Supernumber Theory and Quantum Superposition Phenomena

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Abstract:

Hyperreal numbers are a field encompassing real numbers, infinitesimals, and infinities. In the hyperreal number system, infinitesimals are not equal to zero, and operations can be performed on infinitesimals and infinities.

Based on hyperreal number theory, this paper discovers that multi-level radicals that cannot be simplified have no definite value, varying in size; these can be called quantum numbers or inaccurate numbers. A special type of multi-level radical can exhibit quantum superposition phenomena similar to those in the physical world. This allows the hyperreal number system to be extended to a supernumber system, a field encompassing real numbers, infinitesimals, infinities, and inaccurate numbers.

Confirming the existence and properties of quantum numbers or inaccurate numbers and classifying them as supernumbers provides a new perspective. Within this framework, the negation proves the Riemann hypothesis.

Keywords:

Supernumber theory, Inaccurate numbers, Quantum numbers, Quantum superposition phenomenon, Riemann hypothesis

Hyperreal number theory is a modern mathematical theory system developed within the last hundred years. It was founded in 1960 by American mathematical logician A. Robinson. Hyperreal numbers are a field that includes real numbers, infinitesimals, and infinities[1]. In the hyperreal number system, infinitesimals are not equal to zero, and operations can be performed on infinitesimals and infinities.

Based on hyperreal number theory, this paper finds that multi-level radical numbers that cannot be simplified do not have a definite value, and are sometimes large and sometimes small, and are quantum numbers or inaccurate numbers[2][3]. Among them, special kinds of multi-level radical numbers can exhibit quantum superposition phenomena similar to quantum in the physical world[3]. Thus, the hyperreal number system can be extended to a super-number system, which is a field that includes real numbers, infinitesimals, infinities, and inaccurate numbers.

1. Examples of numerical experiments

Assume:

$$m = \frac{3}{4}\sqrt[3]{-25236 + 4\sqrt{41629613}} - \frac{3}{4}\sqrt[3]{25236 + 4\sqrt{41629613}} - \frac{63}{4}$$

$$t = \frac{30}{4} + \frac{1}{4}\frac{(4\,m + 189)\sqrt{378 + 8m} - \sqrt{-128m^3 - 6048m^2 + (14688\,m + 694008)\sqrt{378 + 8m} + 285768m + 13502538}}{189 + 4m}$$

$$b = \sqrt[9]{9477 + \sqrt{89813529 - t^9}} + \sqrt[9]{9477 - \sqrt{89813529 - t^9}}$$

$$a = \frac{1}{4}\sqrt[3]{-463644 + 2916\sqrt{25781}} - \frac{1}{4}\sqrt[3]{463644 + 2916\sqrt{25781}} - \frac{63}{4}$$

$$u = \frac{15}{2} + \frac{1}{4}\frac{(4\,a + 189)\sqrt{378 + 8a} - \sqrt{(14688\,a + 694008)\sqrt{378 + 8a} - 128a^3 - 6048a^2 + 285768a + 13502538}}{189 + 4a}$$

$$c = \sqrt[9]{6561 + \sqrt{43046721 - u^9}} + \sqrt[9]{6561 - \sqrt{43046721 - u^9}}$$

$$e = \sqrt[3]{-\frac{170371}{16} + \frac{1}{3888}\sqrt{1792632088533909}} - \sqrt[3]{\frac{170371}{16} + \frac{1}{3888}\sqrt{1792632088533909}} - \frac{63}{4}$$

$$v = \frac{15}{2} + \frac{1}{4}\frac{(4\,e + 189)\sqrt{378 + 8e} - \sqrt{(14688\,e + 694008)\sqrt{378 + 8e} - 128e^3 - 6048e^2 + 285768e + 13502538}}{189 + 4e}$$

$$d = \sqrt[9]{9480 + \sqrt{89870400 - v^9}} + \sqrt[9]{9480 - \sqrt{89870400 - v^9}}$$

$$n=\frac{1}{3}\sqrt[3]{-2220+\sqrt{5155381}}-\frac{1}{3}\sqrt[3]{2220+\sqrt{5155381}}-\frac{28}{9}$$

$$h=\frac{10}{3}+\frac{1}{18}\ \frac{(9n+84)\sqrt{168+18n}-\sqrt{-1458n^3-13608n^2+(9792n+91392)\sqrt{168+18n}+127008n+1185408}}{28+3n}$$

$$Z=\frac{\sqrt[9]{247+\sqrt{61009-h^9}}+\sqrt[9]{247-\sqrt{61009-h^9}}-2}{\sqrt[9]{9477+\sqrt{89813529-t^9}}+\sqrt[9]{9477-\sqrt{89813529-t^9}}-3}$$

The calculation results for different floating-point numbers are shown in Table 1 and Table 2.

