			88	16	16	17	17	17	101	
11		5	5	1	34	1	214	40	40	40			44	8	8	52	52	52	304	
12		16	16		17		107	20	20	20			22	4	4	26	26	26	152	
13		8	8		52		322	10	10	10			11	2	2	13	13	13	76	
14		4	4		26		161	5	5	5			34	1	1	40	40	40	38	
15		2	2		13		484	16	16	16			17			20	20	20	19	
16		1	1		40		242	8	8	8			52			10	10	10	58	
17					20		121	4	4	4			26			5	5	5	29	
18					10		364	2	2	2			13			16	16	16	88	
19					5		182	1	1	1			40			8	8	8	44	
20					16		91						20			4	4	4	22	
21					8		274						10			2	2	2	11	
22					4		137						5			1	1	1	34	
23					2		⇓						⇓						⇓	
24					1															

Note in Table 2 above:

1. The integer, 21 in the first column is as special as the integer 5 because it is followed by 64, which is a power of 2
2. The integer, 32, can be written as a power of 2, and its sequence will reach the integer 1 by repeated division by 2
3. The first element in the other columns cannot be written as a power 2, With the exception of 21, the sequence terms reach the integer, 5 whose next term is 16, a power of 2.

Back to Options

Table 3

1	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
2		21		22	136	23		24	148		154	26	160			28		29		30
3		64		11		70		12	74		77	13	80			14		88		15
4		32		34		35		6	37		232	40	40			7		44		46
5		16		17		106		3	112		116	20	20			22		22		23
6		8		52		53		10	56		58	10	10			11		11		70
7		4		26		160		5	28		29	5	5			34		34		35
8		2		13		80		16	14		88	16	16			17		17		106
9		1		40		40		8	7		44	8	8			52		52		53
10				20		20		4	22		22	4	4			26		26		160
11				10		10		2	11		11	2	2			13		13		80
12				5		5		1	34		34	1	1			40		40		40
13				16		16			17		17					20		20		20
14				8		8			52		52					10		10		10
15				4		4			26		26					5		5		5
16				2		2			13		13					16		16		16
17				1		1			40		40					8		8		8
18									20		20					4		4		4
19									10		10					2		2		2
20									5		5					1		1		1
21									16		16									
22									8		8									
23									4		4									
24									2		2									
25									1		1									
26																				

Note in Table 3 above:
1. The integer, 42 in the second column is followed by 21 whose next term is 64 as in Table 2
2. The first element in each column cannot be written as a power 2, With the exception of 42, the sequence terms reach the integer, 5 whose next term is 16, a power of 2.

We will learn in the future that the integers 5 and 21 are special in the sense that $3n+1$ is a power of 2, since if $n=5$, $3n+1=3(5)+1=16$, a power of 2; For $n=21$, $3n+1=3(21)+1=64$, a power of 2. From the observations in the above tables, one would like the sequence of an integer which cannot be written as a power of 2 to reach the integer 5.

Summary of Observations from the above Tables

Two cases are covered.

Case 1: If the integer can be written as a power of 2, 2^k $(k=1,2,3,...)$. the sequence will reach the integer, 1 by repeated division by 2, i.e., $2^k, 2^{k-1}, 2^{k-2}, 2^{k-3}, ... 2^{k-k}$.

Example 1.: $8=2^3$, and $2^3, 2^2, 2^1, 2^0$ or 8.4,2,1.

Example 2: $16=2^4$, and $2^4, 2^3, 2^2, 2^1$, 2^0 or 16,8.4,2,1.

Example3: $32=2^5$, and $2^5, 2^4, 2^3, 2^2, 2^1, 2^0$, or 32,16,8,4,2,1.

Back to Options

Back to Options

Case 2 The integer cannot be written as a power of 2, but the sequence terms of this integer reach the equivalent power, 2^{2k} $(k = 2,\ 3,...)$ which will continue as $2^{2k-1}, 2^{2k-2}, 2^{2k-3},..., 2^{2k-2k}$.

Net Change

Given a sequence of positive integers, one can determine the positive and negative changes from term to term. One can also determine the net change from one integer to another integer in a set of the integers. One would apply the net change in writing formulas to convert a positive integer to a power of 2.

Let n be an integer whose sequence would reach 16 or 2^4 in the $(3n+1)\frac{1}{2}$ process and let $n \pm r = 5$, where r is the net change in the sequence terms before the integer 5; and one uses the positive sign if $n < 5$ but the negative sign if $n > 5$.

Example 1: Let $n = 12$ with the sequence terms $12, 6, 3, 10, 5$. From 12 to 6, the change is -6; from 6 to 3, the change is -3; from 3 to 10, the change is +7; and from 10 to 5, the change is -5, The net change, r= -6 -3 + 7 - 5 = -7; and $n \pm r = 12 - 7 = 5$. Note: The negative sign was used because $n > 5$ $(12 > 5)$. Note: Since $\boxed{3(5-0)+1=16}$, $\boxed{3(12-7)+1=16}$, Also, $\boxed{3(3+2)+1=16}$, Suppose, one wants the sequence for the integer, 27 to reach 16. Then $\boxed{3(27-22)+1=16}$, since .$\boxed{27-22=5-0}$. One confirms the "-22" in $\boxed{3(27-22)+1=16}$, below, where one will determine the net change, beginning with the integer 27 and applying the collatz .$(3n+1)\frac{1}{2}$ process to reach the integer 5.

Example 2 This example shows the sequence of the positive integer 27.
In the table below, the Collatz sequence for the integer 27 was obtained by applying the operation, $f(n) = \begin{cases} \frac{n}{2} \text{ if } n \text{ is even} \\ 3n+1 \text{ if } n \text{ is odd} \end{cases}$, where in this example $n = 27$.

