The enigmas of the numbers 17 and 19 and magic matrices

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Abstract:

This document deals with enigmas related to the numbers 17 and 19, by presenting equations involving squares of natural integers. It'is composed of two parts.

1- Enigmas of the numbers 17 and 19
2- Enigma of the number 17

Key words: Natural integers, prime numbers, square of natural integers, sum of square of two natural integers.

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The enigmas of the numbers 17 and 19 and magic matrices

Summary

This document explores mathematical puzzles related to the numbers 17 and 19, using the squares of natural integers

- Riddles of the numbers 17 and 19

* **Conjecture 1**: Any square of a natural integer N can be written in the form $N^2 = 17u + 19v$, where u and v are integers.

* Examples provided illustrate this conjecture with equations for squares from 6 to 800.

- Riddles of the numbers 17

* **Conjecture 2:** The sum of two natural integers is a perfect square, and the sum of their squares is a multiple of 17.

* Examples demonstrate this conjecture with various equations, including sums and differences of squares.

* Matrices are used to illustrate the relationships between integers and multiples of 17.

I) Enigma of the numbers 17 and 19

Introduction: In this article, I study the enigmas of the numbers 17 and 19, using the square of the natural Integers.

Conjecture 1: Any square of an natural integer N, is written in the form;

$N^2 = 17u + 19v$; u, v Integers

Example of square an natural integers illustrating this conjecture

$6^2 = 17 + 19$

$11^2 = 6*17 + 19$

$12^2 = 4*17 + 4*19$

$18^2 = 9*17 + 9*19$

$19^2 = 19*17 + 2*19$

$20^2 = 9*17 + 13*19$

$21^2 = 17*17 + 8*19$

$22^2 = 24*17 + 4*19$

$23^2 = 30*17 + 1*19$

$24^2 = 16*17 + 16*19$

$25^2 = 20*17 + 15*19$

$26^2 = 23*17 + 15*19$

$27^2 = 25*17 + 16*19$

$28^2 = 26*17 + 18*19$

$29^2 = 26*17 + 21*19$

$30^2 = 44*17 + 8*19$

$31^2 = 42*17 + 13*19$

$32^2 = 58*17 + 2*19$

$33^2 = 54*17 + 9*19$

$34^2 = 49*17 + 17*19$

$35^2 = 62*17 + 9*19$

$36^2 = 74*17 + 2*19$

$37^2 = 66*17 + 13*19$

$38^2 = 76*17 + 8*19$

$39^2 = 85*17 + 4*19$

$40^2 = 74*17 + 18*19$

$41^2 = 81*17 + 16*19$

$42^2 = 87*17 + 15*19$

$43^2 = 73*17 + 32*19$

$44^2 = 96*17 + 16*19$

$45^2 = 99*17 + 18*19$

$46^2 = 101*17 + 21*19$

$47^2 = 102*17 + 25*19$

$48^2 = 102*17 + 30*19$

$49^2 = 101*17 + 36*19$

$50^2 = 99*17 + 43*19$

$52^2 = 149*17 + 9*19$

$53^2 = 163*17 + 2*19$

$54 = 157*17 + 13*19$

$55^2 = 99*17 + 43*19$

$55^2 = 169*17 + 8*19$

$60^2 = 195*17 + 15*19$

$61^2 = 201*17 + 16*19$

$62^2 = 206*17 + 18*19$

$63^2 = 210*17 + 21*19$

$64^2 = 213*17 + 25*19$

$65^2 = 215*17 + 30*19$

$66^2 = 216*17 + 36*19$

$60^2 = 216*17 + 43*19$

$100^2 = 510*17 + 70*19$
$101^2 = 419*17 + 162*19$
$102^2 = 441*17 + 153*19$
$103^2 = 462*17 + 145*19$
$104^2 = 463*17 + 155*19$
$105^2 = 406*17 + 217*19$
$107^2 = 462*17 + 178*19$
$108^2 = 476*17 + 188*19$
$109^2 = 529*17 + 152*19$
$110^2 = 163*17 + 491*19$

$226^2 = 3000*17 + 4*19$
$227^2 = 3030*17 + 1*19$
$228^2 = 3040*17 + 16*19$
$229^2 = 3049*17 + 32*19$
$230^2 = 3095*17 + 15*19$
$231^2 = 3102*17 + 33*19$
$232^2 = 3165*17 + 1*19$
$233^2 = 3189*17 + 4*19$
$234^2 = 3212*17 + 8*19$
$235^2 = 3234*17 + 13*19$

$448^2 = 11805*17 + 1*19$
$449^2 = 11841*17 + 16*19$
$450^2 = 11895*17 + 15*19$
$451^2 = 11948*17 + 15*19$
$452^2 = 12000*17 + 16*19$
$453^2 = 12070*17 + 1*19$
$454^2 = 12120*17 + 4*19$
$455^2 = 12169*17 + 8*19$

$600^2 = 21172*17 + 4*19$
$601^2 = 21246*17 + 1*19$
$602^2 = 21300*17 + 16*19$
$603^2 = 21372*17 + 15*19$
$604^2 = 21443*17 + 15*19$
$605^2 = 21513*17 + 16*19$
$700^2 = 28790*17 + 30*19$
$800^2 = 37637*17 + 9*19$

II) **Enigma of the numbers 17**

Conjecture 2: the sum of two natural integers is a perfect square, and then the sum of their squares is a multiple of 17.

