How to comprehend the unphysical quantum superposition and entanglement

Xianming Meng
Research School of Physics, Australian National University, Canberra, ACT 2601
email: xianming.meng@anu.edu.au

Abstract:

The concepts of quantum superposition and entanglement are at the heart of quantum mechanics, but they often cause confusion. This paper shows that the concepts are intimately related to quantum coherence. Through examining the traditional arguments, the paper reveals that the essence of quantum superposition is a statistical superposition of coherent states, while quantum entanglement results from the coherence of entangled particles as well as the common rule governing the measurements.

Key words: quantum coherence, quantum entanglement, Bell test, phase coherence, coherent states

1. Introduction

Quantum superposition and entanglement are intriguing concepts in quantum mechanics and are very difficult to be explained and tend to cause confusion because of their unphysical nature.

Quantum superposition requires that one particle is in multiple positions or states simultaneously. For example, in his Nobel lecture, Serge Haroche (2012)[1] states:

In order to prepare an electric dipole, a pulse of resonant microwave can be applied to the atom, bringing it in a superposition of the two adjacent e and g states, with respectively 51 and 50 nodes in their wave function. This superposition of states can be referred to as a "Schrödinger cat" because it implies an atom at the same time in two levels, reminding us of the famous cat that Schrödinger imagined suspended between life and death.

How can a cat be both alive and dead? How can a particle be in two places/states at the same time? When being asked this question, no one can explain it logically but relies on the typical answer: this is the nature of a quantum world. If one accepts the multi-world interpretation or the collapse of quantum superposition on measurements, quantum mechanics can explain things well. The issue is that the interpretation or assumption itself is unphysical.

Similarly, quantum entanglement is another mysterious concept. It requires that, when two particles are entangled, they remain connected even if they are separated by vast distance. When the state of one particle changes, it IMMEDIATELY affects the state of the other particle that is millions of lightyears away. This description implies that the two particles can communicate at no time or at least at a speed faster than light travels, or they are internally or 'spiritually' connected without any need of communication.

The strange consequences of these concepts make quantum mechanics mysterious. We may accept that the quantum world is weird and totally different from the macroscopic world we are familiar with. However, the claims embedded in these concepts are not only different from what we know in daily life but are totally unphysical. Understanding the physical implications of these concepts are vital for physics education. As will be seen later, the mystery and confusion about these concepts may hinder the further advance in quantum theory, so it is important to clear up these confusions about these concepts and gain a deeper understanding on a quantum system.

The remaining paper is organized as follows. Section 2 reviews the arguments on quantum superposition and concludes that the essence of superposition is the coherence of time phases of different states. Section 3 uses the quantum explanation of a Bell-test experiment based on polarized light to reveal the cause of correlated behaviours of entangled particles. Section 4 summarizes the paper.

2. quantum superposition and statistical mixture

The quantum superposition concept was popularized by a hypothetic scenario put forward by Schrodinger (1935). Albert Einstein, and his two assistants Boris Podolsky and Nathan Rosen (1935)[2] criticised the uncertainty (statistical) approach of quantum mechanics as being an incomplete theory. This paper was termed as the EPR paper. In response to this paper, Schrodinger (1935) wrote a paper to show that uncertainty (indeterminacy) in quantum mechanics can lead to indeterminacy in macro world, but it can be resolved by direct observation, so it does not affect the validity of the quantum mechanics. He communicated this idea through the following imaginary example:

One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, so small, that perhaps in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.

Apparently, here 'the living and dead cat mixed' with equal probability is an argument for the validity of statistical interpretation, so it is more likely to mean a statistical combination of living and dead cat (i.e. 50% chance of living cat and 50% chance of dead cat) rather than a living and dead cat at the same time physically. This statistical interpretation was formulated by Born (1926)[3].

