RIEMANN HYPOTHESIS VIA WANG'S PEER-REVIEWED PAPER

DMITRI MARTILA
INDEPENDENT RESEARCHER
J. V. JANNSENI 6–7, PÄRNU 80032, ESTONIA

ABSTRACT. I am writing a shortest proof of the Riemann Hypothesis using Wang's paper as a starting point.

Keywords: Functional analysis; number theory.
MSC: 11M26, 11M06.

Nicolas has shown [1] that if the inequality

$$\frac{N_k}{\varphi(N_k)} - \mathrm{e}^{\gamma} \ln\ln N_k > 0 \tag{1}$$

has none or finite amount of violations, the Riemann Hypothesis is true. Here $\gamma \approx 0.577216$ is the Euler–Mascheroni constant, and $\varphi(N)$ is Euler's totient function. $N_k = p_k\#$ is the primorial of order k.

First, the Nicolas inequality (1) for all $k > 44$ seems to be proven in Wang's paper [2], but without citing Nicolas paper that this would make up the preparation for the proof of the Riemann Hypothesis.

Second, Wang refers the Rosser's 1941 paper [3] with $k \geq 6$ in $p_k < k\,(\log k + \log\log k)$. Looking at Theorem 30 on page 212 where $k \geq \exp(2000)$, Wang's paper holds not for the announced $k > 44$, but for $k \geq \exp(2000)$. Of course, it is not easy to check numerically $k < \exp(2000)$ values, but the fact is established now: the amount of hypothetical violations of (1) is finite.

Third, I have realized that the kernel of Wang's argumentation is the reference to Dusart's preprint, however, which as arXiv preprint is not peer-reviewed. This makes the whole result by Wang not peer-reviewed, as it relies entirely on a non-peer-reviewed preprint. Therefore, I am now making one of the major final steps to complete the proof of the Riemann Hypothesis by suggesting a replacement of the reference to Dusart's preprint by the reference to the more precise and

eestidima@gmail.com.

surely peer-reviewed journal paper [4], where the formula of interest is

$$\Big|\frac{\theta(x)-x}{x}\Big| < \frac{3.79\cdot 10^{-5}}{(\log x)^2} < \frac{0.006788}{\log x} \tag{2}$$

for all $x \geq 10^{19}$. However, because of $\theta(x) < x$ for all $x < 10^{19}$ [4], this formula (2) is useful for all x.

Conclusion

In this publication, I have given my approach to provide a proof for the validity of the Riemann Hypothesis. There is a Millennium Prize for solving this puzzle, with the rule: the fame with Prize gets one who makes the final move. Wang has not made the crucial final moves.

References

[1] J.-L. Nicolas, "Petites valeurs de la fonction d'Euler." *J. Number Theory* **17** 3 (1983), 375–388.
https://doi.org/10.1016/0022-314X(83)90055-0

[2] C.-T. Wang, "On an inequality about Euler's totient function." *Notes on Number Theory and Discrete Mathematics* **31** 2 (2025), 299–304,
https://doi.org/10.7546/nntdm.2025.31.2.299-304

[3] Barkley Rosser, "Explicit Bounds for Some Functions of Prime Numbers." *Am. J. Math.* **63** 1 (1941), 211–232,
https://doi.org/10.2307/2371291

[4] Samuel Broadbent, Habiba Kadiri, Allysa Lumley, Nathan Ng, Kirsten Wilk, "Sharper bounds for the Chebyshev function $\theta(x)$," *Math. Comp.* **90** (2021), 2281–2315
https://doi.org/10.1090/mcom/3643