The Absolutely Precise Atomic Unit of Time and its Applications in Determination of Atomic/Nuclear Transition Frequencies and Fundamental Physical Constants

Gang Chen†, Tianman Chen, Tianyi Chen

†Correspondence to: gang137.chen@connect.polyu.hk

Abstract

In our previous paper, we calculated out and determined the atomic unit of time (t_{au}) to be 2.41888432658653284(45)×10^{-17} s or 2.41888432658653280(45)×10^{-17} s. In this paper, using the ^{133}Cs atomic transition frequency which is 9192631770 Hz (a second is defined by this) and its reciprocal in atomic units calculated from our corresponding formula, we calculate out and determine the absolutely precise atomic unit of time to be 2.41888432658653278×10^{-17} s. With this value of t_{au} and the speed of light in atomic units (c_{au}) determined in our previous papers, we calculate out and determine the absolutely precise values of the atomic transition frequencies of some atoms, the nuclear transition frequency of ^{229}Th* and some fundamental physical constants such as the Rydberg constant (R_∞), the Hartree energy (E_h), the Bohr radius (a_0), the classical electron radius (r_e) and the electron mass (m_e).

Keywords: atomic unit of time, the speed of light in atomic units, atomic transition frequencies, nuclear transition frequency of ^{229}Th*, fundamental physical constants.

1. Definitions of the Second, the Speed of light and Planck's Constant

A second is currently defined as the time interval of 9,192,631,770 cycles of radiation associated with the atomic transition between the two superfine levels of the ground state of the ^{133}Cs atom. And the speed of light in vacuum (the speed of light in short) is defined as 299,792,458 m/s and Planck's constant is defined as 6.62607015×10^{-34} J·s. It is worth noting that they are all exact values.

$$f_{^{133}Cs} = 9192631770 \text{ Hz}$$
$$c = 299792458 \text{ m/s}$$
$$h = 6.62607015 \text{ J} \cdot \text{s}$$

2. Determination of the Absolutely Precise Atomic Unit of Time

In our previous paper, we calculated out and determined the atomic unit of time (t_{au}) to be $2.41888432658653284(45)\times10^{-17}$ s [1, 2] or $2.41888432658653280(45)\times10^{-17}$ s [3], and we thought the later was a little more correct. In this paper, we construct reasonable formulas of the reciprocal of the atomic transition frequency in atomic units of ^{133}Cs atom, and hence calculate out and determine the absolutely precise atomic unit of time (t_{au}) to be $2.41888432658653278\times10^{-17}$ s.

$$\frac{1}{f_{^{133}Cs/au}}=\frac{1}{9192631770\times2.41888432658653284(45)\times10^{-17}}$$

$$=4497229.34297061260(84)$$

$$\frac{1}{f_{^{133}Cs/au}}=11(2\cdot3\cdot25\cdot7+1)(2\cdot3\cdot5\cdot13-1)+\frac{1}{2}-\frac{1}{6}+\frac{1}{103}-\frac{1}{45(8\cdot3\cdot13-1)-\frac{7}{8}}$$

$$=4497229.34297061264$$

Relationships with nuclides:

$^{47,48,50}_{22}Ti_{25,26,28}$ $^{55}_{25}Mn_{30}$ $^{56}_{26}Fe_{30}$ $^{89}_{39}Y_{50}$ $^{100}_{44}Ru_{56}$ $^{103}_{45}Rh_{58}$ $^{120}_{50}Sn_{70}$ $^{133}_{55}Cs_{78}$ $^{137}_{56}Ba_{81}$
$^{173}_{70}Yb_{103}$ $^{264,165}_{103}Lr^{*}_{161,162}$

$$\frac{1}{f_{^{133}Cs/au}}=11(2\cdot3\cdot25\cdot7+1)(2\cdot3\cdot5\cdot13-1)+\cfrac{1}{3-\cfrac{1}{12-\cfrac{7}{3\cdot17-\cfrac{7}{173}}}}$$

$$=11(2\cdot3\cdot25\cdot7+1)(2\cdot3\cdot5\cdot13-1)+\cfrac{1}{3-\cfrac{1}{12-\cfrac{7\cdot173}{16\cdot19\cdot29}}}$$

$$=4497229.34297061264$$

Relationships with nuclides:

$^{28,29}_{14}Si_{14,15}$ $^{39}_{19}Kr_{20}$ $^{47,48,50}_{22}Ti_{25,26,28}$ $^{55}_{25}Mn_{30}$ $^{56,58}_{26}Fe_{30,32}$ $^{63,65}_{29}Cu_{34,36}$ $^{78}_{34}Se_{44}$ $^{89}_{39}Y_{50}$
$^{100}_{44}Ru_{56}$ $^{103}_{45}Rh_{58}$ $^{110,112}_{48}Cd_{62,64}$ $^{116,120}_{50}Sn_{66,70}$ $^{126}_{52}Te_{74}$ $^{133}_{55}Cs_{78}$ $^{137}_{56}Ba_{81}$ $^{140,142}_{58}Ce_{82,84}$
$^{173}_{70}Yb_{103}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{188}_{76}\mathrm{Re}_{112}$ $^{194}_{78}Pt_{116}$ $^{209}_{83}Bi^{*}_{126}$ $^{285}_{112}Cn^{*}_{173}$ $^{314}_{126}Ch^{ie}_{188}$ $^{2\cdot173}_{137}Fy^{ie}_{209}$

$$t_{au}=\frac{f_{^{133}Cs/au}}{f_{^{133}Cs}}=\frac{1}{9192631770\times4497229.34297061264}$$

$$=2.41888432658653278\times10^{-17}\ \mathrm{s}$$

The factors in the above formulas of the reciprocal of the atomic transition frequency in atomic units of ^{133}Cs atom are supposed to be related to the nuclides and hence are physically meaningful and reasonable. It is worth noting that the two characteristic factors 173 and 103 which are supposed to be related to some typical nuclides such as ^{173}Yb and ^{185}Cn* appear in the formulas. So we suppose the calculated value of t_{au} is absolutely precise.

3. The Absolutely Precise Values of Some Atomic Transition Frequencies

In our previous papers, we constructed reasonable formulas of the reciprocal of atomic transition frequencies in atomic units of some atoms such as ^{1}H, ^{27}Al^{+}, ^{40}Ca, ^{87}Sr, ^{115}ln^{+}, ^{171}Yb, ^{171}Yb^{+}, ^{199}Hg and ^{199}Hg^{+}. With these formulas and the above calculated absolutely precise atomic unit of time (t_{au}), we can calculate out the absolutely precise atomic transition frequencies of these atoms as follows.

^{1}H 1S - 2S atomic transition frequency ($f_{^1H}$):

$$\frac{1}{f_{^1H-calc/au}} = 17 - \frac{1}{4} + \frac{1}{70} - \frac{1}{37\cdot 173} + \frac{1}{11\cdot 47\cdot 97\cdot 103}$$
$$= 16.7641296822937984$$

$$f_{^1H-calc} = \frac{f_{^1H-calc/au}}{t_{au}}$$
$$= \frac{1}{16.7641296822937984\times 2.41888432658653278\times 10^{-17}}$$
$$= 2466061413187018.02 \text{ Hz}$$

^{27}Al^{+} 3s^2 ^{1}S$_0$ - 3s3p ^{3}P$_0$ atomic transition frequency ($f_{^{27}Al^+}$):

$$\frac{1}{f_{^{27}Al^+-calc/au}} = 37 - \frac{1}{8} + \frac{1}{285} - \frac{1}{3\cdot 7^3\cdot 11\cdot 37 - \frac{9}{16} - \frac{1}{320}}$$
$$= 36.8785063841691099$$

$$f_{^{27}Al^+-calc} = \frac{f_{^{27}Al^+-calc/au}}{t_{au}}$$
$$= \frac{1}{36.8785063841691099\times 2.41888432658653278\times 10^{-17}}$$
$$= 1121015393207859.18 \text{ Hz}$$