For a wave function satisfying a time-independent Schrodinger equation, e.g. $\psi=0.707\varphi_1+0.707\varphi_2$, Born's statistical interpretation (1926) shows that the function ψ is the statistical combination/superposition of two wave functions $\varphi_1$and φ_2, with the probability if each function as the amplitude squared, i.e., $0.707^2=0.5$. The probability of $\varphi_1$and φ_2 can be obtained by finding the expected value of ψ over all positions in the entire space x (for simplification we assume a 1D wave in direction of x):

$$|\psi|^2 = \int \psi^* \psi dx = \int (0.707\varphi_1^* + 0.707\varphi_2^*)(0.707\varphi_1 + 0.707\varphi_2)dx$$

$$= 0.5 \int (\varphi_1^*\varphi_1 + \varphi_2^*\varphi_2 + \varphi_1\varphi_2^* + \varphi_1^*\varphi_2)dx = 0.5|\varphi_1|^2 + 0.5\,|\varphi_2|^2 \qquad (1)$$

Here we used the orthogonal condition that the integrals of $\varphi_1\varphi_2{}^*$ and of $\varphi_1{}^*\varphi_2$ are zero. In this stationary case, the probability to find $\varphi_1$and φ_2 is 50% each.

A wave function satisfying the time-dependent Schrodinger equation includes time phase terms to describe the evolution of the wave function over time, e.g. $\psi = 0.707\varphi_1 e^{-iE_1t/\hbar} + 0.707\varphi_2 e^{-iE_2t/\hbar}$. In this case, the simple statistical description is not good enough because the position probability density in this case is time dependent:

$$\psi^*\psi = \left(0.707\varphi_1^* e^{\frac{iE_1t}{\hbar}} + 0.707\varphi_2^* e^{\frac{iE_2t}{\hbar}}\right)\left(0.707\varphi_1 e^{-\frac{iE_1t}{\hbar}} + 0.707\varphi_2 e^{-\frac{iE_2t}{\hbar}}\right)$$

$$= 0.5\left(\varphi_1^*\varphi_1 + \varphi_2^*\varphi_2 + \varphi_1\varphi_2^* e^{-\frac{i(E_1-E_2)t}{\hbar}} + \varphi_1^*\varphi_2 e^{+\frac{i(E_1-E_2)t}{\hbar}}\right)$$

$$= 0.5\left(|\varphi_1|^2 + |\varphi_2|^2 + 2\varphi_1\varphi_2^* \cos\frac{(E_1-E_2)t}{\hbar}\right) \qquad (2)$$

Above we have used the Hermitian nature of Schrodinger wave functions, i.e. $\varphi_1\varphi_2^* = \varphi_2\varphi_1^*$.

In terms of density matrix ρ, we can write the above equation as:

$$\rho = |\psi><\psi| = \psi^*\psi = \begin{pmatrix} 0.707\varphi_1^* e^{\frac{iE_1t}{\hbar}} \\ \\ 0.707\varphi_2^* e^{\frac{iE_2t}{\hbar}} \end{pmatrix} \begin{pmatrix} 0.707\varphi_1 e^{-\frac{iE_1t}{\hbar}} & 0.707\varphi_2 e^{-\frac{iE_2t}{\hbar}} \end{pmatrix}$$

$$= \begin{pmatrix} 0.5|\varphi_1|^2 & 0.5\varphi_1^*\varphi_2 e^{\frac{i(E_1-E_2)t}{\hbar}} \\ \\ 0.5\varphi_1\varphi_2^* e^{-\frac{i(E_1-E_2)t}{\hbar}} & 0.5|\varphi_2|^2 \end{pmatrix} \qquad (3)$$

In the above expressions, the coefficients related to $|\varphi_1|^2$ and $|\varphi_2|^2$ give the probability of wave functions $\varphi_1$and φ_2 , respectively. However, the terms related to $\varphi_1\varphi_2^*$ and $\varphi_2\varphi_1^*$ vary with time, causing the probability density to oscillate with angular frequency of $\frac{(E_1-E_2)}{\hbar}$.

