

Euclid's inaccessible n -primes $2n+1$ and Peano's unattainable successor number $+1$

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Abstract:

Thales measured the height of the inaccessible pyramid and the distance of the unreachable ship from the harbor, demonstrating that anything that can be plotted on a plane can be measured; Euclid, with the product of known prime numbers, continually generates new primes and demonstrated that prime numbers are infinite; Peano, with the second of his five axioms, affirmed that for every natural number there exists a successor number $+1$. We will never be able to claim to have developed Euclid's inaccessible primes or Peano's unattainable number, but twin primes are two of the infinite primes, one of which is a successor number $+2$ of the other prime, and the sum of the two primes is always a number $6n$; by representing even numbers in the form $6n$ or $6n \pm 2$ and odd numbers in the form $6n \pm 1$ or $6n \pm 2 \pm 1$, we can demonstrate that Euclid's inaccessible primes and Peano's unattainable successor number exist. All prime numbers, all twin primes, all Mersenne primes which are the sum of numbers in double proportion and generate the even perfect numbers, all odd numbers $3n$ of the Collatz algorithm whose successor $+1$ is a power 2^n even which when halved is $2^{(n-1)}$ and ends at $2^0 = 1$ and all even numbers and all odd numbers which are the sum of 2 or 3 primes, all exist and, even if they will never be known, the final digit of the prime numbers and of the successor number which can be a prime or composite number is known.



Prime numbers, twin primes, Mersenne primes, the 3n primes of the Collatz algorithm, the even or odd Goldbach sums of 2 or 3 primes are infinite and will never be known, but their final digit is known.

Euclid in 300 BC and in the Elements, <https://mathcs.clarku.edu/~djoyce/java/elements/Euclid.html> proves that prime numbers are infinite assuming the opposite and, with $2n + 1$, always obtains a contradiction; adding 1 to the result of $2n$ which is the product of the known prime numbers, generates numbers that are not divisible by the n _prime factors indicated by $2n$ and they are the new prime numbers that demonstrate that there is no prime number greater than all but that there is the successor prime number; Euclid stated that among the infinite prime numbers there exist prime numbers such that a successor number +2 is also a prime number; these numbers, known as twin primes, are numbers distant 1 from half of their sum which is always an even number $6n$. In mathematics Peano (1858–1932) defined the natural numbers not with demonstrations like Euclid, but with assertions. In fact, he defined the natural numbers with 5 axioms and with the 2nd he affirmed that every natural number has its successor number +1.. Two twin n _primes are two odd numbers that are consecutive, are prime, are equidistant 1 from half their sum and their sum is a number $6n$. With a display and simulation we can represent all the infinite even numbers ≥ 4 in the form $6n$, $6n+2$ and $6n-2$ and all the odd numbers ≥ 3 , which are numbers preceding and succeeding the even numbers and which in the form $6n$ are $6n\pm 1$, $6n+2+1$ and $6n-2-1$; we can state that the inaccessible and infinite n _even numbers $6n$ and $6n\pm 2$ all have the even successor number +2 and the inaccessible and infinite n _odd numbers all have the odd successor number +2; in one column and among the n _odd numbers $6n-2-1$ there is the prime number 3 and its multiples, in two columns and among the odd numbers $6n-1$ and among the odd numbers $6n+1$ all the other prime numbers ≥ 5 . With this work we do not want to elaborate but to demonstrate that the combinatorics of

the infinite prime numbers known and unknown exists. how it is generated and how it is obtained and where and at what distance. in the three

- ._ a prime number: 3 prime numbers ≥ 5 with successors +6n and prime numbers ≥ 7 with successors +6n $6n-2-1, 6n-1$ e $6n+1$
- ._ twin primes: a prime number with n _prime successor +2 $6n-1 + 2 = 6n+1$
- ._ Mersenne primes: n _prime sum of numbers in double proportion that generates the even n _perfects $2^n n_{primo} - 1 = 6n+1$
- ._ an odd number $3n$ that can be generated with the Collatz algorithm and that +1 and halved ends at 1 $(3n+1)-1 = 2^n n_{pari} - 1 = 6n-2-1$
- ._ an odd n is the sum of the distances between previous odd n is the sum of three n _primes $6n-2-1, 6n-1$ and $6n+1$ or $n_{primo} + n_{pari} (= + 2 \text{ primi})$
- ._ an even n is twice a prime or the sum of two n primes equidistant from half their sum $6n \text{ o } 6n-2 \text{ o } 6n+2 = + 2 \text{ primi}$

