Proof of Polya equivalent of Riemann Hypothesis

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Abstract

[In this paper Polya equivalent of Riemann Hypothesis is proved from Complex analytic expression of Riemann Xi function]

Key words : Riemann Hypothesis equivalent, Riemann xi function, Riemann Zeta function ; Polya equivalent of Riemann Hypothesis .

1. Introduction .

One of the most difficult problem to physics and mathematics community is Riemann Hypothesis which is a Conjecture proposed by Bernhard Riemann which states that all Complex zeros of Riemann Zeta function has a real part $\sigma = ½$. This Conjecture is not yet proved or disproved . There are some equivalent statements whose proofs imply proof of Riemann Hypothesis . One such equivalent is Polya equivalent which is due to G. Polya [1]

This equivalent states that if instead of considering Riemann Xi function $\xi(s)$ we define

$$\Xi(s) = \xi(\frac{1}{2} + is) , s = \text{complex} = x + iy \quad (1.1)$$

Then all zeros of $\Xi(s)$ are real.

2. Proof of Polya statement.

In some papers [2,3] it was shown that complex Riemann Xi function $\xi(s)$ has analytic expression

$$\xi(s) = F_2(l_1) + F_1(l_1) \cosh l_1 (s - \frac{1}{2}) \quad (2.1)$$

where $F_2(l_1)$, $F_1(l_1)$ are two real unknown positive constants and l_1 is a real parameter which is positive also.

It was also shown from the papers [2, 3, 4, 5] that some important results of $\xi(s)$ and some Riemann Hypothesis equivalents follow from the expression of $\xi(s)$ given by (2.1)

In this paper Polya equivalent of Riemann Hypothesis will be proved from the expression of Ξ (s) defined by equation number (1.1)

We have from (1.1)

$\xi(\frac{1}{2} + is) = \Xi(s)$

$= F_2(l_1) + F_1(l_1) \cosh l_1 [\frac{1}{2} + i s - \frac{1}{2}]$ replacing s by $(\frac{1}{2} + is)$ in equation (2.1)

$= F_2(l_1) + F_1(l_1) \cosh l_1 [is]$

$= F_2(l_1) + F_1(l_1) \cosh l_1 [i (x + iy)]$ using (1.1)

i.e $\Xi(s) = F_2(l_1) + F_1(l_1) \text{Cosh } l_1 [ix - y]$

$= F_2(l_1) + F_1(l_1) [\text{Cosh } l_1(ix) \text{ Cosh } l_1y - \text{Sinh } l_1(ix) \text{ Sinh } l_1y]$

$= F_2(l_1) + F_1(l_1) [\text{Cos } l_1x \cdot \text{Cosh } l_1y - i \text{ Sin } l_1x \cdot \text{Sinh } l_1y]$

$$= F_2(l_1) + F_1(l_1) [\text{Cos } l_1x \cdot \text{Cosh } l_1y - i \text{ Sin } l_1x \cdot \text{Sinh } l_1y] \quad (2\cdot2)$$

Now zero of Ξ (s) implies both real (R) and imaginary (I) parts of (2·2) must be zero.

Hence zero of Ξ (s) is given by

$$R = F_2(l_1) + F_1(l_1) \text{ Cos } l_1x \cdot \text{Cosh } l_1y = 0 \quad (2\cdot3)$$

and $$I = F_1 \text{ Sin } l_1x \cdot \text{Sinh } l_1y = 0 \quad (2\cdot4)$$

Now $F_1 \neq 0$, Hence I = 0 implies

either i) $\text{Sin } l_1x = 0$

or ii) $\text{Sinh } l_1y = 0$

But $\text{Sin } l_1x = 0$ implies $\text{Cos } l_1x = \pm1$. Then from (2·3) $\text{Cosh } l_1y = \pm\frac{F_2(l_1)}{F_1(l_1)}$ (2·5)

But $\text{Cosh } l_1y$ is defined always positive, which contradicts equation (2·5) because $F_2(l_1)$ and $F_1(l_1)$ are both positive, consequently r.h.s of (2·5) is always positive only, which is not the only case as the r.h.s of equation (2.5) suggests.

Hence I = 0 implies $\text{Sinh } l_1y = 0$ i.e.

$$y = 0 \quad (2\cdot6)$$

And when y = 0,

$$\text{Cosh } l_1 y = 1 \tag{2.7}$$

Hence from (2.3), R = 0 implies

$$x = \left(\frac{1}{l_1}\right) \text{Cos}^{-1}\left(-\frac{F_2(l_1)}{F_1(l_1)}\right) \tag{2.8}$$

Thus it turns out that zeros of Ξ (s) are of the form [$\frac{1}{l_1}\text{Cos}^{-1}\left(-\frac{F_2}{F_1}\right)$, 0] which are all real.

3. Conclusion,

Therefore the Polya equivalent of Riemann Hypothesis is proved.

4. References.

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