

# The theory of mutual energy flow of transformer, antenna and photonic system is verified by the example of electromagnetic wave of plane-sheet current

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**Abstract**—The author has put forward the viewpoint and theory that electromagnetic energy flow is transmitted by mutual energy flow rather than self energy flow corresponding to the Poynting vector for 7 years, but it has been still in theory, and there is no specific example to verify this theory. This paper studies the examples of electromagnetic field and electromagnetic wave of plane-sheet current. Through this example, the author explains why the theory of mutual energy flow transmitting electromagnetic wave energy should be used to replace the theory of self energy flow transmitting energy. The author studies the magnetic quasi-static electromagnetic field and the radiated electromagnetic field. Corresponding to the magnetic quasi-static electromagnetic field, the power transmitted by self energy flow is reactive power. Energy flow can only be transferred by mutual energy flow. There is no dispute. Since there is no theory of mutual energy flow in the previous classical electromagnetic theory, so far the classical electromagnetic theory can not explain the energy flow from the primary coil to the secondary coil of the transformer. This paper achieves this goal for the first time. For radiated electromagnetic fields, both self energy flow and mutual energy flow are active power, but the author believes that self energy flow does not transfer energy, so Maxwell electromagnetic theory should add a time reversal wave to offset self energy flow. Or modify the electromagnetic theory to make the radiated electromagnetic field consistent with the magnetic quasi-static electromagnetic field, and the self energy flow transmits the electromagnetic wave of reactive power. In short, the classical electromagnetic theory is not a complete theory and must be modified. This example also tells us that the mutual energy flow is generated at the source of the electromagnetic wave and annihilated at the sink of the electromagnetic wave, which is very similar to the photon described by Cramer's transactional interpretation of quantum mechanics. Therefore, the mutual energy theory proposed by the author can be regarded as the concrete realization of the transactional interpretation. The interpretation of mutual energy flow proposed by the author is also a further development of transactional interpretation.

**Index Terms**—Maxwell equation; Reciprocity theorem; Conservation of energy; Poynting theorem; Energy flow; Transformer; Primary coil; Secondary coil; Transmitting antenna; Receiving antenna; Retarded wave; Retarded potential; Advanced wave; Advanced potential; Absorber; Radiator; Emitter; Photons; Quantum; Electromagnetic wave; Electromagnetic field; Transactional interpretation;

## I. INTRODUCTION

The author knows that there are some problems in the classical electromagnetic field theory, such as the classical electromagnetic field theory can not describe photons and the collapse of waves. Wave particle duality problem, whether the wave is a probability wave or an energy wave. These problems seem to be problems of quantum mechanics, which do not belong to classical electromagnetic theory. In fact, these problems have not been solved by quantum mechanics. The author hopes to answer these questions in the framework of classical electromagnetic theory by establishing electromagnetic mutual energy theory. This paper introduces the solution of electromagnetic field plane-sheet current. Answer these questions through an example. The author found a method to solve the retarded vector potential of plane-sheet current. The answer of this method is consistent with the method of calculating magnetic field and electric field by Ampere theorem. In this way, the result is verified, so it should be correct.

### A. Problems of classical electromagnetic theory

The author believes that electromagnetic radiation should not be radiated to the outside of the universe. There should not be a completely outward energy radiation. However, we know that according to the solution of Maxwell's equations, a current element can be regarded as an antenna. If viewed as an antenna, it can produce radiation. This radiation has been toward the outside of the universe.

In the transformer, the energy exchange can reach almost 100%, which is completed through mutual inductance. This conclusion is derived under the theoretical environment of magnetic quasi-static electromagnetic field. The author believes that an antenna system including a transmitting antenna and a receiving antenna should be consistent with the transformer system. Self energy flow does not transfer energy, but mutual energy flow transfers energy. In other words, the transmitting antenna should be consistent with the primary coil of the transformer and be the energy radiation source. The receiving antenna shall be consistent with the secondary coil of the transformer as a radiation sink. However, because radiation © is involved, we must consider the radiated electromagnetic

field satisfying Maxwell's equations, not the magnetic quasi-static field. According to the electromagnetic field determined by Maxwell's equations, self energy flow transfers energy. This can be seen from the fact that the Poynting vector of the electromagnetic field of any antenna is a real number. At this time, the self energy + mutual energy will be greater than the total energy of the system. This is certainly wrong.

What went wrong with Maxwell's theory? this is considered by the author. 1) Maxwell's equations are right, and the solution we get is wrong. 2) The solution of Maxwell's equation is correct, and there is a problem in the physical sense of Maxwell's equations. Firstly, an example is given to illustrate that Maxwell's equations can be solved accurately. Therefore, if there is an error, it can only be an error in the interpretation of the physical meaning of Maxwell's equations.

### B. Physical theory and electromagnetic theory including advanced wave

In the field of physics and quantum physics, Wheeler Feynman put forward the absorber theory [1], [2] in 1945. This theory is based on the action-at-a-distance reaction theory [10], [12], [6]. Stephenson put forward his own advanced wave theory about 1980 [11]. Cramer established the transactional interpretation of quantum mechanical on the basis of absorber theory [3], [4].

In the domain of electromagnetic field, Welch gave Welch's reciprocity theorem in 1960, involving advanced wave [13]. Rumsey gave a new reciprocity theorem in 1963 [9]. In 1987, the author proposed the mutual energy theorem [7], [15], [14], and de Hoop proposed the relevant reciprocal theorem [5] at the end of 1987. All these theorems can be connected by Fourier transform, so they can be regarded as one theorem. The difference between the author and the other three people is that this theorem can be interpreted not only a reciprocity theorem, but also an energy theorem.