Having established how they are generated, where to find them, and that there are accessible ones and inaccessible and unreachable ones, due to lack of space and time, we report the final figure, below cf, of all the n _even sums of two primes and n _odd sums of 2, (2 is n _odd_prime) and 3 primes which in

the infinite even numbers ≥ 4 , put in the form $6n$ or $6n\pm 2$ are:

- ._ twice the number of all prime numbers ≥ 2 ;
- ._ the sum of two prime numbers both in the form $6n+1$ or $6n-1$
- ._ the sum of an n _prime in the form $6n+1$ and an n _prime in the form $6n-1$

The infinite odd numbers ≥ 3 are the numbers preceding or succeeding all even numbers in the form $6n$ and $6n\pm 2$; all odd numbers ≥ 7 are the sum of an even number and a prime number; one of the twin primes is the sum of 2 with the other twin prime.

the cf of the infinite n _even ≥ 4 in the form

①	$6n-2$	$6n$	$6n+2$
↓+6 →+2	4	6	8
↓+6 →+2	10	12	14
↓+6 →+2	16	18	20
↓+6 →+2	22	24	26
↓+6 →+2	28	30	32

the cf of the infinite n _odd ≥ 3 in the form

②	$3n$ $6n-2-1$	$6n-1$ $6n-2+1$	$6n+1$ $6n+2-1$
↓+6 →+2	3	5	7
↓+6 →+2	9	11	13
↓+6 →+2	15	17	19
↓+6 →+2	21	23	25
↓+6 →+2	27	29	31

① = infinite even numbers ② = infinite odd numbers

$6n-2-1$	$6n-2$ ①	$6n-1$ ②	$6n$ ①	$6n+1$ ②	$6n+2$ ①	$6n+2+1$ = ②
3 ed ∞		n_{primo}		n_{primo}		$6n-2-1 \geq 9$
$6n+2+1 \geq 9$			0	1	2	
3	4	5	6	7	8	9
9	10	11	12	13	14	15
15	16	17	18	19	20	21
21	22	23	24	25	26	27
27	28	29	30	31	32	33

n _primo cf x + cf x_6n -1 + cf y_6n -1 = cf z_n_pari 6n -2

n _primo cf y =	cf 9+6 solo 5	cf 5+6 dal	cf 1+6 dal	cf 7+6 dal	cf 3+6 dal
n _pari cf z	5	11	17	23	29
cf 9+6 solo il	5	0	6	2	8
cf 5+6 →1	11	6	2	8	4
cf 1+6 →7	17	2	8	4	0
cf 7+6 →3	23	8	4	0	6
cf 3+6 →9	29	4	0	6	2

① la cf degli ∞ n _pari in forma $6n-2$ è la cf della somma di due primi $6n-1$

the cf of the n _even ∞ in the form $6n-2$, $z=x+y$ is the cf of the double of an n _prime $6n-1$ or of the sum of two n _primes in the form $6n-1$ that are equidistant by $\frac{1}{2}$ of their sum

Prime numbers, twin primes, Mersenne primes, the 3n primes of the Collatz algorithm, the even or odd Goldbach sums of 2 or 3 primes are infinite and will never be known, but their final digit is known.

33	34	35	36	37	38	39
39	40	41	42	43	44	45
45	46	47	48	49	50	51
51	52	53	54	55	56	57
57	58	59	60	61	62	63
63	64	65	66	67	68	69
69	70	71	72	73	74	75
75	76	77	78	79	80	81
81	82	83	84	85	86	87
87	88	89	90	91	92	93
93	94	95	96	97	98	99
99	100	101	102	103	104	105
105	106	107	108	109	110	111
111	112	113	114	115	116	117
117	118	119	120	121	122	123
123	124	125	126	127	128	129
129	130	131	132	133	134	135
$6n-2 + 6n-1 = 6n-2-1$	$6n-2$	$6n + 6n-1 = 6n-1$	$6n$	$6n + 6n+1 = 6n+1$	$6n+2$	$6n+2 + 6n+1 = 6n+2+1$
$\infty 3n$	Collatz $\infty 3n+1 = 2^n n_{\text{pari}}$	$\infty 3n+2$	Gemelli = $\infty \frac{1}{2} (6n-1)$	∞M_p	$2^n n_{\text{primi}}$	$\infty 3n$