Table A: Collatz sequence for the integer 27

27	47	484	137	233	395	668	1132	319	3238	911	9232	433	122	35	10
82	142	242	412	700	1186	334	566	958	1619	2734	4616	1300	61	106	5
41	71	121	206	350	593	167	283	479	4858	1367	2308	650	184	53	16
124	214	364	103	175	1780	502	850	1438	2429	4102	1154	325	92	160	8
62	107	182	310	526	890	251	425	719	7288	2051	577	976	46	80	4
31	322	91	155	263	445	754	1276	2158	3644	6154	1732	488	23	40	2
94	161	274	466	790	1336	377	638	1079	1822	3077	866	244	70	20	1

Note that there are 111 steps to reach the integer, 1, by applying the above operation.
Table B below shows how by addition and subtraction, the sequence reaches the integer, 5

Back to Options

Example 3: Confirmation of the net change for Collatz sequence for the integer 27, from 27 to 5.

Read from top to bottom in the first column, and continue from top of the next column down and repeat the process for the other columns. Numbers with "+" signs are for the positive changes, and numbers with "-" signs are for the negative change. Example, in the first column, 27 + 55 = 82.

Table B:

27 +55	142 -71	121 +243	103 +207	526 -263	445 +891	377 +755	319 +639	1619 +3239	1367 +2735	1154 -577	976 -488	23 +47	20 -10
82 -41	71 +143	364 -182	310 -155	263 +527	1336 -668	1132 -566	958 -479	4858 -2429	4102 -2051	577 +1155	488 -244	70 -35	10 -5
41 +83	214 -107	182 -91	155 +311	790 -395	668 -334	566 -283	479 +959	2429 +4859	2051 +4103	1732 -866	244 -122	35 +71	5 +11
124 -62	107 +215	91 +183	466 -233	395 +791	334 -167	283 +567	1438 -719	7288 -3644	6154 -3077	866 -433	122 -61	106 -53	16
62 -31	322 -161	274 -137	233 +467	1186 -593	167 +335	850 -425	719 +1439	3644 -1822	3077 +6155	433 +867	61 +123	53 +107	
31 +63	161 +323	137 +275	700 -350	593 +1187	502 -251	425 +851	2158 -1079	1822 -911	9232 -4616	1300 -650	184 -92	160 -80	
94 -47	584 -242	412 -206	350 -175	1780 -890	251 +503	1276 -638	1079 +2159	911 +1823	4616 -2308	650 -325	92 -46	80 -40	
47 +95	242 -121	206 -103	175 +351	890 -445	754 -377	638 -319	3238 -1619	2734 -1367	2308 -1154	325 +651	46 -23	40 -20	

Sum of the pluses, "+" = 40,552.; **Sum of the minuses**, "-" = -40,574.

Sum of the pluses and the minuses = $40{,}552 - 40574 = \mathbf{-22}$

The net change, $r = -22$, and $n - r = 27 - 22 = 5$

Example 4: Confirmation of the net change before 5 for Collatz sequence for the integer 33

Read from top to bottom in the first column, and continue from top of the next column down and repeat the process for the other columns. Numbers with "+" signs are for the positive changes, and numbers with "-" signs are for the negative changes.. .

33,100, 50, 25,76,38,19,58,29, 88, 44, 22,11, 34, 17, 52, 26, 13, 40, 20,10, 5

33 +67	50 -25	76 -38	19 +39	29 +59	44 -22	11 +23	17 +35	26 -13	40 -20	10 -5
100 -50	25 +51	38 --19	58 -29	88 -44	22 -11	34 -17	52 -26	13 +27	20 -10	5

Sum of the pluses, "+" = 301.; **Sum of the minuses**, "-" = -329.

Sum of the pluses and the minuses = -28

The net change, $r = -28$, and $n - r = 33 - 28 = 5$

Back to Options

Examples 3 and 4 show that one can write the **net change** without the tedious addition of pluses and minuses in the above tables. For example, for the integer, 33, $n - r = 33 - 28 = 5$.
For the integer, 45, $n - r = 45 - 40 = 5$ One will call the following, the 5-path 2k-converter formula: $\boxed{3(n \pm r) + 1 = 16}$. Using this formula, the sequence of every positive integer which cannot be written as a power of 2, can reach the integer, 16. Once the sequence reaches 16, applying repeated division by 2, the sequence will reach the integer 1. (16,8,4,2,1).
It is worth mentioning that the integers, 1 quadril1ion, 1 trillion, 1 billion, 1 million, and the integer, 123,456,789,012,345, all, use the 5-path to reach the 2k-power forms.

Back to Options

Option 2
Introduction

Collatz conjecture states that beginning with a positive integer, if one repeatedly performs the following operations to form a sequence of integers, the sequence will eventually reach the integer 1; the operations being that if the integer is even, divide it by 2, but if the integer is odd, multiply it by 3 and add 1; and also, use the result of each step as the input for the next step.
To prove Collatz conjecture, one would be guided by a systematic observation of the sequences produced by the Collatz process, $(3n+1)\frac{1}{2}$ process, and note the pattern of the sequence terms as the process reaches the integers, 2^{2k} $(k = 2,\ 3,...)$ and continue straightforwardly as $2^{2k-1},\ 2^{2k-2},\ 2^{2k-3},...,2^{2k-2k}$, reaching the integer 1. For example, 2^4 continues as $2^4, 2^3, 2^2, 2^1$, to 2^0 (16, 8, 4, 2. to 1). Thus, if an integer is equivalent to the form 2^k $(k = 1.2.3,...,)$ or the sequence of positive integers reaches the form 2^{2k} $(2,3,4,...,)$, the sequence will definitely continue to 1. In the numerical sequences of positive integers, in the preliminaries (Option 1: Tables 1, 2, and 3), the following observations were made:

Case 1: If the integer can be written as a power of 2, 2^k $(\text{k} = 1,2,3,...)$, the sequence will reach the integer 1 by repeated division by 2, i.e., $2^k, 2^{k-1}, 2^{k-2}, 2^{k-3},...2^{k-k}$.
Example 1.: $8 = 2^3$, and $2^3, 2^2, 2^1, 2^0$ or 8.4,2,1.
Example 2: $16 = 2^4$, and $2^4, 2^3, 2^2, 2^1,\ 2^0$ or 16,8.4,2,1.
Example 3: $256 = 2^8$, and $2^8, 2^7, 2^6, 2^5, 2^4, 2^3, 2^2, 2^1, 2^0$ or 256,128,64,32,16,8,4,2,1.