It has been argued that, for a statistical mixture of $\varphi_1$and φ_2, the time phases are random, so the terms related to $\varphi_1\varphi_2^*$ and $\varphi_2\varphi_1^*$, i.e., the off-diagonal elements of the density matrix in eq.(3) are zero, implying no oscillation of probability density. As such, statistical mixture is different from the superposition. In other words, quantum superposition is not a statistical superposition with some probability of the particle in the state of φ_1 and some probability of the particle in the state of φ_2 of random time phases, so it has to be a physical superposition where a particle can be in multiple position at the same time. It is further claimed that a superposition of states is a pure state (even though it includes many states) while a statistical mixture is a mixed state, which must have a total probability less than 1.

The above argument has some merits because it highlights the importance of phase term in the evolution of a quantum system, but it suffers from grossing over the details and thus leads to unclear definitions. It is true that randomness is at the heart of the statistical theory, but it not necessary that everything is random in this theory. In terms of statistical superposition of a particle, what we concern is that which state/position the particle occupies is random, and there is no requirement that the phases of the particle at different states/positions are random. Since the time-dependent Schrodinger equation necessitates that the time phase of each state has a frequency determined by the eigenenergy of the state, i.e. the time phase is the state-specific, the stationary states in a truly quantum system that satisfies the time-dependent Schrodinger equation must have fixed phase difference. Namely, the stationary states in a typical quantum system are coherent states. As such, quantum superposition can be viewed as a statistical combination of coherent states, and thus it is a special type of statistical superposition, not necessarily a physical superposition.

Some may argue that the cross-terms in eq.(2) or the off-diagonal elements in eq.(3) indicate the interference of states or wave functions $\varphi_1$and φ_2 , so $\varphi_1$and φ_2 must exist simultaneously, i.e. the particle must be in two states at the same time. Intuitively, interference should occur between two objects. However, we know from the single photon double-slit experiments that the accumulative diffraction pattern appears after a period, which indicates that the single photon can interfere with itself. Using the same analogy, the single atom at different states can interfere with itself and lead to non-zero cross terms. As such, the particle can be in two states $\varphi_1$and φ_2 with coherent time phases, but not necessarily at the same time.

So far we see that statistical superposition is well-equipped to describe quantum superposition of coherent states. Next, we examine the inconsistency of current definition of pure states and statistical mixture, which can be revealed through the density matrix shown in eq.(3) in different cases. Case 1: with fixed phase terms in eq.(3), the off-diagonal elements are non-zero and this case is defined as quantum superposition, which can be demonstrated by well-defined vectors on the Bloch sphere as the total probability (the trace of density matrix or the sum of diagonal elements) is 1. Case 2: if the phase terms in eq.(3) are random, the off-diagonal elements are zero. It can be argued that this case is not statistical mixture because the total probability (the trace of density matrix or the sum of diagonal elements) is 1. this case can be described by not well-defined (due to random phases) vector on the Bloch sphere. Case 3: with random phase terms in eq.(3) and the sum of coefficients for the diagonal elements less than 1 (e.g. with 0.4 and 0.5, instead of 0.5 and 0.5, for two diagonal elements in eq.(3)), which is inside the Bloch sphere (because total probability is less than 1). Based on the current/traditional definition, Case 1 is a pure state and Case 3 is a statistical mixture, so the definition simply ignores or denies the possibility of case 2. The difference between case 2 and case 3 is simply a statistical combination of incoherent states (because the phase terms are random) in a closed system (case 2) or in an open system (case 3). The traditional definition of statistical mixture simply mixes up or bundles the random phase with close/open system. Given that the Cases 1-3 all are statistical combination of different quantum states, they can be called a statistical mixture. A more scientific definition should be: case 1 is statistical mixture/combination of coherent states, which can be demonstrated by definite vectors on Boch sphere; case 2 is statistical mixture/combination of incoherent states in a closed system, which can be shown by random vectors on the Bloch sphere; and case 3 is statistical mixture/combination of incoherent states in an open system, which can be shown as random vectors inside the Bloch sphere.