Welch's reciprocity theorem,

$$-\int_{t=-\infty}^{\infty} dt \iiint_V \mathbf{J}_1 \cdot \mathbf{E}_2 dV = \int_{t=-\infty}^{\infty} dt \iiint_V \mathbf{J}_2 \cdot \mathbf{E}_1 dV \quad (1)$$

de Hoop's correlation reciprocity theorem,

$$\begin{aligned} & -\int_{t=-\infty}^{\infty} dt \iiint_V \mathbf{J}_1(t+\tau) \cdot \mathbf{E}_2(t) dV \\ & = \int_{t=-\infty}^{\infty} dt \iiint_V \mathbf{J}_2(t) \cdot \mathbf{E}_1(t+\tau) dV \end{aligned} \quad (2)$$

The author's mutual energy theorem and Rumsey's reciprocity theorem,

$$-\iiint_V (\mathbf{J}_1 \cdot \mathbf{E}_2^*) dV = \iiint_V (\mathbf{J}_2^* \cdot \mathbf{E}_1) dV \quad (3)$$

### C. Introduction to the mutual energy theory proposed by the author

The author have developed the mutual energy theory [8], [2], [2], [2]. It introduced 3 Axioms and a few theorems.

Axiom 1, law of conservation of energy,

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \int_{t=-\infty}^{\infty} dt \iiint_V (\mathbf{J}_i \cdot \mathbf{E}_j) dV = 0 \quad (4)$$

This axiom is self-evident. It shows that when the power of one current element of a system increase, the power of another current element will decrease, so the total power remains unchanged.

Axiom 2, mutual energy principle, with  $N$  current elements  $\mathbf{J}_i$ ,  $i = 1, \dots, N$ . The following mutual energy principle exists,

$$\begin{aligned} & -\sum_{i=1}^N \sum_{j=1, j \neq i}^N \left( \oiint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma \right) \\ & = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \iiint_V (\mathbf{J}_i \cdot \mathbf{E}_j + \mathbf{E}_i \cdot \frac{\partial}{\partial t} \mathbf{D}_j + \mathbf{H}_i \cdot \frac{\partial}{\partial t} \mathbf{B}_j) dV \end{aligned} \quad (5)$$

The principle of mutual energy is equivalent to  $N$  sets of Maxwell's equations,

$$L\xi_i = \tau_i, \quad i = 1, \dots, N \quad (6)$$

The corresponding electromagnetic field and current elements are,

$$\xi_i = [\mathbf{E}_i, \mathbf{H}_i]^T, \dots, \tau_i = [\mathbf{J}_i, 0]^T \quad (7)$$

Maxwell equation operator is defined as,

$$L = \begin{bmatrix} -\epsilon_0 \frac{\partial}{\partial t}, & \nabla \times \\ -\nabla \times & -\mu_0 \frac{\partial}{\partial t} \end{bmatrix} \quad (8)$$

$$N \geq 2 \quad (9)$$

The principle of mutual energy can be derived from Maxwell's equations, and Maxwell's equations can also be derived from the principle of mutual energy. However, the principle of mutual energy is not completely equivalent to Maxwell's equations. This is because the last formula  $N \geq 2$ . For the principle of mutual energy,  $N = 1$  is impossible. But for Maxwell's equations, of course,  $N$  can be 1. Both retarded wave or advanced wave is the solution of Maxwell's equations. However, only when the retarded wave and the advanced wave form a pair of Maxwell's equations synchronously, it is the solution of the mutual energy principle.

Axiom 3, energy flow must not overflow the universe. Set a sphere with infinite radius  $\Gamma$ ,  $S_{total}$  is the total energy flow, including self energy flow and mutual energy flow.

$$\oiint_{\Gamma} S_{total} \cdot \hat{n} d\Gamma = 0 \quad (10)$$

Note that this axiom is not satisfied for the self energy flow corresponding to the Poynting vector. Therefore, the classical electromagnetic theory is imperfect.

Mutual energy flow theorem,

$$-\int_{t=-\infty}^{\infty} dt \iiint_{V_1} (\mathbf{J}_1 \cdot \mathbf{E}_2) dV = (\xi_1, \xi_2) = \int_{t=-\infty}^{\infty} dt \iiint_{V_2} (\mathbf{J}_2 \cdot \mathbf{E}_1) dV \quad (11)$$

$$(\xi_1, \xi_2) = \int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\mathbf{E}_1 \times \mathbf{H}_2 + \mathbf{E}_2 \times \mathbf{H}_1) \cdot \hat{n} d\Gamma \quad (12)$$

$\Gamma$  is any close surface or infinite surface which is a segmentation of the the two volumes  $V_1$  and  $V_2$  In the frequency domain the mutual energy flow theorem can be written as,

$$-\iiint_{V_1} (\mathbf{J}_1 \cdot \mathbf{E}_2^*) dV = (\xi_1, \xi_2) = \iiint_{V_2} (\mathbf{J}_2^* \cdot \mathbf{E}_1) dV \quad (13)$$

$$(\xi_1, \xi_2) = \oiint_{\Gamma} (\mathbf{E}_1 \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_1) \cdot \hat{n} d\Gamma \quad (14)$$