Case 2 The integer cannot be written as a power of 2, but the sequence terms of this integer reach the equivalent power, 2^{2k} $(k = 2,\ 3,...)$ which will continue as $2^{2k-1}, 2^{2k-2}, 2^{2k-3},...,2^{2k-2k}$. In particular, if the sequence reaches the integer, 16, the previous odd integer would be the integer 5 . This observation was from the sequences of the first 100 positive integers, except for about five integers and the equivalent 2^k $(\text{k} = 1,2,3,...)$-power forms such as 8, 16, and 64. Thus the integer, 5, would be the guiding integer for converting other integers to 2k-power forms, since $3(5) + 1 = 16$. Note that $16 = 2^4$, a 2^{2k} form $(k = 2)$
In Case 2, two main requirements must be satisfied in order for a sequence of positive integers to reach the integer 1. The first requirement is that when the sequence terms reach some particular integers such as 5, 21 and 85, the application of $3n+1$ to these integers will result in the powers. 2^{2k} $(k = 2,\ 3,...)$. One would call these integers, the 2k-power converters. Thus an integer, n,would be called a 2k-power converter if $3n + 1 = 2^{2k}$ $(k = 2,\ 3,...)$. Examples of the converters are 5, 21, and 85 with the respective 2^{2k}- powers, $2^4, 2^6$, and 2^8. There are infinitely many power

Back to Options

converters as there are 2^{2k} powers. To find more power converters, let $3n+1=2^{2k}$.. If $k=5,\ 3n+1=2^{10}$., $3n=2^{10}-1,\ 3n=1024-1$, and $n=341$. Satisfying the first requirement would be straightforward since, if the sequence reaches a 2^{2k} $(k=2,\ 3,...)$ power, the terms will continue as $2^{2k},\ 2^{2k-1},\ 2^{2k-2},\ 2^{2k-3},...,2^{2k-2k}$. For example: 2^4 (k = 2) will continue as 16, 8, 4, 2. to 1. The second requirement is that a term of the sequence of every positive integer must be converted to 2^{2k} $(k=2,\ 3,...)$. Satisfaction of the second requirement involves two main paths, namely, the integer 5-path, and the other infinitely, many 2^{2k} $(k=3,4,...)$-paths. For the integer 5-path, when a sequence terms reach the integer, 5, the next term would be $3(5)+1=16=2^4$. Similarly, for the integer, 21, the next term would be $3(21)+1=64=2^6$. Any integer which is $(21)2^p$ $(p=1,2,3,...)$ can quickly be converted to $64=2^6$. By the substitution axiom, the sequence of every positive integer that cannot be written as a power of 2 can follow the integer 5-path to the 2^4 power as follows: Let n be an integer whose sequence would reach $16=2^4$ in the $(3n+1)\frac{1}{2}$ process, and let $n\pm r=5$, where r is the net change in the sequence terms before the integer 5; and one uses the positive sign if $n<5$, but the negative sign if $n>5$**.** One will call the following, the 5-path 2k-converter formula: $\boxed{3(n\pm r)+1=16 \text{ or } 2^4}$.
Using this formula, as in Case 2, the sequence of every positive integer which cannot be written as a power of 2, can reach the integer, 16. Once the sequence reaches 16, using repeated division by 2, the sequence will reach the integer 1. (16,8,4,2,1). In Case 1**,** the integer can be written as a power of 2 as 2^k $(\text{k}=1,2,3,...)$; and the sequence will reach the integer 1, by repeated division by 2, i.e., $2^{k-1},2^{k-2},2^{k-3},...2^{k-k}$.

In addition to the 2k-power converter, 5, there are other $2k$–power converters such as 21, 85 and 341 which can convert respectively, as follows: 1. $\boxed{3(21)+1=64=2^6}$, 2. $\boxed{3(85)+1=256=2^8}$, and **3**. $\boxed{3(341)+1=1024=2^{10}}$. There are infinitely many $2k$–power converters as there are 2^{2k} powers.
The formula for the ,infinitely, many 2k-converters is $n=\dfrac{2^{2k}-1}{3}$ $(k=2,3,4,...)$
Note: For integer, 5, k = 2; for integer, 21, k = 3; for integer, 85, k = 4; for integer, 341, k = 5,,

Back to Options

Using integer 5-path to convert any positive integer to the 2k-power form, 2^{2k} $(k = 2, 3, ...)$

For integer, 21, since 21 is odd, by applying $3n+1$, one obtains $3(21)+1 = 64 = 2^6$. This path is straightforward. Additionally, for the **integer, 21**, One can go from 21 to 2^4 by applying the 5-path, $n \pm r = 5$, where $n = 21$ and $r = 16$. Then, since $21 - 16 = 5$, $3(21-16)+1 = 16 = 2^4$ (see Figure).