$n_{\text{primo}} \text{ cf } x +$	$\text{cf } x_{6n+1} + \text{cf } y_{6n+1} = \text{cf } z_{n_{\text{pari}}} 6n+2$					the cf of the $n_{\text{even}} \infty$ in the form $6n+2$, $z=x+y$ is the cf of the double of an $n_{\text{prime}} 6n+1$ or of the sum of two n_{primes} in the form $6n+1$ that are equidistant by $\frac{1}{2}$ of their sum
$n_{\text{primo}} \text{ cf } y =$	$\text{cf } 1+6 \text{ dal}$	$\text{cf } 7+6 \text{ dal}$	$\text{cf } 3+6 \text{ dal}$	$\text{cf } 9+6 \text{ solo } 5$	$\text{cf } 5+6 \text{ dal}$	
$n_{\text{pari}} \text{ cf } z$	7	13	19	25	31	
$\text{cf } 1+6 \rightarrow 7$	7	4	0	6	2	8
$\text{cf } 7+6 \rightarrow 3$	13	0	6	2	8	4
$\text{cf } 3+6 \rightarrow 9$	19	6	2	8	4	0
$\text{cf } 9+6 \text{ solo il}$	5	2	8	4	0	6
$\text{cf } 5+6 \rightarrow 1$	11	8	4	0	6	2

① la cf degli ∞n_{pari} in forma $6n+2$ è la cf della somma di due primi $6n+1$

la cf della somma degli infiniti numeri primi gemelli è 0, 4 o 6

$n_{\text{primo}} \text{ cf } x +$	$\text{cf } x_{6n+1} + \text{cf } y_{6n-1} = \text{cf } z_{n_{\text{pari}}} 6n$					the cf of the $n_{\text{even}} \infty$ in the form $6n$, $z=x+y$ is the cf of the double of 3 or of the sum of a number in the form $6n+1$ and the other $6n-1$ which are equidistant from $\frac{1}{2}$ of their sum
$n_{\text{primo}} \text{ cf } y =$	$\text{cf } 1+6 \text{ dal}$	$\text{cf } 7+6 \text{ dal}$	$\text{cf } 3+6 \text{ dal}$	$\text{cf } 9+6 \text{ solo } 5$	$\text{cf } 5+6 \text{ dal}$	
$n_{\text{pari}} \text{ cf } z$	7	13	19	25	31	
$\text{cf } 9+6 \text{ solo il}$	5	$2 \frac{1}{2} 6 \pm 1 = 5$	$8 \frac{1}{2} 4 \pm 1 = 5$	4	$0 \geq \frac{1}{2} 60n \pm 1$	6
$\text{cf } 5+6 \rightarrow 1$	11	$8 \frac{1}{2} 4 \pm 1 = 5$	4	$0 \geq \frac{1}{2} 60n \pm 1$	6	$2 \frac{1}{2} 6 \pm 1 = 5$
$\text{cf } 1+6 \rightarrow 7$	17	4	$0 \geq \frac{1}{2} 60n \pm 1$	6	$2 \frac{1}{2} 6 \pm 1 = 5$	$8 \frac{1}{2} 4 \pm 1 = 5$
$\text{cf } 7+6 \rightarrow 3$	23	$0 \geq \frac{1}{2} 60n \pm 1$	6	$2 \frac{1}{2} 6 \pm 1 = 5$	$8 \frac{1}{2} 4 \pm 1 = 5$	4
$\text{cf } 3+6 \rightarrow 9$	29	6	$2 \frac{1}{2} 6 \pm 1 = 5$	$8 \frac{1}{2} 4 \pm 1 = 5$	4	$0 \geq \frac{1}{2} 60n \pm 1$

① the cf of the $n_{\text{even}} \infty$ in the form $6n$ is the cf of the sum of two equidistant numbers ≥ 1 from $6n$ which is always half of the sum of $6n+1 + 6n-1$

