Gravitational Collapse as Analog of Continuous Spacetime Dimensions

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Abstract

Starting from the Newtonian potential of a two-body system, we recently pointed out that a spacetime endowed with continuous dimensions can be interpreted as analog of classical gravitation in four dimensions. Here we extend the analysis to relativistic two-body systems and suggest that the formation of Black Holes echoes the behavior of continuous spacetime dimensions in primordial cosmology.

Key words: continuous spacetime dimensions, relativistic two-body systems, gravitational collapse, Schwarzschild metric, Cantor Dust.

According to [1], far above the low-energy sector of field theory, the temporal component of the non-relativistic metric $g_{00}(\mu)$ is linearly related to the continuous deviation from four space-time dimensions $\varepsilon(\mu)$ as in

$$g_{00}(\mu) \approx 1 - 2\frac{m^2(\mu)}{M_{Pl}^2} \approx 1 - 2\varepsilon(\mu) \quad (1a)$$

in which

$$\varepsilon(\mu) = 4 - D(\mu) \quad (1b)$$

Here, μ is the observation scale, $m(\mu)$ stands for mass, M_{Pl} is the Planck mass and the Newtonian potential is given by

$$\varphi_N(\mu) \approx \frac{1}{2}[g_{00}(\mu) - 1] \quad (2)$$

It is known that General Relativity (GR) does not allow a straightforward extrapolation of the two-body gravitational potential (2) to curved spacetime. In this instance, one appeals to several *effective models* such as in the Post-Newtonian (PN) and Effective One Body (EOB) approximations. These are typically applied for treating two-body problems (such as binary black holes or neutron star binaries) in a full relativistic context [2 - 5].

With reference to EOB, the two-body problem with masses m_1 and m_2 is mapped onto:

a) A test particle of reduced mass $m_R = m_1 m_2/(m_1+m_2)$,
b) Total mass of the system $M = m_1 + m_2$,
c) A symmetric mass ratio $\eta = m_R/M$.

The effective EOB line element is written in Schwarzschild-like coordinates as

$$ds_{eff}^2 = -A(r)dt^2 + B(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (3)$$

where the metric functions $A(r)$ and $B(r)$ contain all the relativistic corrections to the Newtonian metric. Specifically, the counterpart of (1a) is

$$A(r) = 1 - 2u + 2\eta u^3 + a_4(\eta)u^4 + ... \quad (4)$$

with the Newtonian potential encoded in

$$-2u = -\frac{2G_N M}{r} = -\frac{2M}{M_{Pl}^2 r} \quad (5)$$

and nonlinear corrections included in the series $\sum_n a_n(\eta)u^n$, with $(n \geq 2)$. Likewise, the metric function $B(r)$ assumes the form

$$B(r) = \frac{1}{1-2u} + b_2(\eta)u^2 + b_3(\eta)u^3 + ... \tag{6}$$

All relativistic terms can be safely ignored if $M = m_1 + m_2 << M_{Pl}$ or the radial coordinate approaches infinity (matching flat manifold condition), i.e. $r \to \infty, u \to 0$.

With reference to [1], it is apparent that (4) and (5) recover the expression of minimal dimensional deviation $\varepsilon << 1$ in the limit $m_1 = m_2 = m << M_{Pl}$ and by taking the radial coordinate to be comparable with the Compton wavelength, that is,

$$\boxed{u \to 0 \Leftrightarrow \varepsilon = O(m^2/M_{Pl}^2) \to 0} \tag{7}$$

It is also apparent that demanding the metric (3) to become singular at $A(r)=0,\ B(r)\to\infty$ leads to the well-known expression of the *Schwarzschild radius*, namely,

$$1-2u=0 \Rightarrow r_G = 2G_N M = 4m/M_{Pl}^2 \qquad (8)$$

Let the Schwarzschild radius (8) be again commensurate with the Compton wavelength, $r_G = O(\lambda_c) = O(m^{-1})$. By (1b), (8) yields

$$\boxed{\frac{4m^2}{M_{Pl}^2} \approx 4\varepsilon \approx 1} \qquad (9)$$

Condition (9) signals a *regime of high fractality* $\varepsilon = O(1)$, which denotes a complex dynamic setting where there is a continuous *dimensional reduction* from the ordinary four-spacetime of classical and quantum physics [6].

Several key conclusions may be drawn from the combined use of (7), (9) and ref. [1], namely,

1. As emphasized in [1], (7) sets the stage for the gravitational interpretation of *Cantor Dust* (CD), under the assumption that CD acts like a large-scale cluster of ultralight Dark Matter (DM) particles such as self-interacting bosons, axions, fuzzy condensates or 3D anyons.

2. (9) hints that gravitational collapse and the formation of Black Holes is analogous to the process of *dimensional condensation* and the emergence of Cantor Dust.

3. (9) also hints that dimensional condensation of DM is analogous to DM formation via *primordial Black Holes*.

References

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