For integer, 85, since 85 is odd, by applying $3n+1$, one obtains $3(85)+1 = 256$ or 2^8. This path is straightforward. However, one can go from 85 to 2^4 by applying the 5-path, $n \pm r = 5$, where $n = 85$ and $r = 80$. Then, $85 - 80 = 5$, and $3(85-80)+1 = 16 = 2^4$.(see Figure)

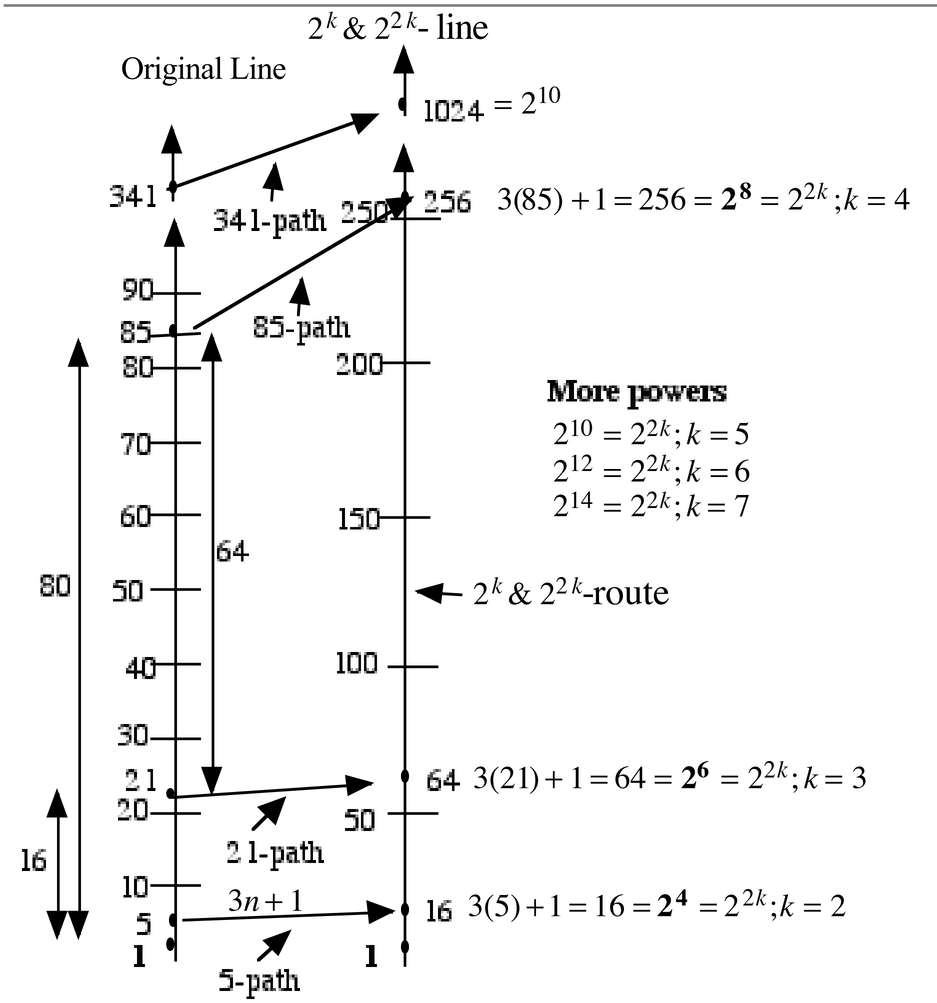
For integer, 341, since 341 is odd, by applying $3n+1$, one obtains $3(341)+1 = 1024$ or 2^{10}. This path is straightforward. However, one can go from 341 to 2^4 by applying the 5-path, $n \pm r = 5$, where $n = 341$ and $r = 336$. Then, $341 - 336 = 5$, and $3(341-336)+1 = 16 = 2^4$.(see Figure)

It is worth noting that the integers, 1 quadril1ion, 1 trillion, 1 billion, and 1 million, all, use the 5-path to convert to the 2k-power forms.

From 21 to 16	**From 85 to 16**	**From 341 to 16**
Step 1: From 21 to 64	Step 1: From 85 to 256	Step 1: From 341 to 1024
$3(21)+1 = 64$ -directly	$3(85)+1 = 256$ directly	$3(341)+1 = 1024$ directly
Step 2: From 64 to 16	Step 2: From 256 to 16	Step 2: From 1024 to 16
$\boxed{\text{net change} = -48}$ (A)	$\boxed{\text{net change} = -240}$ (A)	$\boxed{\text{net change} = -1008}$ (A)
Step 3: From 21 to 5 = - 16	Step 3: From 85 to 5 = - 80	Step 3: From 341 to 5 = - 336
from 5 to 16	from 5 to 16	from 5 to 16
$3(21-16)+1 = 16$	$3(85-80)+1 = 16$	$3(341-336)+1 = 16$
$\boxed{63 - 48 + 1 = 16}$ (B)	$\boxed{255 - 240 + 1 = 16}$ (B)	$\boxed{1023 - 1008 + 1 = 16}$ (B)

Question:
Can a 2k-power converter use the path of another 2k- converter to convert to 2k-power form as above for integers, 21, 85 and 341? .Yes.

