Solving the Riemann Hypothesis Through Shear Force of Will

Author: Ryan Hackbarth
e-mail: rhackbar@purdue.edu

Abstract:

Here I present an equation for the Zeros of the Riemann Zeta Function which connects the distribution of the trivial zeroes with integer inputs to the distribution of the nontrivial zeroes. I demonstrate that this relationship explicitly depends on the critical line where a = ½. I do so in plain language and with replicable calculations, as when I try to write like a mathematician it comes across as inauthentic and bad. Finally, I provide an appendix of calculated solutions.

Introduction:

The zeroes of the Riemann Zeta Function rely on the unique property of the ratio (1):

$$\frac{a+b\,i}{(a+b\,i)-1} = \frac{2\,b-i}{2\,b+i} \text{ where } a = \frac{1}{2} \tag{1}$$

In that for any choice of b, this ratio produces a unit scaled complex number with decreasing exponent c for increasing b.

$$\frac{2\,b-i}{2\,b+i} = e^{-c\,i}$$

$$c = i\log\left(\frac{2\,b-i}{2\,b+i}\right) \tag{2}$$

This exponent c, can be associated with a unique value q:

$$\log(c+2\,\pi) = \log\left(\frac{1}{q}+1\right)$$

$$i\log\left(\frac{2b-i}{2b+i}\right)+2\pi=\frac{1}{q}+1$$

$$q=\frac{1}{i\log\left(\frac{2b-i}{2b+i}\right)+2\pi-1} \tag{3}$$

This value q, can be replicated with the equation (4):

$$q=\frac{1}{-1+e^{i(2\pi+x)}e^{i(2\pi+y)}} \tag{4}$$

Where,

$$\mathrm{Im}\left(\frac{1}{-1+e^{i(2\pi+x)}e^{i(2\pi+y)}}\right)=0$$

Which gives an expression for x in terms of y:

$$x=-2\pi+2\pi\sec(y) \tag{5}$$

Substituting (5) for x in (4).

$$q=\frac{1}{-1+e^{2ie^{iy}\pi\sec(y)}} \tag{6}$$

We can then equating (3) and (6):

$$\frac{1}{i\log\left(\frac{2b-i}{2b+i}\right)+2\pi-1}=\frac{1}{-1+e^{2ie^{iy}\pi\sec(y)}} \tag{7}$$

Is it possible to solve for b in terms of y:

$$i\log\left(\frac{2b-i}{2b+i}\right)+2\pi-1=-1+e^{2ie^{iy}\pi\sec(y)}$$

$$i\log\left(\frac{2b-i}{2b+i}\right)=e^{-2\pi\tan(y)}-2\pi$$

$$b=\frac{i\left(1+e^{i\,e^{-2\pi\tan(y)}}\right)}{2\left(-1+e^{i\,e^{-2\pi\tan(y)}}\right)} \tag{8}$$

Likewise, you can solve for y in terms of b:

$$y=-\tan^{-1}\left(\frac{\log\left(2\pi-i\log\left(\frac{i+2b}{-i+2b}\right)\right)}{2\pi}\right) \tag{9}$$

A list of corresponding y values for the first 40 zeroes of the Zeta Function are included in Appendix A.

Trivial and Nontrivial Zeroes of the Zeta Function

Finally, the point being that it is possible to construct the function (10) from (4) and (5):

$$f(y)=\frac{1}{1-e^{i\,e^{-2\pi\tan(y)}}} \tag{10}$$

Such that it generates the solutions to the zeroes of the zeta function.

$$\zeta(f(y)) = 0$$

This function is particularly useful because it has 2 valid solution sets. The first solution set encodes the trivial zeroes of the Zeta Function through integer inputs:

$$y = \pi - i \tanh^{-1}\left(\frac{1}{2\pi}\left(2\pi - i \log\left(2\pi - i \log\left(\frac{0.5\,(2n+1)}{n}\right)\right)\right)\right)$$

The second solution set encodes the nontrivial zeroes through the ratio (1), where p_n is the nth nontrivial zero of the Zeta Function.

$$y = \pi - i \tanh^{-1}\left(\frac{1}{2\pi}\left(2\pi - i \log\left(2\pi - i \log\left(\frac{\rho\_n - 1}{\rho\_n}\right)\right)\right)\right)$$

The symmetry of these 2 solutions sets indicates an intimate relationship between the distribution of the trivial zeroes and the nontrivial zeroes. We can plot this relationship to confirm the correspondence between them. This relationship is directly dependent on the unique property of (1) where a = 1/2.

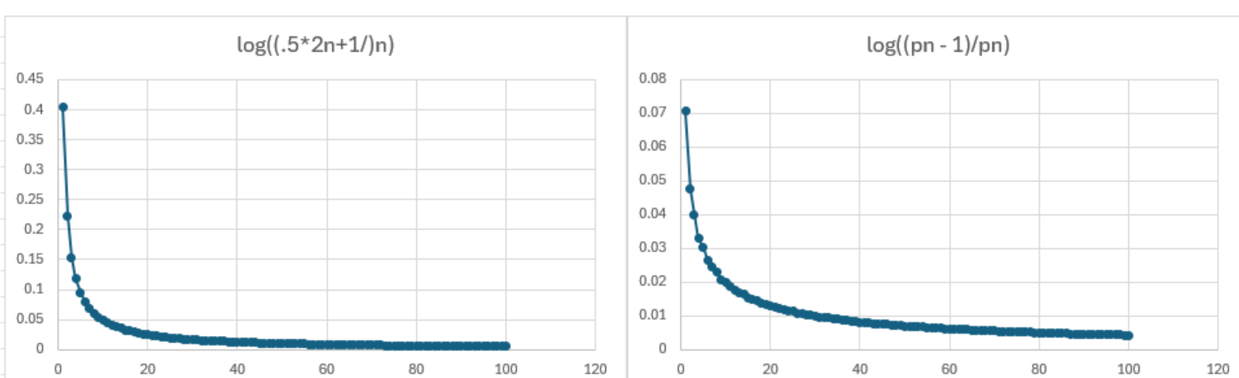

Figure 1. Plots of the symmetrical solution sets for the trivial and nontrivial zeroes of the Riemann Zeta Function

Appendix A. Solutions for (8) and (10)

ZERO	B	Y
1	14.13473	-0.286208711
2	21.02204	-0.285673812
3	25.01086	-0.285498204
4	30.42488	-0.285333282
5	32.93506	-0.285275162
6	37.58618	-0.285187938
7	40.91872	-0.285137607
8	43.32707	-0.285106044
9	48.00515	-0.285053767
10	49.77383	-0.285036558
11	52.97032	-0.285008366
12	56.44625	-0.284981327
13	59.34704	-0.284961183
14	60.83178	-0.284951615
15	65.11254	-0.284926468
16	67.07981	-0.284915986
17	69.5464	-0.284903681
18	72.06716	-0.284891974
19	75.70469	-0.284876454
20	77.14484	-0.284870713
21	79.33738	-0.284862373
22	82.91038	-0.284849726
23	84.73549	-0.284843677
24	87.42527	-0.284835222
25	88.80911	-0.284831071
26	92.4919	-0.28482063
27	94.65134	-0.284814885
28	95.87063	-0.284811755
29	98.83119	-0.284804478
30	101.3179	-0.284798693
31	103.7255	-0.284793357
32	105.4466	-0.284789691
33	107.1686	-0.284786141
34	111.0295	-0.284778582
35	111.8747	-0.284776997
36	114.3202	-0.284772542
37	116.2267	-0.284769199
38	118.7908	-0.284764872
39	121.3701	-0.284760704
40	122.9468	-0.284758242