^{40}Ca ^{1}S$_0$ - ^{3}P$_1$ atomic transition frequency ($f_{^{40}Ca}$):

$$\frac{1}{f_{^{40}Ca-calc/au}} = 90 + \frac{2}{3} - \frac{1}{330} + \frac{1}{2\cdot 9\cdot 5\cdot 19\cdot 101 - \frac{3}{40}}$$
$$= 90.6636421536915675$$

$$f_{^{40}Ca-calc} = \frac{f_{^{40}Ca-calc/au}}{t_{au}}$$

$$= \frac{1}{90.6636421536915675 \times 2.41888432658653278 \times 10^{-17}}$$

$$= 455986240494140.307 \text{ Hz}$$

^{87}Sr 5s^2 ^{1}S$_0$ - 5s5p ^{3}P$_0$ atomic transition frequency ($f_{^{87}Sr}$):

$$\frac{1}{f_{^{87}Sr-calc/au}} = 96 + \frac{1}{3} - \frac{1}{56} + \frac{1}{64 \cdot 91} + \frac{1}{7 \cdot 47(4 \cdot 17(2 \cdot 3 \cdot 5 \cdot 47 - 1) + 1)}$$

$$= 96.3156479254962891$$

$$f_{^{87}Sr-calc} = \frac{f_{^{87}Sr-calc/au}}{t_{au}}$$

$$= \frac{1}{96.3156479254962891 \times 2.41888432658653278 \times 10^{-17}}$$

$$= 429228004229873.001 \text{ Hz}$$

^{115}In$^+$ 5s^2 ^{1}S$_0$ - 5s5p ^{3}P$_0$ atomic transition frequency ($f_{^{115}In^+}$):

$$\frac{1}{f_{^{115}In^+-calc/au}} = 32 + \frac{5}{8} - \frac{1}{2 \cdot 83} + \frac{1}{32 \cdot 3 \cdot 5(2 \cdot 81 \cdot 5 - 1) + \frac{2}{7}}$$

$$= 32 + \frac{5}{8} - \frac{1}{2 \cdot 83} + \frac{1}{2(209(32 \cdot 29 + 1) - 1) + \frac{2}{7}}$$

$$= 32.6189784788082780$$

$$f_{^{115}In^+-calc} = \frac{f_{^{115}In^+-calc/au}}{t_{au}}$$

$$= \frac{1}{32.6189784788082780 \times 2.41888432658653278 \times 10^{-17}}$$

$$= 1267402452901041.04 \text{ Hz}$$

^{171}Yb 6s^2 ^{1}S$_0$ - 6s6p ^{3}P$_0$ atomic transition frequency ($f_{^{171}Yb}$):

$$\frac{1}{f_{^{171}Yb-calc/au}} = 80 - \frac{1}{4} + \frac{1}{70} - \frac{1}{8(2 \cdot 9 \cdot 29 + 1)} + \frac{1}{17 \cdot 73^2(2 \cdot 173 + 1)}$$

$$= 79.7640467403604713$$

$$f_{^{171}Yb-calc} = \frac{f_{^{171}Yb-calc/au}}{t_{au}}$$

$$= \frac{1}{79.7640467403604713 \times 2.41888432658653278 \times 10^{-17}}$$

$$= 518295836590863.634 \text{ Hz}$$

$^{171}Yb^{+}$ $6s^{2}S_{1/2}$ - $5d^{2}D_{3/2}$ atomic transition frequency ($f_{^{171}Yb^{+}-1}$):

$$\frac{1}{f_{^{171}Yb^{+}-1-calc/au}} = 60 + \frac{1}{17} - \frac{1}{3(4\cdot 3\cdot 29+1)} + \frac{1}{5\cdot 131(2\cdot 37\cdot 43-1)+\frac{5}{12}}$$