#### D. Problems of classical electromagnetic theory

We know that Poynting's theorem can be derived from Maxwell's equations,

$$\oiint_{\Gamma} (\mathbf{E} \times \mathbf{H}) \cdot \hat{n} d\Gamma + \iiint_V (\mathbf{J} \cdot \mathbf{E} + \mathbf{E} \cdot \frac{\partial}{\partial t} \mathbf{D} + \mathbf{H} \cdot \frac{\partial}{\partial t} \mathbf{B}) dV = 0 \quad (15)$$

Considering the superposition principle,

$$\mathbf{E} = \sum_{i=1}^N \mathbf{E}_i \quad \mathbf{H} = \sum_{i=1}^N \mathbf{H}_i \quad \mathbf{J} = \sum_{i=1}^N \mathbf{J}_i \quad (16)$$

Therefore, the Poynting theorem of  $N$  current elements is,

$$\begin{aligned} & - \sum_{i=1}^N \sum_{j=1}^N \left( \oiint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma \right) \\ & = \sum_{i=1}^N \sum_{j=1}^N \iiint_V (\mathbf{J}_i \cdot \mathbf{E}_j + \mathbf{E}_i \cdot \frac{\partial}{\partial t} \mathbf{D}_j + \mathbf{H}_i \cdot \frac{\partial}{\partial t} \mathbf{B}_j) dV \end{aligned} \quad (17)$$

But we know that the mutual energy principle of  $N$  current elements is Eq.(5), that is,

$$\begin{aligned} & - \sum_{i=1}^N \sum_{j=1, j \neq i}^N \left( \oiint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_j) \cdot \hat{n} d\Gamma \right) \\ & = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \iiint_V (\mathbf{J}_i \cdot \mathbf{E}_j + \mathbf{E}_i \cdot \frac{\partial}{\partial t} \mathbf{D}_j + \mathbf{H}_i \cdot \frac{\partial}{\partial t} \mathbf{B}_j) dV = 0 \end{aligned} \quad (18)$$

Both are the energy conservation laws of  $N$  current elements. If they are true, their difference should not transfer energy. That is,

$$\begin{aligned} & - \sum_{i=1}^N \left( \oiint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_i) \cdot \hat{n} d\Gamma \right) \\ & + \sum_{i=1}^N \iiint_V (\mathbf{J}_i \cdot \mathbf{E}_i + \mathbf{E}_i \cdot \frac{\partial}{\partial t} \mathbf{D}_i + \mathbf{H}_i \cdot \frac{\partial}{\partial t} \mathbf{B}_i) dV = 0 \end{aligned} \quad (19)$$

Does not transfer energy. Or request,

$$-\oiint_{\Gamma} (\mathbf{E}_i \times \mathbf{H}_i) \cdot \hat{n} d\Gamma = \iiint_V (\mathbf{J}_i \cdot \mathbf{E}_i + \mathbf{E}_i \cdot \frac{\partial}{\partial t} \mathbf{D}_i + \mathbf{H}_i \cdot \frac{\partial}{\partial t} \mathbf{B}_i) dV \quad (20)$$

does not transfer energy. The above formula is Poynting's theorem,  $\mathbf{J}_i$  is the current element of an antenna. For an antenna, the Poynting vector must not be zero. Therefore, the above formula is to transfer energy. This creates a contradiction.

#### E. Solution of electromagnetic field of plane-sheet current

For the solution of the electromagnetic field of the plane-sheet current, the traditional method is to obtain the magnetic field from the Ampere circuital law, and then obtain the electric field from the magnetic field. However, the author is always dissatisfied with the above method, hoping to solve the electromagnetic field by magnetic vector potential, and then compare it with the former method. If the same result is obtained, the answer is considered reliable. In this paper, the author has found an analytical method to solve the electromagnetic field by vector potential. This method increases the author's confidence in Maxwell's equations. Readers may ask why you doubt Maxwell's equations. Because the wave particle duality problem has not been solved. The conservation of energy flow has not been solved, and the problem that radiation does not overflow the universe has not been solved. The author once doubted whether there was a problem with the solution method of Maxwell's equations. That is, Maxwell's equations are correct, but the problem is that we made a mistake in solving them. The solution of electromagnetic field of plane-sheet current tells the author that there is no problem in solving Maxwell equations. At least for this example. That is, there is no problem in mathematics. Then the problem can only be Maxwell's equations or the classical electromagnetic theory itself. We must propose amendments to this physical theory.

## II. MAGNETIC QUASISTATIC ELECTROMAGNETIC FIELD

Consider Maxwell's equations, displacement current  $\frac{\partial}{\partial t} \mathbf{D} \cong 0$ . Maxwell's equations at this time is,

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (21)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (22)$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B} \quad (23)$$

$$\nabla \times \mathbf{H} = \mathbf{J} \quad (24)$$

Above is the magnetic quasi-static Maxwell's equations. For the vector potential and scalar potential of quasi steady electromagnetic field,

$$\phi \cong \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho}{r} dV \quad (25)$$

$$\mathbf{A} \cong \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}}{r} dV \quad (26)$$

magnetic field,

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (27)$$

electric field,

$$\mathbf{E} = -\frac{\partial}{\partial t} \mathbf{A} - \nabla \phi \quad (28)$$

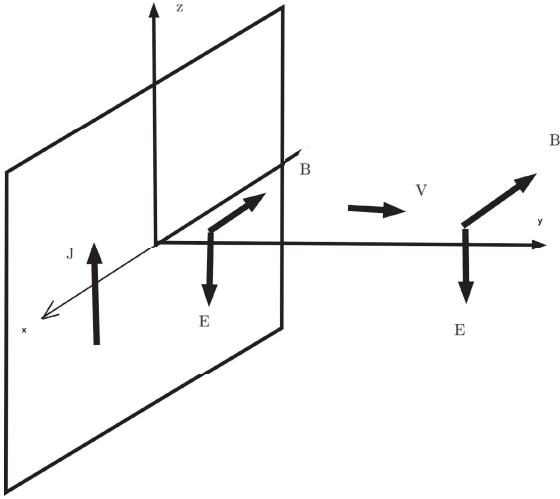


Fig. 1. Electromagnetic field of single plane-sheet current.