In short, the essence of quantum superposition is a probabilistic combination of coherent states, which have fixed time phase differences, so it is a special type of statistical superposition/mixture. In a dynamic quantum system, this statistical combination is reflected on the oscillation between stationary states, e.g. Rabi oscillation, which is experimentally proven by Wineland (2013)[4] and Haroche (2013). On the other hand, a statistical combination of incoherent or decoherent states would wash out the time phase differences and leads to no oscillation. These clearer definitions change nothing essential in quantum mechanics, but they discard the claim of one particle at two positions simultaneously and make the theory more comprehensible.

3. Quantum entanglement and the Bell test

The origin of the quantum entanglement concept started from the EPR paper (1935), which claimed that wavefunctions in quantum theory do not contain complete information of a system. In quantum theory, measuring a property of one particle collapses the wave function and instantaneously influences the totally separated entangled particles. Einstein called this "spooky action at a distance". The forerunners of quantum mechanics, such as Bohr and Schrodinger, responded to Einstein's claim. In his response, Schrodinger (1935)[5] used the term 'entangled' to explain the spooky action at a distance. John Bell (1964)[6] proposed a test for the EPR paradox, CHSH (1969)[7] refined the Bell test, and Aspect et al (1982)[8] convincingly showed that their experimental results violated the Bell theorem but were consistent with the prediction from quantum theory. Afterwards, numerous Bell tests (Tittel et al, 1998[9], Pan et al, 2000[10], Groblacher et al, 2007[11], Ansmann et al, 2009[12], Giustina et al, 2013[13], Hensen et al 2015[14], Thenabadu et al, 2020[15], Storz et al, 2023[16]) have been conducted to close the loopholes in the tests and the results of all tests violates Bell inequality and support quantum mechanics.

The usual assumption for deriving Bell theorem is that at location A, a setting 'a' (e.g. the direction of the spin/polarization analyser) leads to an experimental outcome A(a), while setting 'b' at location B leads to outcome B(b), with the joint outcome being E(a,b) = A(a)B(b). If settings 'a' and 'b' can be changed to a′ and b′, respectively, we can have joint outcomes: E(a,b′) = A(a) B(b′), E(a′, b) = A(a′) B(b), and E(a′, b′) = A(a′) B(b′). The additional assumption is that the detected outcome at any setting is between -1 and +1, namely $|A| \leqslant 1$, $|B| \leqslant 1$.

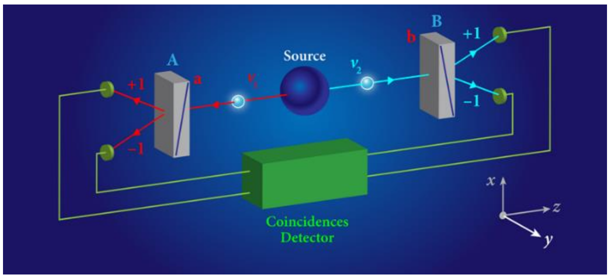
By incorporating a local hidden variable into the inequality, Bell (1964, 1971[17]) and Clauser et al (1969) derived the following the inequality:

$$|E(a,b)+E(a',b')+E(a'b)-E(a,b')| \leqslant 2 \qquad (4)$$

This inequality becomes the test criterium to differentiate the local hidden variable effect from the quantum entanglement effect. If the inequality holds, the local hidden variable hypothesis is verified, otherwise, the spooky action of quantum entanglement from a distance is regarded as proven.

The experimental setups to test the above inequality varies. In this section, We use the Bell test experimental setting of Aspect (1982) as the base for discussion. For the convenience of the reader, we illustrate the experimental setting in Fig.1.

Fig. 1 A four-way coincidence Bell test based on polarised light



The source in the middle produces correlated/entangled polarized photon pairs that move to left (A) and right (B), i.e. the polarization of photon pair has a fixed angle (e.g. the same polarization for a special case). The polarizers at A and B can be adjusted to any directions (Aspect used an optical switch so as to change the polarizer direction randomly. This closes the loophole of possible hidden local variables related to polarizers). The photon passing through the polarizer in horizontal direction (in state |H>) is recorded a value of +1, in the vertical direction (in state |V >) is recorded as -1.