$6n$	$6n-1^a$	$6n-1^*2$	$6n-1^b$		$6n-2$
①	②	①	②		①
0					
6	5	10	5	Combinatorics of the infinite $6n-1a$ with the infinite $6n-1b \rightarrow$	10
12	11	22	11		22
18	17	34	17		34
24	23	46	23		46
30	29	58	29		58
36	35	70	35		70
42	41	82	41		82
48	47	94	47		94
54	53	106	53		106
60	59	118	59		118
66	65	130	65		130
72	71	142	71		142

successore +12

A →

The even numbers in the form $6n-2$ are infinite and are:

- _ the double of all the prime numbers in the form $6n-1$ or the result of the combinatorics of two of the infinite prime numbers in the form $6n-1$;
 - _ among the numbers in the form $6n-1$ there are the prime numbers ≥ 5 and infinite prime successors $+6n$ in the form $6n-1$, the even numbers in the form $6n-2$ are also: double the infinite prime numbers in the form $6n-1$ and the sum of the combinatorics of the infinite prime numbers, known and unknown, in the infinite even numbers are the double of an $n_{\text{prime}} 6n-1$ but also the sum of two n_{primes} that are equidistant from $\frac{1}{2}$ of their sum; the same $n_{\text{even}} 6n-2$ is the sum of n_{primes} distant ≥ 0 equidistant from $\frac{1}{2}$ of $6n-2$
- double and combinatorics of prime numbers $\updownarrow 6n-1$**

Prime numbers, twin primes, Mersenne primes, the 3n primes of the Collatz algorithm, the even or odd Goldbach sums of 2 or 3 primes are infinite and will never be known, but their final digit is known.

6n	6n+1 ^a	6n-1*2	6n+1 ^b		6n+2
①	②	①	②		①
	n_primi		n_primi		
0					
6	7	14	7	→	14
12	13	26	13	Combinatorics of the infinite 6n+1a with the infinite 6n+1b	26
18	19	38	19		38
24	25	50	25		50
30	31	62	31		62
36	37	74	37		74
42	43	86	43		86
48	49	98	49		98
54	55	110	55		110
60	61	122	61	→	122
66	67	134	67		134
72	73	146	73		146
78	79	158	79		158
84	85	170	85		170

successor e +12

B →

The even numbers in the form 6n+2 are infinite and are:

- the double of all odd numbers in the form 6n+1 or the result of the combinatorics of two of the infinite odd numbers in the form 6n+1;
- among the numbers in the form 6n+1 there are the prime numbers ≥ 7 and infinite prime successors +6n in the form 6n+1, the even numbers in the form 6n+2 are also: double the infinite prime numbers in the form 6n+1 and the sum of the combinatorics of the infinite prime numbers, known and unknown, in the form 6n+1.

the infinite even numbers are the double of an n_prime 6n+1 but also the sum of two n_primes 6n+1 that are equidistant from ½ of their sum; the same n_even 6n-2 is the sum of n_primes distant ≥ 0 equidistant from ½ of 6n-2

double and combinatorics of prime numbers ⇕ 6n+1

6n	6n-1 ^a	6n-1*2	6n+1 ^b		6n
①	②	①	②		①
	n_primi		n_primi		
0					
6	5	12	7	→	12
12	11	24	13	Combinatorics of the infinite 6n+1a with the infinite 6n+1b	24
18	17	36	19		36
24	23	48	25		48
30	29	60	31		60
36	35	72	37		72
42	41	84	43		84
48	47	96	49		96
54	53	108	55		108
60	59	120	61	→	120
66	65	132	67		132
72	71	144	73		144
78	77	156	79		156
84	83	168	85		168

successor e +12

C →

Even numbers in the form 6n are the sum of the infinite combinatorics of two numbers that are ≥ 2n apart and equidistant from half their sum; one of the two numbers is one of the infinite numbers in the form 6n-1, the other one of the infinite numbers in the form 6n+1; their sum is a number 6n generated by:

- infinite pairs of "twin" primes 6n-1 + 6n+1, 2 distant from each other and 1 equidistant from half of their sum which is 6n
- infinite pairs of first "cousins" 6n-1 + 6n+1 distant from each other 4 and equidistant 2 from half of their sum which is 6n
- infinite coppie di primi sexy 6n-1 + 6n+1 tra loro distanti 6 ed equidistanti 3 dalla metà della loro somma che è 6n
- infinite pairs of n_primes 6n-1 with n_primes 6n+1 distant from each other ≥ 8 and equidistant, ½ of their distance, from half of their sum

no double but combinatorics of prime numbers 6n-1 ↔ 6n+1

Prime numbers, twin primes, Mersenne primes, the 3n primes of the Collatz algorithm, the even or odd Goldbach sums of 2 or 3 primes are infinite and will never be known, but their final digit is known.