*A. Magnetic quasi-static electromagnetic field of plane-sheet current*

As shown in the figure 1, the finite plane  $Oxz$  has current density  $\mathbf{J} = \mathbf{J}_0 \exp(-j\omega t)$ , now calculate the electromagnetic field intensity at  $(0, y, 0)$ . Therefore, it is necessary to find the vector potential  $\mathbf{A}$ , then  $\mathbf{B}$ , and finally  $\mathbf{E}$ . On the plane of  $Oxz$ , a small section of current  $\mathbf{J} dx dz$  which has the distance  $l$  from the origin. This current changes with time. Therefore, the phase delay from this point to  $(0, y, 0)$  is  $jk\sqrt{y^2 + l^2}$ . The retarded vector potential in integral form can be obtained,

$$\mathbf{A} = \iint_{\sigma} J \frac{1}{r} \exp(-jkr) d\sigma \quad (29)$$

Using polar coordinates on the  $Oxz$  plane,

$$\mathbf{A} = \frac{\mu_0}{4\pi} J_0 \exp(j\omega t) \int_{\phi=0}^{2\pi} \int_0^R \frac{\exp(-jk\sqrt{y^2 + l^2})}{\sqrt{y^2 + l^2}} l dl \hat{z} \quad (30)$$

Under the assumption of magnetic quasi-static situation, the retarded factor can be regarded as 1

$$\exp(-jk\sqrt{y^2 + l^2}) \cong 1 \quad (31)$$

The condition of that is,

$$k\sqrt{y^2 + l^2} \ll 2\pi \quad (32)$$

or

$$\frac{2\pi f}{c} \sqrt{y^2 + l^2} \ll 2\pi \quad (33)$$

or

$$\frac{f}{c} \sqrt{y^2 + l^2} \ll 1 \quad (34)$$

or

$$\frac{1}{\lambda} \sqrt{y^2 + r^2} \ll 1 \quad (35)$$

or

$$\sqrt{y^2 + l^2} \ll \lambda \quad (36)$$

Where  $\lambda$  is the wavelength, and  $\mathbf{J} = \mathbf{J}_0 \exp(j\omega t)$ , so,

$$\begin{aligned} \mathbf{A} &= \frac{\mu_0}{4\pi} J \int_{\phi=0}^{2\pi} \int_0^R \frac{1}{\sqrt{y^2 + l^2}} l dl \hat{z} \\ &= \frac{\mu_0}{4\pi} J 2\pi \int_0^R d\sqrt{y^2 + l^2} \hat{z} \\ &= \frac{\mu_0}{2} J \sqrt{y^2 + r^2} \Big|_0^R \hat{z} \\ &= \frac{\mu_0}{2} J (\sqrt{y^2 + R^2} - y) \hat{z} \end{aligned} \quad (37)$$

We know,

$$\mathbf{E} = -j\omega \mathbf{A} - \nabla \phi \quad (38)$$

In this case, consider  $\nabla \phi = 0$ , hence there is,

$$\mathbf{E} = -j\omega \frac{\mu_0}{2} J (\sqrt{y^2 + R^2} - y) \hat{z} \quad (39)$$

$$\mathbf{E}(y=0) = -j\omega \frac{\mu_0}{2} J R \hat{z} \sim jJ(-\hat{z}) \quad (40)$$

” $\sim$ ” is symbol does not care the value, but only care the phase and direction.

$$\begin{aligned} \mathbf{B} = \nabla \times \mathbf{A} &= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & A_z \end{vmatrix} = \hat{x} \frac{\partial A_z}{\partial y} \\ &= \frac{\mu_0}{2} J \frac{\partial}{\partial y} (\sqrt{y^2 + R^2} - y) \hat{x} \\ &= \frac{\mu_0}{2} J \left( \frac{y}{\sqrt{y^2 + R^2}} - 1 \right) \hat{x} \\ &= -\frac{\mu_0}{2} J \left( 1 - \frac{y}{\sqrt{y^2 + R^2}} \right) \hat{x} \end{aligned} \quad (41)$$

$$\mathbf{H} \sim J(-\hat{x}) \quad (42)$$

The direction phase of the magnetic field obtained by the above formula is consistent with that obtained by Ampere circuital law. Poynting vector is,

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}^* \sim jJ(-\hat{z}) \times J^*(-\hat{x}) = jJJ^* \hat{y} \sim j\hat{y} \quad (43)$$

Therefore, Poynting vector is an imaginary number and is reactive power. If Poynting's theorem represents the transmission of energy, there is no energy transmission in this case. However, according to the author's mutual energy theory, Poynting vector is not energy flow. The real energy flow is mutual energy flow. The next section studies mutual energy flow.