Back to Options



Above, the vertical line on the left is the original line and the line on the right is the $2^k, 2^{2k}$ line

From 21 on the left to 16 on the right

Step 1: From 21 to 64

$3(21)+1 = 64$ directly

Step 2: From 64 to 16

$\boxed{\text{net change} = -48}$ (A)

From 21 on the left to 5 on the left, followed by to 16 on the right

Step 1: From 21 to 5 = - 16

Step 2 From 5 to 16

$3(21-16)+1 = 16$

$\boxed{63-48+1=16}$ (B)

From 85 on the left to 16 on the right

Step 1: From 85 to 256

$3(85)+1 = 256$ directly

Step 2: From 256 to 16

$\boxed{\text{net change} = -240}$ (A)

From 85 on the left to 5 on the left, followed by to 16 on the right

Step 1 From 85 to 5 = - 80

Step 2 From 5 to 16

$3(85-80)+1 = 16$

$\boxed{255-240+1=16}$ (B)

From 341 on the left to 16 on the right

Step 1: From 341 to 1024

$3(341)+1 = 1024$ directly

Step 2: From 1024 to 16

$\boxed{\text{net change} = -1008}$ (A)

From 341 on the left to 5 on the left, followed by to 16 on the right

Step 1: From 341 to 5 = - 336

Step 2 From 5 to 16

$3(341-336)+1 = 16$

$\boxed{1023-1008+1=16}$ (B)

Back to Options

Justification for using the 5th-path to convert every positive integer to a 2k-power

From the observations of hundreds of sequences of positive integers it was observed that most of the sequences used the integer 5-path to convert to a 2k-power form. The exceptions were either the other infinitely 2k-power converter such as the integers, 21, 85 and 341. and the descendants of these 2k-power converters. The 2k-power converters 21, 85, and 341 do not normally use the integer 5-path to convert to 2k-power forms.. The question the author was curious about was " can a 2k-power converter use the path of another 2k- converter to convert to a 2k-power form The answer is Yes as shown below. This verification would imply that every positive integer can use the integer 5-path to convert to a 2k-power form.

Direct path to 2^{2k} $(k=3)$ for the 2k-power converter 21

Since 21 is odd, by applying $3n+1$, one obtains $3(21)+1=64$ or 2^6. This path is straightforward. However, one can go from 21 to 2^4 by applying the 5-path, $n \pm r = 5$, where $n=21$ and $r=16$ Then, $21-16=5$, and $3(21-16)+1=16=2^4$(see Fig.).

It is worth noting that the integers, 1 quadrillion, 1 trillion, 1 billion, and 1 million, all, use the 5-path to convert to the 2k-power forms.

Direct path to 2^{2k} $(k=4)$ for the 2k-power converter 85

Since 85 is odd, by applying $3n+1$, one obtains $3(85)+1=256$ or 2^8. This path is straightforward. However, one can go from 85 to 2^4 by applying the 5-path, $n \pm r = 5$, where $n=85$ and $r=80$ Then, $85-80=5$, and $3(85-80)+1=16=2^4$.(see Fig.)

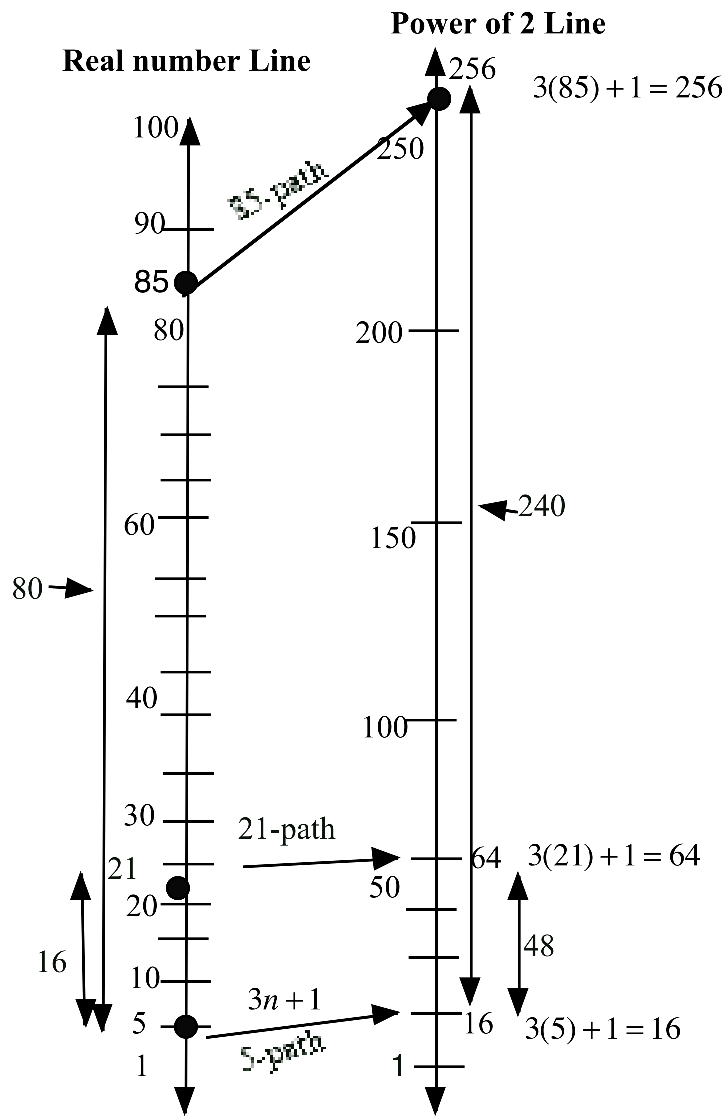
Direct path to 2^{2k} $(k=5)$ for the 2k-power converter 341

Since 341 is odd, by applying $3n+1$, one obtains $3(341)+1=1024$ or 2^{10}. This path is straightforward.

However, one can go from 341 to 2^4 by applying the 5-path, $n \pm r = 5$, where $n=341$ and $r=336$ Then, $341-336=5$, and $3(341-336)+1=16=2^4$.(see Fig.)