Table 1. Numerical results of calculations for numbers c, b, and d with different floating-point numbers.

floating-point bits	number	values calculated with different floating-point numbers	size sorting
20	c	2.9999999999755191029	1
	b	2.9874462303672778244	3
	d	2.9875512926586440631	2
25	c	2.999999999999999725360181	2
	b	2.999999993416757061156282	3
	d	3.000000019522847836846675	1
27	c	3.00000000000000000002082201	3
	b	3.00000000062160912158934078	1
	d	3.00000000025821781065876453	2
30	c	2.999999999999999999999693848224	3
	b	3.00000000000019193490985194345	1
	d	3.00000000000007014894418886736	2
35	c	2.9999999999999999999999999930410922	2
	b	2.9999999999999999980155680532155316	3
	d	3.0000000000000000059804360522162267	1

Table 2. Numerical results of number Z with different floating-point bit depths.

floating-point bits	calculated values
21	-21.9429087298718998705
22	-4.284823725411146851439
23	3.4162294585227679825466
24	-0.314468726752117288753167
25	51.29214950619176697363662
26	-2.2822204892585746002806458
27	1.47857411967326984415973813
28	2.631798468723020563965562491
29	0.24381752023828507818009397867
30	11.6843224176921837525605096455
31	10.73433128310351709125692223886

2. Discussion and Conclusion of Calculated Results

The numerical calculations above show that, within the framework of the hyperreal number system, numbers c, b, d, and z do not have definite values; they only exhibit fixed values when calculated using a selected number of floating-point bits. The calculated values of numbers c, b, and d are sometimes greater than the integer 3 and sometimes less than 3. The sorting results of the values of numbers c, b, and d calculated with different floating-point bits vary with the number of floating-point bits. Therefore, it is impossible to determine the relative sizes of numbers c, b, and d; that is, these numbers do not possess the ordered nature of the real number system and the hyperreal number system, and cannot be sorted.

The values of numbers c, b, d, and z depend on the precision chosen for their calculation, and these numbers cannot be represented by series. Because a convergent series approaches a certain limit value in a single direction, while the calculated values of the above numbers with different floating-point bits jump around a certain integer value, this indicates a need to further expand the number system.

3. Practical Applications

Proof of the negation of the Riemann Hypothesis.

3.1 Calculation method for nontrivial zeros of Riemann zeta function ζ(s)

The Riemann zeta function ζ(s) is a series expression [4]

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^{s}}$$

where R(s) > 1.
The analytic continuation in the complex plane,

$$\zeta(s)=\frac{\Gamma(1-s)}{2\pi i}\int_{-\infty}^{+\infty}\frac{-z^{s}}{e^{z}-1}\frac{dz}{z}$$

Riemann used integral expressions to prove that the Riemann zeta function satisfies the following algebraic relation:

$$\zeta(s)=2^{s}\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)$$

Riemann introduced an auxiliary function ξ(s),

$$\xi(s)=\Gamma(\frac{s}{2}+1)(s\text{-}1)\pi^{-\frac{s}{2}}\zeta(s)$$

Its zeros coincide with the nontrivial zeros of the Riemann zeta function. This leads to the Riemann-Siegel formula.

Let s = 1/2 + it (t is a real number), and prove it using the definition of ξ(s) [4]

$$\xi(s)=[e^{\mathrm{Re}\ln\Gamma(\frac{s}{2})}\pi^{-\frac{1}{4}}\frac{(-t^{2}-\frac{1}{4})}{2}][e^{i\mathrm{Im}\ln\Gamma(\frac{s}{2})}\pi^{-\frac{it}{2}}\zeta(s)]$$

Clearly, since the expression within the first square brackets is always negative, it can be ignored when calculating the zeros of ξ(s). This shows that to determine the nontrivial zeros of the Riemann zeta function, it is only necessary to study the expression within the second square brackets. Let Z(t) denote this expression, i.e.

$$Z(t)=e^{iIm\ln\Gamma(\frac{s}{2})}\pi^{-\frac{it}{2}}\zeta(\frac{1}{2}+it)]$$

Thus, the nontrivial zeros of the Riemann zeta function are reduced to the study of the zeros of Z(t).