$= 60.0578688995229708$

$$f_{^{171}Yb^{+}-1-calc} = \frac{f_{^{171}Yb^{+}-1-calc/au}}{t_{au}}$$

$$= \frac{1}{60.0578688995229708\times 2.4188843265865 3278\times 10^{-17}}$$

$= 688358979309308.247$ Hz

$^{171}Yb^{+}$ $6s^{2}$ $S_{1/2}$ - $4f^{13}6s^{2}$ $^{2}F_{7/2}$ atomic transition frequency ($f_{^{171}Yb^{+}-2}$):

$$\frac{1}{f_{^{171}Yb^{+}-2-calc/au}} = 64 + \frac{7}{2\cdot 9} - \frac{1}{4\cdot 3\cdot 13} - \frac{1}{2\cdot 3\cdot 37(4\cdot 19\cdot 181+1)+\frac{13}{27}}$$

$= 64.3824783050450500$

$$f_{^{171}Yb^{+}-2-calc} = \frac{f_{^{171}Yb^{+}-2-calc/au}}{t_{au}}$$

$$\frac{1}{64.3824783050450500\times 2.41888432658653278\times 10^{-17}}$$

$= 642121496772645.126$ Hz

^{199}Hg $6s^{2}$ $^{1}S_{0}$ - 6s6p $^{3}P_{0}$ atomic transition frequency ($f_{^{199}Hg}$):

$$\frac{1}{f_{^{199}Hg-calc/au}} = 37 - \frac{1}{2} + \frac{1}{7} - \frac{1}{87} + \frac{1}{9\cdot 25\cdot 41} - \frac{1}{2\cdot 7\cdot 47\cdot 137\cdot(2\cdot 209+1)}$$

$= 36.6314712645923742$

$$f_{^{199}Hg-calc} = \frac{f_{Hg-calc/au}}{t_{au}}$$

$$= \frac{1}{36.6314712645923742\times 2.41888432658653278\times 10^{-17}}$$

$= 1128575290808154.12$ Hz

^{199}Hg $5d^{10}6s^{2}$ $S_{1/2}$ - $5d^{9}6s^{2}$ $^{2}D_{5/2}$ atomic transition frequency ($f_{^{199}Hg^{+}}$):

$$\frac{1}{f_{^{199}Hg^{+}-calc/au}} = 39 - \frac{1}{5} + \frac{1}{35} - \frac{1}{4\cdot 3\cdot 19^{2}} + \frac{1}{4\cdot 9\cdot 7\cdot 11^{2}(4\cdot 3\cdot 5\cdot 17-1)}$$

$= 38.8283406204968790$

$$f_{^{199}Hg^{+}-calc} = \frac{f_{^{199}Hg^{+}-calc/au}}{t_{au}}$$

$$= \frac{1}{38.8283406204968790 \times 2.41888432658653278 \times 10^{-17}}$$

$$= 1064721609899146.96$$

The above calculated results for the atomic transition frequencies are summarized in comparison with the BIPM recommended values [4] in the following table (**Table 1**). Among them, the result for H 1S-2S atomic transition frequency should be the most important.

Table 1. The Values of the Atomic Transition Frequencies of Some Atoms

Atoms	Recommended by BIPM (Hz)	Calculated by this work (Hz)
^{1}H	2 466 061 413 187 018(11)	2 466 061 413 187 018.02
$^{27}Al^{+}$	1 121 015 393 207 859.16(21)	1 121 015 393 207 859.18
^{40}Ca	455 986 240 494 140(8)	455 986 240 494 140.307
^{87}Sr	429 228 004 229 872.99(8)	429 228 004 229 873.001
$^{115}In^{+}$	1 267 402 452 901 041.3(54)	1 267 402 452 901 041.01(24)
^{171}Yb	518 295 836 590 863.63(10)	518 295 836 590 863.634
$^{171}Yb^{+}$-1	688 358 979 309 308.24(14)	688 358 979 309 308.247
$^{171}Yb^{+}$-2	642 121 496 772 645.12(12)	642 121 496 772 645.126
^{199}Hg	1 128 575 290 808 154.32(21)	1 128 575 290 808 154.12
$^{199}Hg^{+}$	1 064 721 609 899 146.96(23)	1 064 721 609 899 146.96