Example of square an natural integers illustrating this conjecture

Conjecture 2.1: the sum of two natural integers is a perfect square, and then the sum of their squares is a multiple of 17. With the use of matrix operations on integers that make up two or more equalities

Example 1 :

$20^2+29^2=73*17\rightarrow 20+59=7^2$
$5^2+31^2=58*17\rightarrow 5+31=6^2$
$20^2+5^2=15*17\rightarrow 20+5=5^2$
$31+20=3*17$
$20+29=2*17$
$20*29-5*31=25*17$
$20*31+5*29=45*17$
$29*31-5*20=47*17$
$20*31-5*5=35*17$
$20*20-29*5=15*17$

The matrice is $\begin{vmatrix} 20 & 29 \\ 5 & 31 \\ 5 & 20 \end{vmatrix}$

Example 2:

$3^2+46^2=125*17\rightarrow 3+46=7^2$
$5^2+31^2=58*17\rightarrow 31+5=6^2$

the matrice is $\begin{vmatrix} 3 & 46 \\ 5 & 31 \end{vmatrix}$

$3+31=2*17$
$5+46=3*17$
$3*31+5*46=19*17$
$31*5-3*46=17$
$46*31-3*5=83*17$
$3^2+31^2+5^2+46^2=183*17$
$31^2+46^2=181*17$
$3^2+5^2=2*17$

Conjecture 2.2: For any natural integer p, there exists at least, one natural integer q , such that the sum of p and q equal a perfect square and the sum of their squares is a multiple of 17 .

$\forall p,q\in N^2 \,/\, p+q=k^2 \Rightarrow p^2+q^2=17\lambda$

$3^2+46^2=125*17\rightarrow 3+46=7^2$

$5^2+31^2=58*17\rightarrow 31+5=6^2$

$6^2+10^2=8*17\rightarrow 6+10=4^2$

$7^2+57^2=194*17\rightarrow 7+57=8^2$

$11^2+38^2=92*17\rightarrow 11+38=7^2$

$12^2+37^2=89*17\rightarrow 12+37=7^2$

$13^2-4^2=-9*17\rightarrow 13-4=-3^2$

$14^2+22^2=40*17\rightarrow 14+25=6^2$

$15^2-19^2=-8*17\rightarrow 15-19=-4^2$

$16^2-118^2=-1089*17\rightarrow 16-137=-11^2$

$18^2-137^2=-800*17\rightarrow 18-118=-10^2$

$19^2-15^2=-8*17\rightarrow 19-15=-2^2$

$23^2+41^2=13*17\rightarrow 23+41=8^2$

$24^2+40^2=128*17\rightarrow 24+40=8^2$

$28^2+197^2=2329*17\rightarrow 28+197=15^2$

$29^2+20^2=73*17\rightarrow 29+20=7^2$

$45^2+180^2=2025*17\rightarrow 45+180=15^2$

$58^2+6^2=200*17\rightarrow 58+6=8^2$

$58^2+198^2=2504*17\rightarrow 58+198=16^2$

$29^2+71^2=346*17\rightarrow 29+71=10^2$

$58^2+23^2=229*17\rightarrow 58+23=9^2$

$99^2+22^2=605*17\rightarrow 99+22=11^2$

$107^2+37^2=784*17\rightarrow 107+37=12^2$

$129^2+40^2=1073*17\rightarrow 129+40=13^2$

$150^2+46^2=1448*17\rightarrow 150+46=14^2$

$164^2+61^2=1801*17\rightarrow 164+61=15^2$

$194^2+62^2=2440*17\rightarrow 194+62=16^2$

$249^2+75^2=3978*17\rightarrow 249+75=18^2$

$250^2+150^2=5000*17\rightarrow 250+150=20^2$

$251^2-55^2=3528*17\rightarrow 251-55=14^2$

$252^2+148^2=5024*17\rightarrow 252+148=20^2$

$300^2 + 24^2 = 5328*17 \rightarrow 300 + 24 = 18^2$

$575^2 + 386^2 = 28213*17 \rightarrow 575 + 386 = 31^2$

$600^2 + 184^2 = 23168*17 \rightarrow 600 + 184 = 28^2$

$610^2 + 1890^2 = 23200*17 \rightarrow 610 + 1890 = 50^2$

$615^2 + 114^2 = 23013*17 \rightarrow 615 + 114 = 27^2$

$438^2 - 38^2 = 112*17 \rightarrow 438 - 38 = 20^2$