Back to Options

Real number and Power of 2 Lines



A. Clockwise from 21 on the real number line to 16 on the Power of 2 Line

Step 1: From 21 to 64

$3(21)+1=64$ directly

Step 2: From 64 to 16

$\boxed{\text{net change} = -48}$ (A)

Summary:

$63+1-48=16$

B. Counterclockwise, from 21 on the real number Line to 5 on the real number Line, followed by to 16 on the Power of 2 Line

Step 1: From 21 to 5 = - 16

Step 2 From 5 to 16

$3(21-16)+1=16$

$\boxed{63-48+1=16}$ (B

A. Clockwise From 85 on the real number line to 16 on the Power of 2 line

Step 1: From 85 to 256

$3(85)+1=256$ directly

Step 2: From 256 to 16

$\boxed{\text{net change} = -240}$ (A)

B. Counterclockwise, from 85 on the real number line to 5 on the real number line, followed by to16 on the Power of 2 line

Step 1 From 85 to 5 = - 80

Step 2 From 5 to 16

$3(85-80)+1=16$

$\boxed{255-240+1=16}$ (B)

A. Clockwise, from 341 on the real number line to 16 on the Power of 2 line

Step 1: From 341 to 1024

$3(341)+1=1024$ directly

Step 2: From 1024 to 16

$\boxed{\text{net change} = -1008}$ (A)

B. Counterclockwise, from 341 on the real number line to 5 on the real number line , followed by to 16 on the Power of 2 line

Step 1: From 341 to 5 = - 336

Step 2 From 5 to 16

$3(341-336)+1=16$

$\boxed{1023-1008+1=16}$ (B)

Back to Options

Converting a Sequence term of the integers, 21, 85, and 341 to the integer 16

A. From 21 on the Real number Line to 16 on the Power of 2 Line

Step 1: From 21 to 64

$3(21)+1=64$ directly

Step 2: From 64 to 16

$\boxed{\text{net change}=-48}$ (A)

Summary:

$63+1-48=16$

B. From 21 on the Real number Line to 5 on the Real number Line , followed by to 16 on the Power of 2 Line

Step 1: From 21 to 5 = - 16

Step 2 From 5 to 16

$3(21-16)+1=16$

$\boxed{63-48+1=16}$ (B)

C.

The expression in A =

$3(21)+1-48=16$

$63+1-48=16$

In B

$3(21)+1=64$

$63+1-48=16$

A. From 85 on the Real number Line to 16 on the Power of 2 Line

Step 1: From 85 to 256

$3(85)+1=256$ directly

Step 2: From 256 to 16

$\boxed{\text{net change}=-240}$ (A)

B. From 85 on the Real number Line to 5 on the Real number Line , followed by to 16 on the Power of 2 Line

Step 1 From 85 to 5 = - 80

Step 2 From 5 to 16

$3(85-80)+1=16$

$\boxed{255-240+1=16}$ (B)

A. From 341 on the Real number Line to 16 on the Power of 2 Line

Step 1: From 341 to 1024

$3(341)+1=1024$ directly

Step 2: From 1024 to 16

$\boxed{\text{net change}=-1008}$ (A)

B. From 341 on the Real number Line to 5 on the Real number Line , followed by to 16 on the Power of 2 Line

Step 1: From 341 to 5 = - 336

Step 2 From 5 to 16

$3(341-336)+1=16$

$\boxed{1023-1008+1=16}$ (B)

Back to Options

Back to Options

Option 3

Proving & Teaching Collatz Conjecture

Given: 1. A positive integer, n

2. The function $f(n) = \begin{cases} \frac{n}{2} \text{ if } n \text{ is even} \\ 3n+1 \text{ if } n \text{ is odd} \end{cases}$

Required: Begin with the positive integer, n, and form a sequence by applying the above operation repeatedly, using the result of each step as input for the next step and prove that eventually, the sequence reaches the integer 1.

Proof

Case 1: The integer can be written as a power of 2 as 2^k $(\text{k} = 1, 2, 3, ...)$
In this case, the sequence will reach the integer 1 by repeated division by 2, i.e., $2^{k-1}, 2^{k-2}, 2^{k-3}, ... 2^{k-k}$

Case 2: The integer cannot be written as a power of 2, but the sequence terms of the integer reach the equivalent power , 2^{2k} $(k = 2,\ 3, ...)$ which will continue as $2^{2k-1}, 2^{2k-2}, 2^{2k-3}, ..., 2^{2k-2k}$. Particularly, if the sequence reaches the integer, 16, the previous odd integer would be the integer 5 .
One would call the integer, 5, a 2k-power converter. Thus, an integer, n, would be called a 2k-power converter if $3n + 1 = 2^{2k}$ $(k = 2,\ 3, ...)$.
In Case 2, the sequence of every integer can reach the power, 2^{2k} $(k = 2,\ 3, ...)$. There are infinitely many paths for reaching 2^{2k} $(k = 2,\ 3, ...)$ powers. Of these paths, the integer 5–path, is the nearest 2^{2k} $(k = 2)$ converter path to the integer 1 on the 2^{2k}–route. For the 5–path, when a sequence terms reach the integer, 5, the next term would be $3(5) + 1 = 16$. Any sequence which reaches 5, will continue as 16, 8, 4, 2. to 1. Other integers can follow the integer 5–path to the 2^4 power as follows: Let n be an integer whose sequence terms are to reach 16 or 2^4 in the $(3n+1)\frac{1}{2}$ process; and let $n \pm r = 5$, where r is the net change in the sequence terms before the integer 5; and one uses the positive sign if $n < 5$, but the negative sign if $n > 5$. Since $5 = 5 - 0$, one can write $3(5 - 0) + 1 = 16$. One can also view $5 - 0$ as coming from single term sequence whose first term is 5 and whose net change is zero in the above $n \pm r = 5$ definition. If a sequence begins with for example, the positive integer 27, one can write $3(27 - 22) + 1 = 16 = 2^4$, by the substitution axiom, since $27 - 22 = 5$. Here, $n = 27$ and $r = 22$. Therefore, the sequence terms of every positive integer can reach the positive integer 16 or the power, 2^4. Thus, $3(n \pm r) + 1 = 16$ is for the integer 5–path. When the sequence reaches the integer, 16, repeated division by 2 will proceed as $8, 4, 2, 1$. One will call $\boxed{3(n \pm r) + 1 = 16 \text{ or } 2^4}$ the 5–path 2k-converter formula. Using this formula, the sequence of every positive integer which cannot be written as a power of 2, can reach the integer, 16. Once the sequence reaches 16, applying repeated division by 2, the sequence will reach the integer 1. $(16, 8, 4, 2, 1)$.
In Case 1**,** the integer can be written as a power of 2 as 2^k $(\text{k} = 1, 2, 3, ...)$; and the sequence will reach the integer 1, by repeated division by 2, i.e., $2^{k-1}, 2^{k-2}, 2^{k-3}, ... 2^{k-k}$. Therefore, the sequence of every positive integer would reach the integer 1, and the proof is complete.