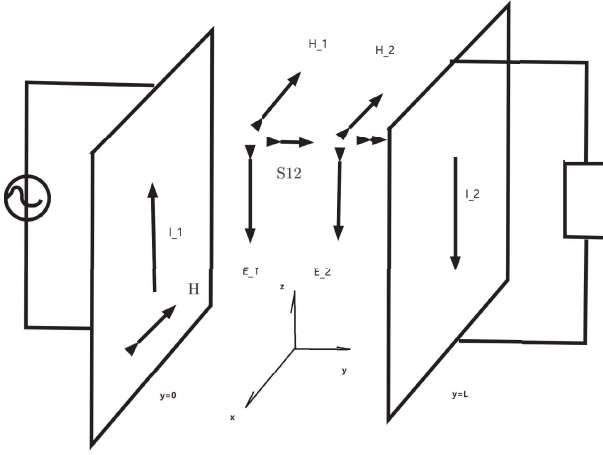


Fig. 2. Double plate transformer.

### B. Electromagnetic field between two coils of transformer

Assume there is a two plate transformer as following figure 2. The primary coil and the secondary coil are all plane-sheet currents. The primary coil has connected to current source, secondary coil has connected to a resistance.

The last section told us for the primary coil of the transformer,

$$\mathbf{E}_1 \sim jJ(-\hat{z}) \quad (44)$$

$$\mathbf{H}_1 \sim J(-\hat{x}) \quad (45)$$

Suppose a plane-sheet current is the primary coil, and another plane-sheet current is the secondary coil on the right side of the primary coil. Current of secondary coil is,

$$I_2 = \frac{\mathcal{E}_1}{R_2 + j\omega L_2} \quad (46)$$

$\mathcal{E}_1$  is the induced electromotive force generated by the primary coil on the secondary coil,  $R_2 + j\omega L_2$  is the impedance of the secondary coil.  $L_2$  is the secondary coil inductance.  $R_2$  is the resistance of the secondary coil, if

$$R \gg \omega L_2 \quad (47)$$

In this case, the secondary coil current is in phase with the electromotive force of the primary coil,

$$I_2 \sim \mathcal{E}_1 \quad (48)$$

or

$$J_2 \sim E_1 \quad (49)$$

According to the formula,  $\mathbf{E}_2 = -\frac{\partial}{\partial t}\mathbf{A}_2 = -j\omega\mathbf{A}_2$  and  $\mathbf{A}_2 \sim \mathbf{J}_2$  we have,

$$\mathbf{E}_2 \sim (-j)J_2 \sim (-j)(jJ) \sim J_1 \quad (50)$$

Same as Eq.(42), the magnetic field can be obtained by using the Ampere circuital law

$$\mathbf{H}_2 \sim J_2 \sim jJ_1 \quad (51)$$

$$\mathbf{E}_2 \times \mathbf{H}_2^* = (J_1)(jJ_1)^*\hat{z} \sim -j\hat{z} \quad (52)$$

The Poynting vector on the secondary coil is reactive power. Mixed Poynting vector corresponding to mutual energy flow,

$$\begin{aligned} \mathbf{S}_m &= \mathbf{E}_1 \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_1 \\ &\sim (E_1 H_2^* + E_2^* H_1) \hat{z} \\ &\sim ((j)(j)^* + (1)^*(1)) J_1 J_1^* \hat{z} = \hat{z} \end{aligned} \quad (53)$$

The mutual energy flow is from the primary coil to the secondary coil. Therefore, we can explain the energy flow from the primary coil to the secondary coil by the mutual energy flow. This cannot be explained by the self energy flow corresponding to the Poynting vector.

### C. Mutual energy flow outside two coils of transformer

Since the magnetic field is reversed on both sides of the plane-sheet current, when the mutual energy flows to the right of the secondary coil, the mutual energy flow cancels out because the magnetic field of the secondary coil is reversed.

$$\mathbf{H}_2(y = L + \delta) = -\mathbf{H}_2(y = L - \delta) \quad (54)$$

The secondary plane-sheet current is at the place of  $y = L$ .  $\delta$  is a very small amount.  $L + \delta$  is at the right side of the plane-sheet current.  $L - \delta$  is at the left side of the plane-sheet current. We know,

$$\mathbf{S}_{12} = \mathbf{E}_1 \times \mathbf{H}_2^* \quad (55)$$

$$\mathbf{S}_{12}(y = L + \delta) = -\mathbf{S}_{12}(y = L - \delta) \quad (56)$$

Similarly, on the left side of the primary coil, due to the change of magnetic field direction,

$$\mathbf{H}_1(y = 0 - \delta) = -\mathbf{H}_1(y = 0 + \delta) \quad (57)$$

Considering,

$$\mathbf{S}_{21} = \mathbf{E}_2^* \times \mathbf{H}_1 \quad (58)$$

There is,

$$\mathbf{S}_{21}(y = 0 - \delta) = -\mathbf{S}_{21}(y = 0 + \delta) \quad (59)$$

Therefore, the mutual energy flow is,

$$\mathbf{S}_m = \mathbf{S}_{12} + \mathbf{S}_{21} \sim \begin{cases} 0 & -\infty < y < 0 \\ \hat{z} & 0 \leq y \leq L \\ 0 & 0 < y < \infty \end{cases} \quad (60)$$

Therefore, it is clear from the above formula that the transformer mutual energy flow is generated on the primary coil and annihilated on the secondary coil. Mutual energy flow is the only energy flow of transformer. The self energy flow corresponding to the Poynting vector is reactive power,

$$\Re(\mathbf{S}_{11}) = \Re(\mathbf{E}_1 \times \mathbf{H}_1^*) = 0$$

$$\Re(\mathbf{S}_{22}) = \Re(\mathbf{E}_2 \times \mathbf{H}_2^*) = 0$$

Here  $\Re$  means taking the real part of a complex number. Therefore, the self-energy flow corresponding to Poynting vector does not transfer energy. At least it is correct for this example.