6n	6n-2-1 e 6n-2-2 due primi	6n-1 2+6n-2-1	6n+1 2+6n-1	3 + 6n-1 ① 6n+2	3 + 6n+1 ① 6n-2	successore +6 4
6	2			8	10	
6	3	5	7	14	16	
12	9	11	13	20	22	
18	15	17	19	26	28	
24	21	23	25	32	34	
30	27	29	31	38	40	
36	33	35	37	44	46	
42	39	41	43	50	52	
48	45	47	49	56	58	
54	51	53	55	62	64	
60	57	59	61	68	70	
66	63	65	67	74	76	
72	69	71	73	80	82	
78	75	77	79	86	88	
84	81	83	85			

the odd cf ② of the sum of 2 with n_6n-1 or 2 with n_6n+1 repeats

The double of the prime number 2 is 4 and in the form 6n = 6n-2; the results of the sum of the prime number 2 with the prime number 3 and infinitely many odd numbers in the form 6n-2-1 = 6n-2-1+2 = 6n+1 = 5 and infinitely many successors +6; the results of the sum of the prime number 2 with prime and odd numbers in the form 6n-1 = 6n-1+2 = 6n+1 = 7 and infinitely many successors +6; the twin prime numbers are: the prime numbers 6n+1 -2 = prime numbers 6n-1 with cf successor +30.

the n_even cf ① of the sum of 3 with n_6n-1 or 3 with n_6n+1 repeats

The double of the prime number 3 is 6 and in the form 6n is 6n+1; the results of the sum of the prime number 3 with the infinite prime numbers and odd numbers in the form 6n-1 are even numbers in the form 6n-2 = 10 with successors +6; the results of the sum of the prime number 3 with the infinite prime numbers and odd numbers in the form 6n+1 are even numbers in the form 6n+2= 8 with successors +6

Twin primes: we will never be able to compute all the prime numbers that, when added to 2, are a new prime number because there are infinite prime numbers and there exists a prime number that succeeds the nth prime number in 6n-1; in the numbers in the format 6n+1 there are all the numbers that are the result of the sum of the prime numbers 6n-1 +2; the prime numbers are not multiples of the primes ≤ the square root of the given number, the multiples will not eliminate all the infinite pairs of n_odd successors + 2 which are the twin primes

Mersenne primes that generate all even perfect numbers: we will never be able to process all the prime numbers that are the result of a 2^n_prime-1 because the prime numbers are infinite and the prime number successor of all prime numbers exists; in the numbers in the format 6n+1 there are all the results of the 2^n_odd-1 among which there are the results of all the infinite 2^n_prime-1 that * 2^(n_prime-1) = n_perfecteven.

Collatz's 3n: we will never be able to use the 3n+1 algorithm to find all the odd numbers 3n that, when added to 1, are the result of a power of 2 whose result, when halved, is 2^(n-1) and the nth halving is 2^0 = 1; among the infinite even numbers in the 6n-2 format there are the results of all the infinite 2^n_even numbers that can be generated with the 3n+1 algorithm because the infinite 2^n_even numbers are 6n-2; all the infinite 2^n_even numbers -1 are 6n-2-1 and are the infinite n_odd numbers 3n that can be generated with the Collatz algorithm which, +1 is a 2^n_even 6n-2, which when halved ends at 2^0 =1; the smallest 2^n_even-1 is 2^2-1 from which the smallest 3n which is 3 and which +1 = 4 which is the smallest 2^2 whose 1/2 is 2^0 = 2^0 =1.

in A, B, C and D the infinite even numbers which are the sum of two different prime numbers or the double of the same prime number, known or unknown; the ∞ odd numbers ≥ 7 are the sum of a prime number plus an even number which in A, B, C and D is the sum of two prime numbers.

Bibliographic and website references:
a) <https://vixra.org/pdf/2206.0084v1.pdf> b) <https://vixra.org/pdf/2210.0090v1.pdf> c) <https://vixra.org/abs/2202.0145>
d) <https://vixra.org/abs/2212.0170> e) <https://www.dimostriamogoldbach.it/wp-content/contributi/it/G.%20Di%20Savino>