The Zeroes of the Riemann Zeta Function Encode Rational Approximations of Pi

Author: Ryan Hackbarth

Abstract:

In this paper, I present a formula for the zeroes of the Riemann Zeta Function and highlight their dependence on a rational integer ratio. I connect these ratios with a hyperbola reminiscent of Pell's equation which approximates pi and provide a table of calculated ratios and their corresponding Zero. Finally, I demonstrate the requirement of the critical line at ½ in producing these integer approximations.

Zeroes of the Zeta Function

The imaginary part of the argument resulting in a Zero of the Zeta function, b in s=(a+bi) can be characterized by the formula:

$$b_n = \frac{i\left(e^{p_n/q_n} + e^{2\pi}\right)}{2\left(e^{2\pi} - e^{p_n/q_n}\right)} \tag{1}$$

Where p,q are integers and f(p,n) ≈ π from (2).

$$f(p,\, q) = 0.5 \times \frac{p}{q} \tag{2}$$

Calculated Ratios

Zero	p	q	f(p,q)
1	1186779	186779	3.1769
2	1186077	186077	3.1871
3	593929	93929	3.1616
4	118811	18811	3.1580
5	1188117	188117	3.1579
300	297129	47129	3.1523

Critical Line Dependence

$$\frac{a + b\,i}{(a + b\,i) - 1} = \frac{2\,b - i}{2\,b + i} \text{ where } a = \frac{1}{2}$$