Simple (possibly simplest) Fibonacci cycle when sign is minus instead of plus.

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Abstract.

A famous Fibonacci sequence is forming a simple cycle when sign plus is replaced to minus. A simple proof for any numbers is outlined.

Fibonacci sequence is a marvel of mathematics and thoroughly investigated [1]. There are numerous generalizations of Fibonacci numbers (tribonacci, tetrabonacci etc), but I found no mention of simplifications of Fibonacci sequence, mainly Fibonacci cycle.

The classical Fibonacci sequence is generated as follows:

$F(n)=F(n-1)+F(n-2)$

Now let's play around and replace sign "plus" to sign "minus". The result is unusual - it will generate a simple cycle.

$F(n-2)=0$

$F(n-1)=1$

$F(n)=F(n-1)-F(n-2)=1-0=1$

$F(n+1)=F(n)-F(n-1)=1-1=0$

$F(n+2)=F(n+1)-F(n)=0-1=-1$

$F(n+3)=F(n+2)-F(n+1)=-1-0=-1$

$F(n+4)=F(n+3)-F(n+2)=-1-(-1)=-1+1=0$

$F(n+5)=F(n+4)-F(n+3)=0-(-1)=0+1=1$

$F(n+6)=F(n+5)-F(n+4)=1-0=1$

$F(n+7)=F(n+6)-F(n+5)=1-1=0$

$F(n+8)=F(n+7)-F(n+6)=0-1=-1$

$F(n+9)=F(n+8)-F(n+7)=-1-0=-1$

....

The sequence is obviously a simple cycle: 0,1,1,0,-1,-1,0,1,1,0,-1,-1,.......

This simple cycle is easily proved for any original numbers:

$F(n-2)=a$

$F(n-1)=b$

$F(n)=F(n-1)-F(n-2)=b-a$

$F(n+1)=F(n)-F(n-1)=b-a-b=-a$

$F(n+2)=F(n+1)-F(n)=-a-(b-a)=-a-b+a=-b$

$F(n+3)=F(n+2)-F(n+1)=-b-(-a)=-b+a=a-b$

$F(n+4)=F(n+3)-F(n+2)=a-b-(-b)=a-b+b=a$

$F(n+5)=F(n+4)-F(n+3)=a-(a-b)=a-a+b=b$

$F(n+6)=F(n+5)-F(n+4)=b-a$

$F(n+7)=F(n+6)-F(n+5)=b-a-(b)=-a$

$F(n+8)=F(n+7)-F(n+6)=-a-(b-a)=-a-b+a=-b$

$F(n+9)=F(n+8)-F(n+7)=-b-(-a)=-b+a=a-b$

The sequence is a cycle for any input numbers: a;b;b-a;-a;-b;a-b;a;b;b-a;-a;-b;a-b.......

The original Fibonacci sequence has a lot of implications in physical world [1]: golden ratio, spirals of pine cones, honeybees etc.

The cycle Fibonacci sequence has even more implications for physical world - oscillations are everywhere and time itself is based on the presence of repeating sequences (cycles). Yet the cycle sequence is very easy to obtain - the great question on the exam in elementary school to illustrate how small change may lead to great difference in behavior.

References.

1.Fibonacci sequence - Wikipedia