When the polarizer filter is rotated by angle θ, the horizontal direction H and vertical direction V will change to $\widetilde{H}$ and $\widetilde{V}$, respectively. The new state $|\widetilde{H}>$ and $|\widetilde{V}>$ can be obtained by rotating the initial states |H> and |V> by angle θ. Based on the 2D rotation matrix, we can obtain:

$|\widetilde{H}> = |H> cos\theta\ + |V> sin\theta$ and

$$|\widetilde{V}> = -|H> sin\theta + |V> cos\theta \qquad (5)$$

Following the tradition, we assume orthonormal states, so we have:

$<\widetilde{H}|\widetilde{H}> = <H\big|H> = <\widetilde{V}\big|\widetilde{V}> = <V|\mathrm{V}> = 1$, and

$$<\widetilde{H}|\widetilde{V}> = <\widetilde{V}|\widetilde{H}> = <H|V> = <V|\mathrm{H}> = 0 \qquad (6)$$

For the photon pair arriving at A and B from the source, we assume they have the same polarization. In terms of a normalized wave function, it can be expressed as:

$$|\psi> = (|H_A>|H_B> + |V_A>|V_B>)/\sqrt{2} \qquad (7)$$

This is one of the 4 maximumly entangled states. Here the entanglement is implicitly expressed as the correlation between the polarization angle of photon A and that of photon B, i.e. $|H_A>$ goes hand in hand with $|H_B>$, and $|V_A>$ and $|V_B>$.

For the photons passing through both the $\widetilde{H}$ direction at A (rotated by θ_a degrees) and the $\widetilde{H}$ direction at B (rotated by θ_b degrees), i.e. with +1 values, the amplitude of the wave function can be calculated as:

$$\mathcal{L}(++) = \big(<\widetilde{H}_A|<\widetilde{H}_B|\big)|\psi> = (<H_A|cos\theta_a + <V_A|sin\theta_a)(<H_B|cos\theta_b + <V_B|sin\theta_b)|\psi>$$

$$= \{(<H_A|<H_B|cos\theta_a cos\theta_b + <V_A|<V_B|sin\theta_a sin\theta_b + <H_A|<V_B|cos\theta_a sin\theta_b + <H_B|<V|cos\theta_b sin\theta_a)\}\frac{(|H_A>|H_B> + |V_A>|V_B>)}{\sqrt{2}} = \frac{\cos(\theta_a - \theta_b)}{\sqrt{2}} \qquad (8)$$

In the same way, we can calculate the following:

Similarly, for photons passing through $\widetilde{V}$ at both A and B, we have:

$$\mathcal{L}(--) = (<\widetilde{V}_A|<\widetilde{V}_B|)|\psi> = \frac{\cos(\theta_a - \theta_b)}{\sqrt{2}} \qquad (9)$$

$$\mathcal{L}(+-) = (<\widetilde{H}_A|<\widetilde{V}_B|)|\psi> = \frac{\sin(\theta_a - \theta_b)}{\sqrt{2}} \qquad (10)$$

$$\mathcal{L}(-+) = (<\widetilde{V}_A|<\widetilde{H}_B|)|\psi> = \frac{-\sin(\theta_a - \theta_b)}{\sqrt{2}} \qquad (11)$$

The probability of photons passing through each combination of axis is the amplitude squared, so we have:

$$p(+\,+) = p(-\,-) = \frac{\cos^2(\theta_a - \theta_b)}{2}, \qquad and\ p(+\,-) = p(-\,+) = \frac{\sin^2(\theta_a - \theta_b)}{2} \tag{12}$$

The expected values of photons passing through all combinations of axes of setting (a,b) of rotating polarizer at A by θ_a at B by θ_b is:

$$E(a,b) = (+1)(+1)p(+\,+) + (-1)(-1)p(-\,-) + (-1)(+1)p(-\,+) + (+1)(-1)p(+\,-)$$
$$= \cos^2(\theta_a - \theta_b) - \sin^2(\theta_a - \theta_b) = \cos 2(\theta_a - \theta_b) = cos2\theta_{ab} \tag{13}$$

Where θ_{ab} is the angle of polarization directions of two polarizers.