4. The Absolutely Precise Values of Nuclear Transition Frequencies of $^{229}Th^{*}$

In our previous paper, we constructed the formula of the reciprocal of the nuclear transition frequency of $^{229}Th^{*}$. With this formula and the above calculated absolutely precise value of the atomic unit of time (t_{au}), we can calculate out and determine the absolutely precise value of the nuclear transition frequency of $^{229}Th^{*}$ as follows.

$$\frac{1}{f_{^{229}Th^{*}-nuclear-calc/au}} = 20 + \frac{1}{2} - \frac{1}{26} + \frac{1}{16 \cdot 173 - \cfrac{1}{16 - \cfrac{1}{6}}}$$

$$= 20.4618997414581433$$

$$f_{^{229}Th^{*}-nuclear-calc} = \frac{f_{^{229}Th^{*}-nuclear-calc/au}}{t_{au}}$$

$$= \frac{1}{20.4618997414581433 \times 2.41888432658653278 \times 10^{-17}}$$

$$= 2020407384335167.06 \text{ Hz}$$

By *Nature* reporting on Sept. 4, 2024, the breakthrough experimental value of the nuclear transition frequency of $^{229}Th^{*}$ was $2.020407384335(2)\times10^{15}$ Hz [5]. Now we also give the calculation and prediction for the value of this frequency (**Table 2**).

Table 2. The Values of the Nuclear Transition Frequency of $^{229}Th^{*}$

Nucleus	Measured (Hz)	Calculated by this work (Hz)
$^{229}Th^{*}$	$2.020407384335(2)\times10^{15}$	2.020407384335167.06

5. The Absolutely Precise Values of Some Fundamental Physical Constants

In our previous papers, we constructed formulas of the fine-structure constant and the speed of light in atomic units (c_{au}) [6-20]. With the calculated value of c_{au} and the above calculated absolutely precise value of the atomic unit of time (t_{au}) , we can calculate out the absolutely precise values of some fundamental physical constants such as the Rydberg constant (R_∞), the Hartree energy (E_h), the Bohr radius (a_0), the classical electron radius (r_e) and the electron mass (m_e) as follows.

$2\pi - e$ formula:

$$2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots$$

$$(2\pi)_{Chen-k} = \left(\frac{e}{e^{\gamma_{c-k}}}\right)^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

Formulas of the fine-structure constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7(2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}}$$

$$= 1/137.035999037415379$$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13(2\pi)_{Chen-278}}{100} \frac{1}{112 - \frac{1}{64\cdot 3\cdot 29}}$$

$$= 1/137.035999111872963$$

$$c_{au} = \frac{c}{v_e} = \frac{4\pi\varepsilon_0 \hbar c}{e^2} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \alpha_1}}$$

$$= \sqrt{112(168 - \frac{1}{3} + \frac{1}{4\cdot 141} - \frac{1}{14\cdot 112(2\cdot 173+1) + 13 + \frac{7}{72}})}$$

$$= 2\sqrt{56(83 + \frac{157}{188} - (\frac{1}{8\cdot 141} + \frac{1}{56^2(2\cdot 173+1) + 26 + \frac{7}{36}}))}$$

$$= 137.035999074644171$$

$c = 299792458$ m/s; $h = 6.62607015\times10^{-34}$ J·s, $\hbar = \frac{h}{2\pi}$

$t_{au} = 2.41888432658653278 \times 10^{-17}$ s

$$t_{au} = \frac{\hbar}{E_h} = \frac{\hbar}{2R_\infty hc} = \frac{1}{4\pi R_\infty c}$$

Rydberg constant:

$$R_\infty = \frac{1}{4\pi c t_{au}} = \frac{1}{4\pi \times 299792458 \times 2.41888432658653278 \times 10^{-17}}$$