Back to Options

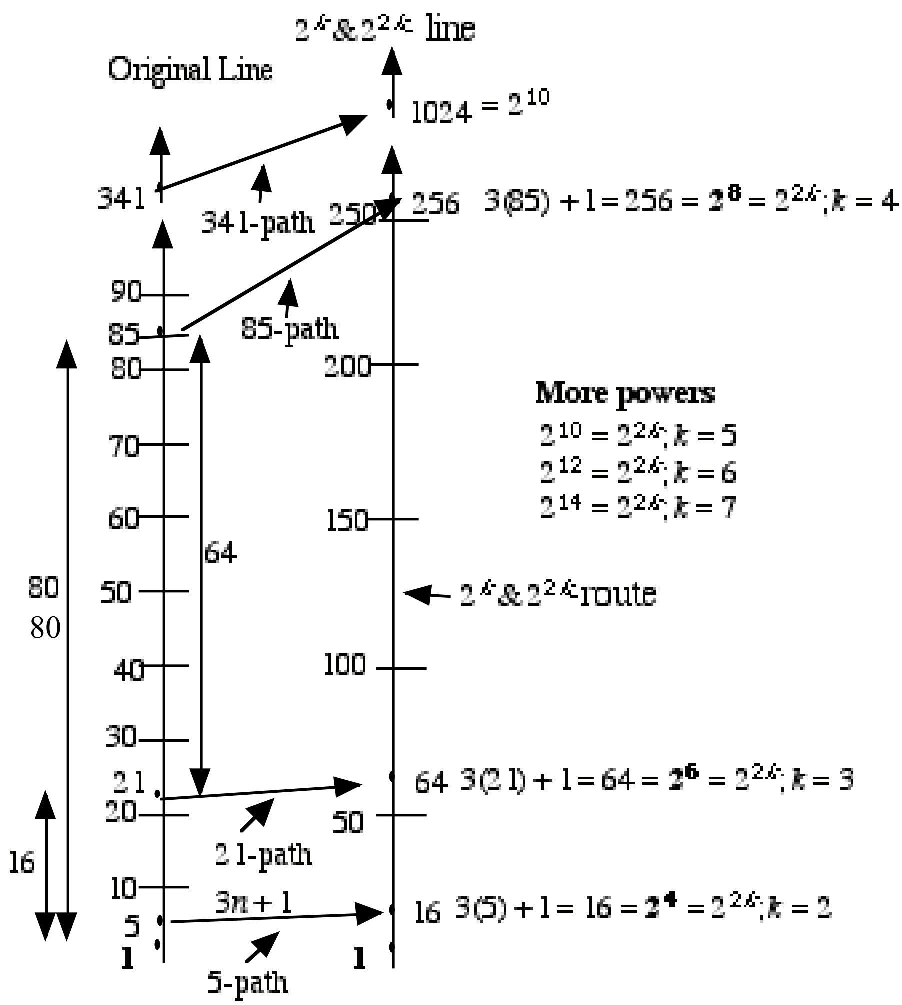
Back to Options

Option 4
Discussion

Justification for using the integer 5-path to convert a positive integer to a 2k-power

It is worth noting that the integers, 1 quadrillion, 1 trillion, 1 billion, 1 million, and the integer 123,456,789,012,345, all, use the 5-path to convert to the 2k-power forms.

Question:
Can a 2k-power converter use the path of another 2k- converter to convert to 2k-power form as above for integers, 21, 85 and 341
The answer is Yes as shown below