### III. RADIATED ELECTROMAGNETIC FIELD

Maxwell's equations at this case is,

$$\nabla \cdot \mathbf{e} = \frac{\rho}{\epsilon_0} \quad (61)$$

$$\nabla \cdot \mathbf{b} = 0 \quad (62)$$

$$\nabla \times \mathbf{e} = -\frac{\partial}{\partial t} \mathbf{b} \quad (63)$$

$$\nabla \times \mathbf{h} = \mathbf{J} + \frac{\partial}{\partial t} \mathbf{d} \quad (64)$$

Considering current density  $\mathbf{J} = \mathbf{J}_0 \exp(-j\omega t)$ ,  $\rho = \rho_0 \exp(-j\omega t)$ . In the above formula, we use lowercase letters to represent the radiated electromagnetic field. Capital letters indicate magnetic quasi-static electromagnetic fields. The author believes that the magnetic quasi-static electromagnetic field and the radiated electromagnetic field are two different systems. Their properties are very different. So use different symbols to distinguish them.

#### A. Retarded potential

The retarded potential can be written as,

$$\phi^{(+)} = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho}{r} \exp(-jk \cdot \mathbf{r}) dV \quad (65)$$

$$\mathbf{a}^{(+)} = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}}{r} \exp(-jk \cdot \mathbf{r}) dV \quad (66)$$

Above is the magnetic quasi-static Maxwell's equations. As shown in the figure 1, the finite plane  $Oxz$  has current density  $\mathbf{J} = \mathbf{J}_0 \exp(j\omega t)$ , now calculate the electromagnetic field intensity at  $(0, y, 0)$ . Therefore, it is necessary to find the vector potential  $\mathbf{a}^{(+)}$ , then  $\mathbf{b}$ , and finally  $\mathbf{e}$ . On the plane of  $Oxz$ , a small section of current  $\mathbf{J} dx dz$  which has the distance  $l$  from the origin. This current changes with time. Therefore, the phase delay from this point to  $(0, y, 0)$  is  $jk\sqrt{y^2 + l^2}$  to obtain the retarded vector potential in integral form,

$$\begin{aligned} \mathbf{a}^{(+)} &= \frac{\mu_0}{4\pi} J_0 \exp(j\omega t) \int_{\phi=0}^{2\pi} \int_0^R \frac{\exp(-jk\sqrt{y^2 + l^2})}{\sqrt{y^2 + l^2}} l dl \hat{z} \\ &= \frac{\mu_0}{4\pi} J_0 \exp(j\omega t) 2\pi \int_0^R \exp(-jk\sqrt{y^2 + l^2}) \frac{l}{\sqrt{y^2 + l^2}} dl \hat{z} \\ &= \frac{\mu_0}{2} J_0 \exp(j\omega t) \int_0^R \exp(-jk\sqrt{y^2 + r^2}) d\sqrt{y^2 + r^2} \hat{z} \\ &= \frac{\mu_0}{-jk2} J_0 \exp(j\omega t) \int_0^R \exp(-jk\sqrt{y^2 + l^2}) d(-jk\sqrt{y^2 + l^2}) \hat{z} = \frac{\mu_0}{2} J_0 [-\exp(j(\omega t - ky)) + \exp(j(\omega t - k\sqrt{y^2 + R^2}))] \hat{z} \\ &= \frac{1}{-jk} \frac{\mu_0}{2} J_0 \exp(j\omega t) \exp(-jk\sqrt{y^2 + l^2}) \Big|_0^R \hat{z} \\ &= \frac{\mu_0}{-jk2} J_0 \exp(j\omega t) [\exp(-jk\sqrt{y^2 + R^2}) - \exp(-jk y)] \hat{z} \end{aligned} \quad (67)$$

or

$$\mathbf{a}^{(+)} = \frac{1}{jk} \frac{\mu_0}{2} J_0 [\exp(j(\omega t - ky)) - \exp(j(\omega t - k\sqrt{y^2 + R^2}))] \hat{z} \quad (68)$$

electric field,

$$\begin{aligned} \mathbf{e} &= -\frac{\partial}{\partial t} \mathbf{a}^{(+)} = -j\omega \mathbf{a}^{(+)} \\ &= \frac{-j\omega}{jk} \frac{\mu_0}{2} J_0 [\exp(j(\omega t - ky)) - \exp(j(\omega t - k\sqrt{y^2 + R^2}))] \hat{z} \\ &= -\frac{\omega}{k} \frac{\mu_0}{2} J_0 [\exp(j(\omega t - ky)) - \exp(j(\omega t - k\sqrt{y^2 + R^2}))] \hat{z} \end{aligned} \quad (69)$$

In the latter term, when  $R \rightarrow \infty$ , the rapid vibration is divergent, but the average value is zero. Therefore, it can be ignored.

$$\begin{aligned} \mathbf{e} &= -\frac{\omega}{k} \frac{\mu_0}{2} J_0 \exp(j(\omega t - ky)) \hat{z} \\ &= -\frac{\omega}{k} \frac{\mu_0}{2} J_0 \exp(j(\omega t - ky)) \hat{z} \\ &= -\frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} \frac{\mu_0}{2} J_0 \exp(j(\omega t - ky)) \hat{z} \\ &= -\frac{J_0}{2} \sqrt{\frac{\mu_0}{\epsilon_0}} \exp(j(\omega t - ky)) \hat{z} \end{aligned} \quad (70)$$