$= 10973731.5681561329$ m^{-1}

Hartree Energy:

$$E_h = H_a = \frac{\hbar}{t_{au}} = \frac{6.62607015 \times 10^{-34}}{2\pi \times 2.41888432658653278 \times 10^{-17}}$$

$= 4.35974472220563338 \times 10^{-18}$ J

Bhor radius:

$$v_e = \frac{c}{c_{au}} = \frac{299792458}{137.035999074644171} = 2187691.26378756577 \text{ m s}^{-1}$$

$$a_0 = v_e t_{au} = 2187691.26378756577 \times 2.41888432658653278 \times 10^{-17}$$

$= 5.2917721093860269 \times 10^{-11}$ m

Classical electron radius:

$$r_e = \frac{a_0}{c_{au}{}^2} = \frac{5.2917721093860269 \times 10^{-11}}{137.035999074644171^2}$$

$= 2.81794032676732026 \times 10^{-15}$ m

Electron mass:

Consider an electron becoming a photon with frequency of ν_e

$$E_e = h\nu_e = m_e c^2 \;\Rightarrow\; \nu_e = \frac{m_e c^2}{h}, \quad \nu_{e/au} = \frac{m_{e/au} c_{au}{}^2}{h_{au}} = \frac{c_{au}{}^2}{2\pi}$$

$$\mathrm{t}_{au} = \frac{1}{4\pi R_\infty c} = \frac{a_0 c_{au}}{c}$$

$$\nu_e = \frac{\nu_{e/au}}{t_{au}} = \frac{c_{au}{}^2}{2\pi \mathrm{t}_{au}} = 2R_\infty c c_{au}{}^2 = \frac{c c_{au}}{2\pi a_0}$$

$$m_e = \frac{h\nu_e}{c^2} = \frac{h c_{au}{}^2}{2\pi \mathrm{t}_{au} c^2} = \frac{2hR_\infty c_{au}{}^2}{c} = \frac{h c_{au}}{2\pi a_0 c}$$

$$= \frac{6.62607015 \times 10^{-34} \times 137.035999074644171^2}{2\pi \times 2.41888432658653278 \times 10^{-17} \times 299792458^2}$$

$= 9.1093837003111156 \times 10^{-31}$ kg

The above calculated values of the fundamental physical constants and their corresponding CODATA recommended values are listed in the following table (**Table 3**) for comparison. We can notice that our calculated values are absolutely precise with more digits than the CODATA recommended values.

Table 3. Values of the Fundamental Physical Constants

	CODATA 2022 Recommended	Calculated by this work
c_{au} (1/α)	137.035999177(21)	137.035999074644171
t_{au}	$2.4188843265864(26)\times10^{-17}$ s	$2.41888432658653278\times10^{-17}$ s
R_∞	$10973731.568157(12)$ m^{-1}	$10\,973\,731.5681561329$ m^{-1}
E_h	$4.3597447222060(48)\times10^{-18}$ J	$4.35974472220563338\times10^{-18}$ J
a_0	$5.29177210544(82)\times10^{-11}$ m	$5.2917721093860269\times10^{-11}$ m
r_e	$2.8179403205(13)\times10^{-15}$ m	$2.81794032676732026\times10^{-15}$ m
m_e	$9.1093837139(28)\times10^{-31}$ kg	$9.1093837003111156\times10^{-31}$ kg

6. Discussion and Conclusion

In the above formulas of the reciprocal of atomic transition frequencies in atomic units of some atoms such as ^{1}H, ^{87}Sr, ^{115}ln^{+}, ^{133}Cs, ^{171}Yb and ^{199}Hg, there are some characteristic factors such as 29/58/87, 45, 56, 70, 83, 91, 103, 137, 173 and 209. In the above formula of the reciprocal of nuclear transition frequency in atomic units of ^{229}Th*, there is the characteristic factor of 173. In the formula of the speed of light in atomic units, there are some characteristic factors such as 26, 56, 47/141/188, 83, 112, 137, 157, 168 and 173. It is supposed that these characteristic factors are related to nuclides according to our theories as follows.