The Riemann-Siegel formula is the asymptotic expansion of Z(t), specifically expressed as [4]

$$Z(t)=2\sum_{n^2<\frac{t}{2\pi}} n^{-\frac{1}{2}}\cos(\theta(t)-t\ln(n))+R(t) \quad (1)$$

where:

$$\theta(t)=\frac{t}{2}ln(\frac{t}{2\pi})-\frac{t}{2}-\frac{\pi}{8}+\frac{1}{48t}+\frac{7}{5760t^3}+...,... \quad (2)$$

$$R(t)\sim(-1)^{N-1}(\frac{t}{2\pi})^{-\frac{1}{4}}[C_0+C_1(\frac{t}{2\pi})^{-\frac{1}{2}}+C_2(\frac{t}{2\pi})^{-\frac{2}{2}}+C_3(\frac{t}{2\pi})^{-\frac{3}{2}}+C_4(\frac{t}{2\pi})^{-\frac{4}{2}}] \quad (3)$$

R(t) in the formula is called the remainder, where N is the integer part of $(\frac{t}{2\pi})^{\frac{1}{2}}$, and the coefficients of each term in R(t) are respectively

$$C_0=\Psi(p)\equiv\frac{cos(2\pi(p^2-p-\frac{1}{16}))}{cos(2\pi p)}$$

$$C_1=-\frac{1}{2^5\times3\times\pi^2}\Psi^{(3)}(p)$$

$$C_2=\frac{1}{2^{11}\times3^2\times\pi^4}\Psi^{(6)}(p)+\frac{1}{2^6\times\pi^2}\Psi^{(2)}(p)$$

$$C_3=-\frac{1}{2^{16}\times3^4\times\pi^6}\Psi^{(9)}(p)+\frac{1}{2^8\times3\times5\times\pi^4}\Psi^{(5)}(p)-\frac{1}{2^6\times\pi^2}\Psi^{(1)}(p)$$

$$C_4=\frac{1}{2^{23}\times3^5\times\pi^8}\Psi^{(12)}(p)+\frac{11}{2^{17}\times3^2\times5\times\pi^6}\Psi^{(8)}(p)+\frac{19}{2^{13}\times3\times\pi^4}\Psi^{(4)}(p)+\frac{1}{2^7\times\pi^2}\Psi(p)$$

Where p is the fractional part of $(\frac{t}{2\pi})^{\frac{1}{2}}$, and $\Psi^{(n)}(p)$ is the n-th derivative of Ψ(p).

3.2 Proof of the Rejection of the Riemann Hypothesis

The Riemann Hypothesis states that when s = (1/2 + it), (t is a real number), the value calculated by function (1) is zero, and s = (1/2 + it) is a non-trivial zero.

Because the formula (3) for calculating the remainder R(t) contains p and $C_1(\frac{t}{2\pi})^{-\frac{1}{2}}$,

$C_2(\frac{t}{2\pi})^{-\frac{2}{2}}$, $C_3(\frac{t}{2\pi})^{-\frac{3}{2}}$, and $C_4(\frac{t}{2\pi})^{-\frac{4}{2}}$ etc., there are numerous roots of the transcendental number π, and function (1) contains a large number of trigonometric function operations involving irrational numbers, which are similar to multi-level radical numbers. The test results in this paper show that such numbers do not have definite values and are quantum numbers or inaccurate numbers. The result of calculations involving multiple such numbers that cannot cancel each other out is definitely not zero. As a result, the value calculated by function (1) cannot be zero regardless of the real number t. That is, for any real number t, s = (1/2 + it) is not a non-trivial zero.

In summary, it can be proven that there are no "nontrivial zeros" and that there are no roots of polynomials whose real part has a value of 1/2. This negates the Riemann hypothesis, or proves that the Riemann hypothesis does not hold within the framework of supernumber systems.

4 Discussions

The only correct way to prove whether two numbers are exactly the same is to simplify them to their simplest form and then compare them. If the simplest forms are exactly the same, then they are the same number. Real numbers under the classical definition have density, order and continuity, and there is a unique one-to-one correspondence with points on the number line.

It is generally believed that numbers on the number line are real numbers, while infinitesimals and infinity introduced in hyperreal numbers are not fixed "points" on the number line. The test results in this paper prove that it is impossible to determine whether numbers c, b and d are greater than or less than the integer 3, and they do not correspond to fixed "points" on the number line. Therefore, such numbers can be named quantum numbers [3]. We give the following definition of quantum numbers:

Quantum numbers: a class of multi-level radical numbers that do not have ordering.

Quantum numbers have the same phenomenon as quanta in the physical world, both of which have uncertainty. Quantum numbers do not have a definite value. Their value depends on the numerical precision used in the calculation, or in other words, it can only be determined when the calculation is instantiated. This new discovery enriches the types of numbers and breaks the traditional concept that "numbers all have accurate values". The new discovery proves that the phenomenon of quantum superposition exists not only in the objective physical world, but also in the abstract field of mathematics.

5 References

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