Back to Options

Option 5
Conclusion

Back to Options

By applying a systematic observation of the sequences produced by the Collatz process, the $(3n+1)\frac{1}{2}$ process, the author has shown that Collatz conjecture is true. Particularly, one noted the patterns of the sequence terms as the process reaches the equivalent powers, 2^{2k} $(k=2,\ 3,...)$ and continues straightforwardly as $2^{2k-1},\ 2^{2k-2},\ 2^{2k-3},\ \ ,...,2^{2k-2k}$. The approach used consists of two main cases, namely, Case 1 and Case 2. In Case 1, the integer can be written as a power of 2 as 2^k $(\mathrm{k}=1,2,3,...)$, and the sequence would reach the integer one by repeated division by 2, i.e., $2^{k-1},2^{k-2},2^{k-3},...2^{k-k}$. In Case 2, the integer cannot be written as a power of 2, but the sequence terms of the integer reach the equivalent power, 2^{2k} $(k=2,\ 3,...)$, and by repeated division by 2, the sequence will reach the integer 1, i.e., $2^{2k-1},2^{2k-2},2^{2k-3},,...,2^{2k-2k}$.. Thus the sequence will definitely continue to 1. In Case 2, two main requirements must be satisfied in order for a sequence of positive integers to reach the integer 1. The first requirement is that when the sequence terms reach some particular integers such as 5, 21 and 85, the application of 3n +1 to these integers would result in the powers, 2^{2k} $(k=2,\ 3,...)$. One would call these integers, the 2k-power converters. Examples of the converters are 5, 21, and 85 with the respective 2^{2k}- powers, $2^4,2^6,$ and 2^8. There are infinitely many power converters as there are 2^{2k} powers. The second requirement is that a term of the sequence of positive integers must be converted to 2^{2k} $(k=2,\ 3,...)$. There are infinitely many paths for converting integers to 2^{2k} $(k=2,\ 3,...)$ powers. Of these conversion paths, the integer 5-path, is the nearest 2^{2k} $(k=2)$ converter path to the integer 1 on the 2^{2k}–route. For the 5-path, when a sequence terms reach the integer, 5, the next term would be $3(5)+1=16$ or 2^4. Similarly, for the 2k-power converter, 21, the next term would be $3(21)+1=64=2^6$. Other integers can follow the integer 5-path to the 2^4 power as follows: Let n be an integer whose sequence terms would reach 16 or 2^4 in the $(3n+1)\frac{1}{2}$ process, and let $n\pm r=5$, where r is the net change in the sequence terms before the integer 5; and one uses the positive sign if $n<5$, but the negative sign if $n>5$. One will call the following, the 5-path 2k-converter formula: $\boxed{3(n\pm r)+1=16 \text{ or } 2^4}$. By the substitution axiom, using this formula, the sequence of every positive integer that cannot be written as a power of 2, would reach the integer, 16, as in Case 2; and once the sequence reaches 16, by repeated division by 2, the sequence would reach the integer 1. ((16,8,4,2,1). In Case 1**,** the integer can be written as a power of 2 as 2^k $(\mathrm{k}=1,2,3,...)$; and the sequence will reach the integer 1, by following $2^k,2^{k-1},2^{k-2},2^{k-3},...2^{k-k}$. Therefore, the sequence of every positive integer would reach the integer 1 It is worth noting that the integers, 1 quadrillion, 1 trillion, 1 billion, and 1 million, all, use the 5-path to convert to the 2k-power forms. The approach used in this paper has applications in civil engineering, especially in road design, road construction. as well as town and country planning.

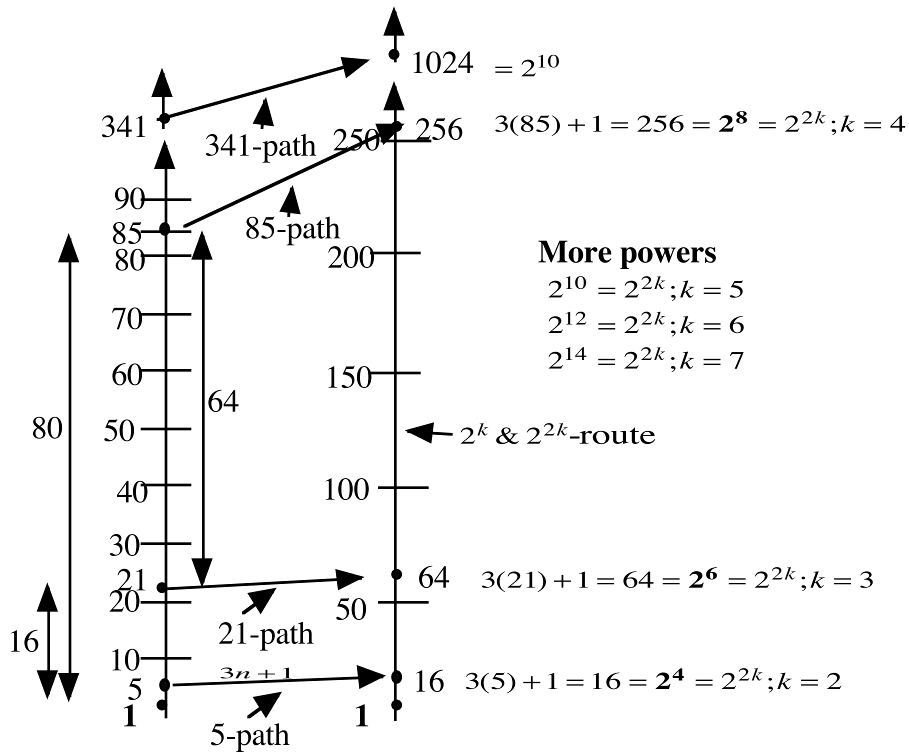
References:

1.. https://www.dcode.fr/collatz-conjecture
2. https://www.goodcalculators.com/collatz-conjecture-calculator

Back to Options

Back to Options

Option 6
Integer Humor



Integer 5 speaks: Integer 27, the integer 21-path is near you. Why did you not use the 21-path to cross to the 2^{2k}–route, but instead, you went through the jungle to use my path?

Integer 27 answers: I am not a descendant of integer 21, For me to use the 21-path my name should be $(21)2^p$ $(p = 1,2,...)$. Perhaps, when I reached the entrance to the 21-path, the Kamikaze typhoon blew me away to your path.

Integer 5 speaks to Integer 21: What should I write on the path formula if I want to use your path to go to the 2^{2k}–route**?**

Integer 21 answers: Write $3(5+16)+1 = 64 = 2^6$ on the formula.

Integer 21 speaks to Integer 32: Integer 32, When are you going to use my path to cross to the $2^k, 2^{2k}$-route?

Integer 32 answers: I, 2^5, do not need to use your path. I am already on the $2^k, 2^{2k}$-route**,** and I am on my way to attend the UN meeting on 1st Avenue, .

Adonten

Back to Options