Riemann Hypothesis [Proving] Obfuscatedly Easy

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ABSTRACT

This paper proposes a 'naïve,' 'three-line' demonstration for RH building on a functional-equation reduction. It is suggested, *inter alia*, how prematurely restricting an exposition to the conjectured critical band *a priori* may usher in some paradoxic singularities that unduly restrict the complex candidate (imaginary extension) domain, albeit without questioning the real core. It remains to be judged whether the 'gray area,' as implied or straddled, qualifies as a 'constructive' completion of the RH around its frontier of inference.

Keywords: RH, functional reduction, weak/ordinal equivalence

Initial Exposition

We begin with an open-form representation of the zeta function as in Riemann (1859).

$$\pi^{-\frac{s}{2}} * \Gamma\left(\frac{s}{2}\right) * \zeta(s) = \pi^{-(1-s)/2} * \Gamma\left(\frac{1-s}{2}\right) * \zeta(1-s) \quad (Z0)$$

It is henceforth treated as a *functional* (difference, recurrent) *equation*, with (Z0) taking on a Phi-form as in (1), then solved for zeta taken, in turn, to a zero value. (Arguably, one would arrive at a somewhat arcane construct such as a higher-order Gamma functional, were it for a direct zeta reduction ventured.)

Demonstration

$$\Phi(s) = \Phi(1-s) = \Phi(0) * 1^{\frac{s}{2s-1}} = \Phi(0) * e^{\frac{2s}{2s-1} * in\pi} = -\Phi(0) * e^{\frac{in\pi}{2s-1}} \qquad (1)$$

$$\zeta(\bar{s}) = \Phi(\bar{s}) * \pi^{\frac{\bar{s}}{2}} * \frac{1}{\Gamma\left(\frac{\bar{s}}{2}\right)} \equiv -\Phi(0) * e^{\frac{in\pi}{2\bar{s}-1}} * T \equiv 0 \quad (1a)$$

$$\Phi(0) = 1 * \Gamma(0) * \zeta(0) = -\infty * \begin{cases} \infty \\ -1/2 \end{cases} \qquad (1b)$$

[1] Owing a debt of gratitude to Profs. Mearsheimer & Sachs for their principled stances & well-thought, all-grounded points informing an otherwise shallow magpie mainstream.

$$\Phi(1)=\frac{\sqrt{\pi}}{\sqrt{\pi}}*\zeta(1)=\begin{cases}\infty\\ \gamma\end{cases}\quad(1b')$$

Whatever the *Phi*-initial may imply (which can and will be endogenized in [1b]-[1b*'*]), as (1a) suggests aside from the trivial *Gamma* zeros, and those less so as per the *T* factor aggregate, the IFF to ensure a zero zeta value would be a *negative infinity* in the exponent power. Suffice it to generalize a [lower neighborhood of] zero as $-2n\pi i$, which befits [peer/within] powers though not [across/beyond] levels, and the candidate argument pick—alongside a *constructive* specification for *t=Im(s)*—obtains as in (2):

$$2s-1\sim 0^{-}=-2n\pi i\quad\rightarrow\bar{s}=\frac{1}{2}-n\pi i\quad QED\ (2)$$

Now, one somewhat more rigorous way of approaching (1) would be to embark on a complex reciprocal, yet in a *lax/generalized* rather than standard & prior-restricted fashion. In effect,

$$\exists\varepsilon,\delta:\frac{\delta}{\varepsilon}=\frac{2t}{2a-1}\ s.t.\ \frac{1}{2\bar{s}-1}\equiv\frac{1}{(2a-1)+2ti}\equiv\frac{1}{\hat{a}+\hat{t}i}=\frac{1}{\hat{a}}*(\varepsilon-\delta i)$$
$$=\frac{\varepsilon}{\hat{a}}*\left(1-\frac{\hat{t}}{\hat{a}}i\right)\sim\begin{cases}-\varepsilon\infty-\hat{t}\infty^{2}i\ IFF\ \hat{a}\equiv 0^{-}\\ -\varepsilon\hat{t}\infty^{2}i\ IFF\ \hat{a}\equiv 0\end{cases}\quad(3)$$

Should a –n period apply (in the end, $1=1^{\pm}$) just like in (2), (1a) would appear as follows:

$$-\Phi(0)*e^{\frac{in\pi}{2\bar{s}-1}}*T=-\Phi(0)*T*\begin{cases}e^{-n\pi*\infty^{2}}\\ e^{n\pi\infty i-n\pi*\infty^{2}}=0^{2}\end{cases}=0\quad QED\ \ (3a)$$