$${}^{56,58}_{26}Fe_{30,32}\quad {}^{63,65}_{29}Cu_{34,36}\quad {}^{83,84}_{36}Kr_{47,48}\quad {}^{100}_{44}Ru_{56}\quad {}^{107,109}_{47}Ag_{60,62}\quad {}^{103}_{45}Rh_{58}$$

$${}^{112}_{48}Cd_{64}\quad {}^{137}_{56}Ba_{81}\quad {}^{140,142}_{58}Ce_{82,84}\quad {}^{173}_{70}Yb_{103}\quad {}^{167,168}_{68}Er_{99,100}\quad {}^{185,187}_{75}\mathrm{Re}_{110,112}$$

$${}^{188}_{76}\mathrm{Re}_{112}\quad {}^{209}_{83}Bi^{*}_{126}\quad {}^{223,224}_{87}Bi^{*}_{136,137}\quad {}^{231,8\cdot29}_{91}Pa^{*}_{140,141}\quad {}^{257}_{100}Fm^{*}_{157}\quad {}^{2\cdot137}_{107}Bh^{*}_{167}$$

$${}^{276}_{108}Hs^{*}_{168}\quad {}^{264,265}_{103}Lr^{*}_{161,162}\quad {}^{285}_{112}Cn^{*}_{173}\quad {}^{2\cdot157}_{126}Ch^{ie}_{188}\quad {}^{2\cdot173}_{137}Fy^{ie}_{209}\quad {}^{437,438}_{173}Ch^{ie}_{264,265}$$

${}^{173}_{70}Yb_{103}$: ${}_{70}Yb$ is the end of the 5f elements

${}^{285}_{112}Cn^{*}_{173}$: ${}_{112}Cn^{*}$ is the end of the 6d elements

${}^{2\cdot173}_{137}Fy^{ie}_{209}$: ${}_{137}Fy^{ie}$ is the end of the H-like ideal extended elements

${}^{437,438}_{173}Ch^{ie}_{264,265}$: ${}_{173}Ch^{ie}$ is the end of the ideal extended elements

In particular, the characteristic factor 173 appears frequently and dramatically in the above formulas. And the particularity of 173 exists on its corresponding to the 70th element Yb which is the end of the 5f elements, the 112th element Cn* which is the end of the 6d elements and the natural end of all elements according to our theories [21], the 137th element which is the end of the H-like ideal extended elements (ie) according to Feynman's calculation and the 173th element which is the end of the ideal extended elements (ie in short) according to Dirac equation. It also corresponds to the centigrade approximation of the square root of 3 according to our theories [17]. So we suppose that the above formulas and methodology to calculate and determine the atomic/nuclear transition frequencies and the fundamental physical constants should be reasonable and absolutely precise.

Reference

1. E-preprint: vixra.org/abs/2501.0095
2. E-preprint: vixra.org/abs/2502.0111
3. E-preprint: vixra.org/abs/2502.0128
4. ttps://www.bipm.org/en/publications/mises-en-pratique/standard-frequencies
5. C-K Zhang, et al. Nature 633, 63–70 (2024).
6. E-preprint: vixra.org/abs/2002.0203
7. E-preprint: vixra.org/abs/2002.0020
8. E-preprint: vixra.org/abs/2012.0107
9. E-preprint: vixra.org/abs/2102.0162
10. E-preprint: vixra.org/abs/2103.0088
11. E-preprint: vixra.org/abs/2104.0053
12. E-preprint: vixra.org/abs/2106.0042
13. E-preprint: vixra.org/abs/2106.0151
14. E-preprint: vixra.org/abs/2308.0168
15. E-preprint: vixra.org/abs/2407.0038
16. E-preprint: vixra.org/abs/2409.0044
17. E-preprint: vixra.org/abs/2501.0003
18. E-preprint: vixra.org/abs/2501.0095
19. E-preprint: vixra.org/abs/2503.0172
20. E-preprint: vixra.org/abs/2504.0130
21. E-preprint: vixra.org/abs/2312.0055