In the above we have considered that,

$$k = \frac{\omega}{c} = \omega\sqrt{\mu_0\epsilon_0} \quad (71)$$

hence,

$$\mathbf{e} = -\frac{J_0\eta_0}{2} \exp(j(\omega t - ky)) \hat{z} \quad (72)$$

where  $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ . The magnetic field,

$$\begin{aligned} \mathbf{b} &= \nabla \times \mathbf{a}^{(+)} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & \mathbf{a}_z^{(+)} \end{vmatrix} \\ &= \hat{x} \frac{\partial \mathbf{a}_z^{(+)}}{\partial y} \\ &= \frac{1}{jk} \frac{\mu_0}{2} J_0 \frac{\partial}{\partial y} [\exp(j(\omega t - ky)) - \exp(j(\omega t - k\sqrt{y^2 + R^2}))] \hat{x} \\ &= \frac{1}{jk} \frac{\mu_0}{2} J_0 [(-jk) \exp(j(\omega t - ky)) \\ &\quad - (-jk) \exp(j(\omega t - k\sqrt{y^2 + R^2})) \frac{y}{\sqrt{y^2 + R^2}}] \hat{x} \\ &= \frac{\mu_0}{2} J_0 [-\exp(j(\omega t - ky)) + \exp(j(\omega t - k\sqrt{y^2 + R^2})) \frac{y}{\sqrt{y^2 + R^2}}] \hat{x} \end{aligned} \quad (73)$$

When  $R \rightarrow \infty$  is considered, the latter term of magnetic field tends to zero, so

$$\mathbf{b} = -\frac{\mu_0}{2} J_0 \exp(j(\omega t - ky)) \hat{x} \quad (74)$$

We calculate electric field from  $\mathbf{b}$ ,

$$\nabla \times \mathbf{h} = \frac{\partial}{\partial t} \epsilon_0 \mathbf{e} \quad (75)$$

$$-j\mathbf{k} \times \mathbf{h} = j\omega\epsilon_0 \mathbf{e} \quad (76)$$

$$-\frac{\mathbf{k}}{\omega} \times \mathbf{h} = \epsilon_0 \mathbf{e} \quad (77)$$

$$\mathbf{k} \times \hat{\mathbf{x}} = k\hat{\mathbf{y}} \times \hat{\mathbf{x}} = -k\hat{\mathbf{z}} \quad (78)$$

$$\begin{aligned} \mathbf{e} &= -\frac{1}{\epsilon_0} \frac{\mathbf{k}}{\omega} \times \mathbf{h} = \frac{1}{\epsilon_0} \frac{k}{\omega} h \hat{\mathbf{z}} \\ &= \frac{1}{\epsilon_0} \frac{k}{\omega} \left( -\frac{1}{2} J_0 \exp(j(\omega t - ky)) \right) \hat{\mathbf{z}} \\ &= -\frac{1}{2} J_0 \frac{1}{\epsilon_0} \frac{k}{\omega} \exp(j(\omega t - ky)) \hat{\mathbf{z}} \end{aligned} \quad (79)$$

considering,

$$\exp(j\omega(t - r/c)) = \exp(j(\omega t - \frac{\omega}{c}r)) \quad (80)$$

$$k = \frac{\omega}{c} = \omega \sqrt{\mu_0 \epsilon_0} \quad (81)$$

or

$$\begin{aligned} \mathbf{e} &= -\frac{1}{2} J_0 \frac{1}{\epsilon_0} \frac{\omega \sqrt{\mu_0 \epsilon_0}}{\omega} \exp(j(\omega t - ky)) \hat{\mathbf{z}} \\ &= -\frac{1}{2} J_0 \sqrt{\frac{\mu_0}{\epsilon_0}} \exp(j(\omega t - ky)) \hat{\mathbf{z}} \end{aligned} \quad (82)$$

Formula Eq.(76) is consistent with formula Eq.(72).

$$\begin{aligned} \mathbf{S} &= \mathbf{e} \times \mathbf{h}^* \\ &= \left( -\frac{1}{2} J_0 \eta_0 \exp(j(\omega t - ky)) \hat{\mathbf{z}} \right) \times \left( -\frac{1}{2} J_0 \exp(j(\omega t - ky)) \hat{\mathbf{x}} \right)^* \\ &= \frac{1}{4} |J_0|^2 \eta_0 \hat{\mathbf{z}} \times \hat{\mathbf{x}}^* = \frac{1}{4} |J_0|^2 \eta_0 \hat{\mathbf{y}} \end{aligned} \quad (83)$$

Above is the energy flow to the right of the plane-sheet current, so it can be written,

$$\mathbf{S}_{right} = \frac{1}{4} |J_0|^2 \eta_0 \hat{\mathbf{y}} \quad (84)$$

Poynting power flowing to the right

$$\hat{\mathbf{y}} \cdot \mathbf{S}_{right} = \frac{1}{4} |J_0|^2 \eta_0 \quad (85)$$

Similarly, the power flowing out to the left is,

$$\mathbf{S}_{left} \cdot (-\hat{\mathbf{y}}) = \frac{1}{4} |J_0|^2 \eta_0 \quad (86)$$

and

$$\begin{aligned} \mathbf{S}_{right} \cdot \hat{\mathbf{y}} + \mathbf{S}_{left} \cdot (-\hat{\mathbf{y}}) \\ &= \frac{1}{4} |J_0|^2 \eta_0 + \frac{1}{4} |J_0|^2 \eta_0 = \frac{1}{2} |J_0|^2 \eta_0 \end{aligned} \quad (87)$$

