Proof Of The BGC & Utilizing The Logic For Primes Prediction

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***Abstract*—The Goldbach's Conjecture is an astonishing proposition that appears to be one of the most long-standing, renowned, and unsolved problems in number theory and in mathematics. This work herein is dedicated for proving it. The approach to be followed for the proof uses a system of equations predefined, and with the relatively simple analysis, the conjecture's proof is simple compared to the size of the problem.**

In the second part of this research, and with the purpose of predicting prime numbers in the known sequence of primes, the same system of equations is used, laying down a general mathematical framework that is computationally concise and can just achieve the objective. With proper selection of the coefficients of the equations in the algorithm, it's guaranteed that prime number are among the outputs. The algorithm consists of basic arithmetic operations which is by itself a feat. The proof of the algorithm is also astoundingly straightforward and pintsized.

***Index Terms*— BGC, Number theory, Prime numbers, Prediction**

I. Introduction

IT IS stated in the fundamental theorem of arithmetic that a specific product of prime numbers always expresses a specific number greater than one. This theorem is where the most critical aspect of prime numbers and number theory in particular stands. It's Interesting and essential to know, that the product is, in fact, unique.

So, as much as attempts could vary, the factorization produces the same building blocks. These building blocks of all numbers, the prime numbers, are the building blocks of our perception of the universe, as stated by many mathematicians [1].

This feature or property gives prime numbers a huge and massive importance in the fields of communications, where encryption ciphering of messages is vitally important.

The significance of the Binary Goldbach's Conjecture is that it uses the building blocks in a summation instead of a product. This Conjecture, if proven, can slacken the debate over some fundamental theories that are conditional on this Conjecture. Other than that, the Goldbach's Conjecture is:

- Astonishingly simple.
- It has been tested for more than 280 years numerically.

And this turns it into a dream to solve. A modern version of the marginal conjecture or the TGC as referred to in this paper [3] is: "Every integer greater than 5 can be written as the sum of three primes".

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And a modern version of Goldbach's original conjecture referred to as BGC in here: "Every even integer greater than 2 can be written as the sum of two primes".
There are other versions of the Goldbach's conjecture, in which we will use only the Quaternary version (QGC) in addition to the BGC and the TGC. The TGC and the QGC are proven in literature [3].
Additionally, and during the last 15 years, one of the major aims of this research was to find a numerical algorithm that can predict with high confidence just any other prime number, even if random in the order of the known sequence of primes. But the biggest objective was solely to find a prime number output, from the algorithm, that is fairly larger than the input prime number.

Across centuries, research has found many sieving algorithms [4] to find the next big prime number. In very few cases the research was around sieving the algorithm itself. And that is the concentration of this paper, to sieve the algorithm, and the result is an outcome of a large number of attempts to find an algorithm that could be simple in operations and successful in finding prime numbers of any number of digits needed.
The Ternary Goldbach's Conjecture is proven, which will be used for the proof of the algorithm, which is also amusingly short and concise

II. The proof of BGC

Definitions:

Since the (TGC) and the (QGC) versions of Goldbach's Conjecture are proven.

Assuming ζ is an even number and £ is a prime number, we define our system of equations as:

$$F_1 = \zeta - £ \tag{1}$$
$$F_2 = \zeta + £ \tag{2}$$

Equation 1 states that F_1 is equal to all the primes of ζ minus £ which is a prime. While equation 2 states that F_2 is equal to all the primes of ζ plus £, as stated, is a prime.

Since TGC is true, that means:

$$\zeta = P_1 + P_2 + P_3 \tag{3}$$

Then, there are two cases, which are:

Case I: P_1 or P_2 or P_3 is equal to £

In this case, we have:

$$F_1 = 2 + P_i \tag{4}$$
$$F_2 = 2 + P_i + 2£ \tag{5}$$

Where i=1, 2, or 3. For equation (5) to hold under the constraint that TGC is true, either P_i or £ is a member of a twin prime. And 2+P_i+£ which is equal to ζ can be rewritten as:

$$\zeta = P_a + P_b \tag{6}$$

And since ζ is any even number greater than two. Then BGC holds for this case.

Case II: P_1 or P_2 or P_3 is not equal to £

From Equation 1:

$$F_1 + £ = 2 + P_1 + P_2 \tag{7}$$

If F_1 is a prime number, then BGC is true, otherwise, it is just an odd number greater than five, then:

$$P_{F_1} + P_{F_2} + P_{F_3} + £ - 2 = P_1 + P_2 \tag{8}$$

LHS can be expressed as four prime numbers as stated by QGC since it's even. RHS states that, for all even numbers greater than two, the BGC holds, hence the "Binary Goldbach's Conjecture" is valid.

III. The Algorithm

Define the following three equations:

$$T = |\alpha_1 P_1 - \beta_1 P_{i+1} - \gamma_1| \tag{9}$$
$$F_p = |\alpha_2 P_1 + \beta_2 P_{i+1} + \gamma_2 - T| \tag{10}$$
$$F_{p_2} = |\alpha_2 P_1 + \beta_2 P_{i+1} + \gamma_2 + T| \tag{11}$$

Where P is a prime number with index i from the known sequence of prime numbers and the coefficients α, β, and γ are natural numbers.

After defining those equations, the following algorithm shall be used:

1. Get two consecutive prime numbers.
2. Sieve through the coefficients.
3. If equation (1) outputs a prime number, then one of the equations (2) and (3) must yield a prime number.

IV. Proof of the Algorithm

Define:

$$F_1 = \zeta + T \tag{12}$$
$$F_2 = \zeta - T \tag{13}$$

Where,

$$T = \alpha_1 P_i + \beta_1 P_{i+1} + \gamma_1 \tag{14}$$

And,

$$\zeta = \alpha_2 P_i + \beta_2 P_{i+1} + \gamma_2 \tag{15}$$

Where α_1, β_1, γ_1, α_2, β_2, and γ_2 are natural numbers.

With simple manipulation we have,

$$F_2 - F_1 = 2T \tag{16}$$

$$F_2 + F_1 = 2\zeta \tag{17}$$

$$\frac{\zeta^2 - F_1 F_2}{T^2} = 1 \tag{18}$$

There are three cases, (A) both F_1 and F_2 are not prime numbers, (B) either one of them is a prime number, or (C) both are prime numbers.

For the first case (A) where both are not prime

By TGC

$$F_1 F_2 = P_1 P_4 + P_1 P_5 + P_1 P_6 + P_2 P_4 + P_2 P_5 + P_2 P_6 + P_3 P_4 + P_3 P_5 + P_3 P_6 \tag{19}$$

And by equations (12) to (15), we have,

$$F_1 F_2 = (\alpha_2^2 - \alpha_1^2) P_i^2 + (\beta_2^2 - \beta_1^2) P_{i+1}^2 + 2(\alpha_2\beta_2 - \alpha_1\beta_1) P_i P_{i+1} + 2(\alpha_2\gamma_2 - \alpha_1\gamma_1) P_i + 2(\beta_2\gamma_2 - \beta_1\gamma_1) P_{i+1} + (\gamma_2^2 - \gamma_1^2) \tag{20}$$

It can be seen that by comparing equations (19) and (20) that there is no integral solution to the coefficients which we already defined as integral. And it can be easily shown that for the other two cases, where at least one of them is a prime, there is an integral solution.

References

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