One other way of saying this would be to surmise that, *n* tending to infinity, the sign-reversal of $\exp(n\pi i)$ could be seen as resulting in 0. Or, $0^{i\varphi}*0=0\ though\ 0^{i}\neq 0$. Under at least some of these scenarios, a zero results invariably, or *invariantly* wrt any conventions, e.g. be this 1+1+1…=-1/2 versus infinity or, when centered around *Phi(1)* rather than *Phi(0)* as an initial condition implying zeta as a matter of an infinite *1/N* series, divergent or presumed at .5772…(Cauchy-Euler-Mascheroni, [*1b'*]). Please note that (1) could readily be generalized over an arbitrary *interim* lag/lead *k*:

$$\Phi(s)=\Phi(k)*e^{\frac{2(s-k)}{2s-1}*in\pi}=-\Phi(k)*e^{\frac{1-2k}{2s-1}*in\pi}\quad(1c)$$

For instance, per *k=1*, one need not impose any assumptions on the sign of *n*.

Afterthoughts

Prior simplifications may usher in undue paradoxes. For starters, (2) may subsist on a zero imaginary expansion—which may not violate RH. For the same token, if we were to *start* with

RH, or test *Re(s)=1/2* as a *sufficient* condition as per (3), that too could have been prone to a degenerate, all-real case:

$$\frac{1}{\hat{a}+\hat{t}i} = -\frac{i}{\hat{t}} \rightarrow -\infty i \;\; IFF \; t=0 \quad (3d)$$

Strictly speaking, our relaxed generalization of complex inverting would follow the standardized reduction or calibrating path:

$$\frac{1}{\hat{a}+\hat{t}i} = \frac{1}{\hat{a}^2+\hat{t}^2} * (\hat{a}-\hat{t}i) = -\frac{i}{\hat{t}} \qquad (3e)$$

While this is fully in line with (3d) above, nor is it structurally (ordinally) violated by the (epsilon, delta) notation as in (3), which avoids prior RH style restriction so as to substantiate, or induce it *a posteriori*. Yet, if one were to try on *t-hat* equal 0 as above, the ordinal implications could prove somewhat fuller-blown:

$$\frac{1}{\hat{a}+\hat{t}i} \rightarrow \frac{0}{20^2} - \frac{0}{20^2} \sim \frac{\infty}{2}(1-i) \quad (3f)$$

This could either be a finitely inconclusive real value or a complex infinity on the order of (3d) and (3e) alike, with complexity ironically emerging *a posteriori* from *prior* denial, i.e. *Im(s)=0*.

A similar phenomenon shines through in scrutinizing the *pi^s* term: Though representable additively along the (*epsilon*, *delta*) lines, these would suggest the *Im*, or *delta*, slope being effectively imaginary, for the whole thing to net out to zero; however, *pi^i* is known to have taken on, or been defined as, a rather specific value, with the *Im(pi^i)* leaving no room for embedded or hidden complexity.

On the other hand, while embarking on this and the (2) notation, one might stumble into peculiar real-case extensions around the $0^{\pm} \sim \pm 2n\pi i$. More so when it comes to strengthened/powered zero values acting to offset any reasonable infinity/convention-related indeterminacies.

$$2s-1 \sim 0^2 \sim (\pm 2n\pi i)^2 \;\; \leftrightarrow \;\; \bar{s} = \frac{1}{2} - 4n^2\pi^2 \quad (2a)$$

Whilst any such solution pertains to the real domain phenomenologically, 1/2 now refers to a *non-transcendental* kernel. For that matter, the selfsame *pi^s* factor posits:

$$\pi^{s/2} \sim 0 \;\; \leftrightarrow \frac{s}{2} \sim \frac{1}{0^-} \sim \frac{1}{-2n\pi i} \;\; \leftrightarrow \;\; \bar{s} = \frac{i}{n\pi} \quad (2b)$$

Seemingly imaginary, (2b) straddles *Im(s)* in (2) & *1/(2s-1)* per (3d-3e). Ironically, with *n* taken to 0, (2a) & (2b) could be rethought in *indefinite reduplicating* terms, respectively as *additive* versus *multiplicative*:

$$\bar{s} = \frac{1}{2} + (-1)^k 4^k n^2 \pi^{2k} \quad (2a')$$

$$\bar{s} = \frac{1}{2^{k-1}} * \frac{i^{1+3k}}{n\pi^k} \quad (2b')$$

Outlook

In passing, while no explicit mention of *entire (meromorphic) functions* is being made under the intentionally 'naïve,' if parsimonious, treatment, such possibilities are implied & reserved as future exercise (less-expected applications invocable). Not least, some peculiar parallels, as revealed under unrelated approaches, are forthcoming.

References

Riemann, Bernhard. (1859). Ueber die Anzahl der Primzahlen unter einer gegebenen Groesse. *Monatsberichte der Koeniglich Preussischen Akademie der Wissenschaften zu Berlin*, November.