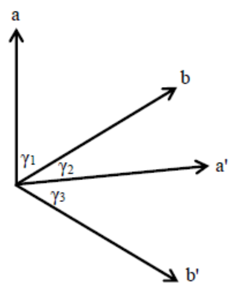
If we change the rotation angle at A by $\theta_a{}'$ and/or change the rotation angle at B by $\theta_b{}'$, we can calculate the expected value at each setting as:

$$E(a,b') = cos2\theta_{ab'} \quad E(a',b) = cos2\theta_{a'b} \quad E(a',b') = cos2\theta_{a'b'} \tag{14}$$

Following Clauser and Shimony (1978)[18] and Aspect et al (1982), we can illustrate polarizer rotation setting as coplanar vectors in Fig.2 and obtain:

$$\theta_{ab} = \gamma_1, \qquad \theta_{a'b} = \gamma_2 \qquad \theta_{a'b'} = \gamma_3 \qquad \theta_{ab'} = \gamma_1 + \gamma_2 + \gamma_3 \tag{15}$$

Fig.2 Coplanar presentation of polarisation analysers setting in the Bell test



In considering the Bell inequality of eq.(4), our task is to evaluate if $S = cos2\gamma_1 + cos2\gamma_2 + cos2\gamma_3 - cos2(\gamma_1 + \gamma_2 + \gamma_3)$ is greater than 2 or less than -2. Using the first and second order conditions, we can find the maximum and minimum value of S are:

At $\gamma_1 = \gamma_2 = \gamma_3 = \frac{\pi}{8}, S_{max} = 2\sqrt{2}$; at $\gamma_1 = \gamma_2 = \gamma_3 = \frac{3\pi}{8}, S_{min} = -2\sqrt{2}$ (16)

This is the quantum prediction for the Bell test, and it is indeed proven to be correct.

What are the key factors in obtaining the quantum prediction? First, a maximumly entangled state like eq.(7) is the basis for maximum violation of the Bell inequality. Using other 3 maximumly entangled states we can also obtain the same results. The key requirement of maximumly entangled states is that the polarization angle difference of the entangled photon pairs is fixed. This requirement reveals the nature or essence of quantum entanglement.

Second, the quantum state transformation based on the rotation matrix eqs (5) and the dot product for measuring the wavefunction amplitude in eqs.(8) – (11) are vital. The transformation rule and dot product rule are used at both locations A and B for all settings, so they are the common rule governing the measurements. As the common measurement rule is applied to the photon pairs with fixed difference in polarization angles, one can expect that the measurement results at A and B are correlated. It is apparent that these common rules underpin the results for E(a,b), E(a,b'), E(a',b) and E(a',b'), leading to the maximum violation of the Bell theorem.

Third, the rule that an operator must act on the same basis in quantum mechanics plays an implicit but significant role. This rule requires that a Bra at location A (e.g. $<H_A|$ or $<V_A|$) can act only on a Ket at location A (e.g. $|H_A>$ or $|V_A>$) of the entangled wave function, and vice versa for location B. In terms of the experiment shown in Fig.1, the quantum rule simply states the fact that an operator/measurement at A cannot act on the photon at B. With this operation rule, we can rewrite the measurement of amplitude $\mathcal{L}(+\,+)$ as:

$$\begin{aligned}\mathcal{L}(+\,+) &= (<\widetilde{H}_A|<\widetilde{H}_B|)(\frac{|H_A>|H_B>+|V_A>|V_B>)}{\sqrt{2}} \\ &= \frac{(<\widetilde{H}_A|<\widetilde{H}_B|H_A>|H_B>+<\widetilde{H}_A|<\widetilde{H}_B|V_A>|V_B>)}{\sqrt{2}} \\ &= \frac{(<\widetilde{H}_A|H_A><\widetilde{H}_B|H_B>+<\widetilde{H}_A|V_A><\widetilde{H}_B|V_B>)}{\sqrt{2}} \qquad (17)\end{aligned}$$

