The oscillations of a particle in an anharmonic potential with damping.

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The natural processes equations are nonlinear and have dissipating terms. In many cases they do not have an exact analytical solution. Using perturbation theory we solved the equation of an oscillating particle in an anharmonic potencial.

The equations of natural processes are nonlinear and in many cases do not have an exact analytical solution. The perturbation theory is useful in solving equations with small nonlinearities. In this work, approximate solutions were found for the nonlinearity x^3 in the equation, taking into account the resistance of the environment. Equation

$$\ddot{x} + g\dot{x} + \omega_0^2 x = \varepsilon a x^3 \quad (1)$$

The solution for the part ax^3 without damping is given in the literature [1]. We find approximate solutions taking into account the resistance of the medium. To do this, we expand the oscillation function into a series as follows:

$$x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \cdots,$$

and we put it into equation (1).

$$\ddot{x}_0 + \varepsilon\ddot{x}_1 + gx\dot{0} + g\varepsilon x\dot{1} + w(x_0 + \varepsilon x_1) = \varepsilon a(x_0 + \varepsilon x_1 + \cdots)^3 \quad (2)$$

we separate equations by degrees and we find the following equations system

$$\varepsilon^0 : \quad \ddot{x}_0 + \delta\dot{x}_0 + \omega^2 x_0 = 0$$

$$\varepsilon^1 : \quad \ddot{x}_1 + \delta\dot{x}_1 + \omega^2 x_1 = n x_0^3$$

$$\varepsilon^2 : \quad \ddot{x}_2 + \delta\dot{x}_2 + \omega^2 x_2 = 3n x_0^2 x_1$$

here *n =1* . The solution of the system first equation is found .

$$x_0(t) = Ae^{-\frac{\delta}{2}t}\sin(\Omega t + \varphi)$$

$$\Omega = \sqrt{\omega^2 - \frac{\delta^2}{4}}$$

Putting this solution to the system second equation we find

$$\ddot{x}_1 + \delta\dot{x}_1 + \omega^2 x_1 = n\left(Ae^{-\frac{\delta}{2}t}\sin(\Omega t + \varphi)\right)^3$$

$$nA^3 e^{-\frac{3\delta}{2}t}\sin^3(\Omega t+\varphi) = nA^3 e^{-\frac{3\delta}{2}t}\left[\frac{3}{4}\sin(\Omega t+\varphi) - \frac{1}{4}\sin(3\Omega t+3\varphi)\right]$$

we are looking for the particular solution as

$$x_{p_1}(t) = e^{-\frac{3\delta}{2}t}\left(a\cos(\Omega t+\varphi) + b\sin(\Omega t+\varphi)\right)$$

we find the coefficients a and b

$$\begin{cases} a\left(\Omega^2 - \frac{9\delta^2}{4}\right) - \frac{3\delta\Omega}{2}b = 0, \\ b\left(\Omega^2 - \frac{9\delta^2}{4}\right) + \frac{3\delta\Omega}{2}a = -\frac{3\delta^3}{4} \end{cases}$$

This equation solution

$$b = \frac{9A^3\delta\Omega n\left(\Omega^2 - \frac{9\delta^2}{4}\right)}{8\left[\left(\Omega^2 - \frac{9\delta^2}{4}\right)^2 + \left(\frac{3\delta\Omega}{2}\right)^2\right]}$$

$$a = \frac{27A^3\delta^2\Omega^2 n}{16\left[\left(\Omega^2 - \frac{9\delta^2}{4}\right)^2 + \left(\frac{3\delta\Omega}{2}\right)^2\right]}$$

The second particular solution

$$X_{p_2}(t) = e^{-\frac{3\delta t}{2}}\left(a\cos(3\theta) + b\sin(3\theta)\right)$$

we will find its coefficients

$$\begin{cases} a\left(-8\Omega^2 - \frac{9\delta^2}{4}\right) - \frac{9\delta\Omega}{2}b = 0, \\ b\left(-8\Omega^2 - \frac{9\delta^2}{4}\right) + \frac{9\delta\Omega}{2}a = -\frac{A^3}{4}n. \end{cases}$$

This equation solution

$$a = \frac{9\delta\Omega}{2\left(-8\Omega^2 - \frac{9\delta^2}{4}\right)}b$$

$$b = -\frac{A^3}{4}n\,\frac{\left(-8\Omega^2 - \frac{9\delta^2}{4}\right)}{\left(-8\Omega^2 - \frac{9\delta^2}{4}\right)^2 + \left(\frac{9\delta\Omega}{2}\right)^2}$$

The final solution will be

$$X(t) = Ae^{-\frac{\delta}{2}t}\sin(\Omega t+\varphi)$$

$$+\varepsilon\left[e^{-\frac{3\delta}{2}t}\left(\frac{27A^3\delta^2\Omega^2 n\cos(\Omega t+\varphi)}{16\left[(\Omega^2-\frac{9\delta^2}{4})^2+\left(\frac{3\delta\Omega}{2}\right)^2\right]}\right)\right]$$

$$+\frac{9A^3\Omega n\delta\left(\Omega^2-\frac{9\delta^2}{4}\right)}{8\left[(\Omega^2-\frac{9\delta^2}{4})^2+\left(\frac{3\delta\Omega}{2}\right)^2\right]}-\frac{9n\delta A^3\cos(3\Omega t+3\varphi)}{8\left[(-8\Omega^2-\frac{9\delta^2}{4})^2+\left(\frac{9\delta\Omega}{2}\right)^2\right]}$$

$$-\frac{nA^3(-8\Omega^2-\frac{9\delta^2}{4})}{4\left[(-8\Omega^2-\frac{9\delta^2}{4})^2+\left(\frac{9\Omega\delta}{2}\right)^2\right]}\sin(3\Omega t+3\varphi)$$

[1] A. A. Abdumalikov , Nonlinear wave equations , 2010