Here the dot product of both $<\widetilde{H}_A|H_A>$ and $<\widetilde{H}_A|V_A>$ are the measurement at analyser A, while $<\widetilde{H}_B|H_B>$ and $<\widetilde{H}_B|V_B>$ are measurement at analyser B. The dot product of both $<\widetilde{H}_A|H_A><\widetilde{H}_B|H_B>$ and $<\widetilde{H}_A|V_A><\widetilde{H}_B|V_B>$ are correlation between the measurement at A and B. The dot product gives a cosine function, which underpins the quantum results eq.(24). In fact, the dot product of measurement at both A and B gives the same result as the Malus's law in optics, and the dot product for correlation resembles the statistical correlation calculation, so it is not surprising that the statistical approach by Meng (2021)[19] and Annila (2024)[20] give the same results of eq.(24).

Since an operation at A cannot act on a quantum state at B, an operation at A cannot change the wavefunction at location B. As such, this quantum rule effectively rules out the possibility that a measurement at location A can change the quantum state of location B, i.e., rules out any possibility of spooky action at a distance.

To get the state of an entangled system, a joint measurement such as the one shown in eq.(8) must be performed. A one-location measurement can obtain only indeterminate results. For example, measuring only the polarisation at location A with setting 'a', we have:

$$\mathcal{L}(A+) = (<\tilde{H}_A|)|\psi>= (<H_A|cos\theta_a + <V_A|sin\theta_a)\frac{|H_A>|H_B>+|V_A>|V_B>}{\sqrt{2}}$$

$$=\frac{cos\theta_a|H_B> + sin\theta_a|V_B>}{\sqrt{2}} \qquad (18)$$

On the surface, one may interpret eq.(18) as that the measurement at location A can change the amplitude or probability of wavefunction $|H_B>$ and $|V_B>$ at location B. However, one needs to realize that calculation in eq.(18) is for the amplitude measured at A, i.e., $\mathcal{L}(A+)$. The resultant $\cos^2\theta_a$ is the probability of detecting photons on horizontal (H_A) axis, and $\sin^2\theta_a$ the probability of detect photons on the vertical (V_A) axis. The squared coefficients in front of the wavefunction at the location B, i.e., $\frac{\cos^2\theta_a}{2}$ for $|H_B>$ and $\frac{\sin^2\theta_a}{2}$ for $|V_B>$, is not the probability of photon polarization at location B, but the intended joint probability of measurement both at A and later at B. As a joint probability needs inputs from both locations, it is logical that the measurement at A will affect this joint probability. Since eq.(18) indicates no impact on the probability of incidence measurement at B other than the joint probability of coincidence measurements at A and B, it does not show any spooky action over a distance.

In summary, the quantum prediction on the Bell test does not mean any spooky action at a distance. On the contrary, the quantum operation rule prohibits this possibility. The seemingly coordinated results stem from both the initial entangled state and the common rules governing the measurement process.

4. Conclusions

From the foregone discussion, we can conclude that quantum superposition does not require a physical superposition of one particle at two states/positions at the same time. Quantum superposition or pure states are simply a statistical superposition/combination of coherent states, while the so-called statistical mixture or mixed states are a statistical superposition/combination of incoherent or decoherent states.

Similarly, the fact that experimental results of Bell tests violate Bell inequality does not prove any impact of entangled particles on distant counterparts. In fact, the quantum rule of operating on wave function of the same basis effectively rules out any spooky action from a distance. The correlated behaviour of entangled particles can be explained by the correlated initial quantum states of particles as well as the same measurement rules applied to the entangled particles. As such, the essence of quantum entanglement is the coherence among the entangled particles.

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