The total output power of plane-sheet current is,

$$\begin{aligned} &-\mathbf{e}(y=0) \cdot \mathbf{J} \\ &= -\left( -\frac{1}{2} J_0 \sqrt{\frac{\mu_0}{\epsilon_0}} \hat{\mathbf{z}} \right) \cdot J_0 \hat{\mathbf{z}} \end{aligned}$$

$$= \frac{1}{2} \eta_0 |J_0|^2 \quad (88)$$

The above two formulas show that Poynting's theorem is satisfied

$$\oiint_{\sigma} \mathbf{S} \cdot \hat{\mathbf{n}} d\sigma = - \iint_{\sigma} \mathbf{e} \cdot \mathbf{J}^* d\sigma \quad (89)$$

Here  $\sigma$  is the area of plane-sheet current. The above formula can be written as.

$$\mathbf{S}_{right} \cdot \hat{\mathbf{y}} + \mathbf{S}_{left} \cdot (-\hat{\mathbf{y}}) = -\mathbf{e} \cdot \mathbf{J}^* \quad (90)$$

The above formula tell us the output power of the current  $\mathbf{J}$  is same as the power of the Poynting energy flow.

### B. The advanced wave

The advanced potential can be written as following,

$$\phi^{(-)} = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho}{r} \exp(+j\mathbf{k} \cdot \mathbf{r}) dV \quad (91)$$

$$\mathbf{a}^{(-)} = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}}{r} \exp(+j\mathbf{k} \cdot \mathbf{r}) dV \quad (92)$$

As shown in the figure 1, the finite plane  $Oxz$  has current density  $\mathbf{J} = J_0 \exp(j\omega t)$ , now calculate the electromagnetic field intensity at  $(0, y, 0)$ . Therefore, it is necessary to find the vector potential  $\mathbf{a}^{(-)}$ , then  $\mathbf{b}$ , and finally  $\mathbf{e}$ . On the plane of  $Oxz$ , a small section of current  $\mathbf{J} dx dz$  which has the distance  $l$  from the origin. This current changes with time. Therefore, the phase delay from this point to  $(0, y, 0)$  is  $k\sqrt{y^2 + l^2}$  to obtain the retarded vector potential in integral form,

$$\begin{aligned} \mathbf{a}^{(-)} &= \frac{\mu_0}{4\pi} J_0 \exp(j\omega t) \int_{\phi=0}^{2\pi} \int_0^R \frac{\exp(+jk\sqrt{y^2 + l^2})}{\sqrt{y^2 + l^2}} l dl \hat{\mathbf{z}} \\ &= \frac{\mu_0}{4\pi} J_0 \exp(j\omega t) 2\pi \int_0^R \exp(+jk\sqrt{y^2 + l^2}) \frac{l}{\sqrt{y^2 + l^2}} dl \hat{\mathbf{z}} \\ &= \frac{\mu_0}{4\pi} J_0 \exp(j\omega t) 2\pi \int_0^R \exp(+jk\sqrt{y^2 + r^2}) d\sqrt{y^2 + r^2} \hat{\mathbf{z}} \\ &= \frac{\mu_0}{4\pi} J_0 \exp(j\omega t) 2\pi \int_0^R \frac{1}{+jk} \exp(+jk\sqrt{y^2 + l^2}) d(+jk\sqrt{y^2 + l^2}) \hat{\mathbf{z}} \\ &= \frac{1}{+jk} \frac{\mu_0}{2} J_0 \exp(j\omega t) \exp(+jk\sqrt{y^2 + l^2}) \Big|_0^R \hat{\mathbf{z}} \\ &= \frac{1}{+jk} \frac{\mu_0}{2} J_0 \exp(j\omega t) [\exp(+jk\sqrt{y^2 + R^2}) - \exp(+jky)] \hat{\mathbf{z}} \end{aligned} \quad (93)$$

or

$$\mathbf{a}^{(-)} = -\frac{1}{jk} \frac{\mu_0}{2} J_0 [\exp(j(\omega t + ky)) - \exp(j(\omega t + k\sqrt{y^2 + R^2}))] \hat{\mathbf{z}} \quad (94)$$

$$\mathbf{e} = -\frac{\partial}{\partial t} \mathbf{a}^{(-)} = -j\omega \mathbf{a}^{(-)}$$

$$= \frac{-j\omega}{-jk} \frac{\mu_0}{2} J_0 [\exp(j(\omega t + ky)) - \exp(j(\omega t + k\sqrt{y^2 + R^2}))] \hat{\mathbf{z}}$$

$$= \frac{\omega}{k} \frac{\mu_0}{2} J_0 [\exp(j(\omega t + ky)) - \exp(j(\omega t + k\sqrt{y^2 + R^2}))] \hat{\mathbf{z}}$$

$$\begin{aligned}
 &= \frac{\omega}{k} \frac{\mu_0}{2} J_0 \exp(j(\omega t + ky)) \hat{z} \\
 &= \frac{\omega}{k} \frac{\mu_0}{2} J_0 \exp(j(\omega t + ky)) \hat{z} \\
 &= \frac{\omega}{\omega \sqrt{\mu_0 \epsilon_0}} \frac{\mu_0}{2} J_0 \exp(j(\omega t + ky)) \hat{z} \\
 &= \frac{J_0}{2} \sqrt{\frac{\mu_0}{\epsilon_0}} \exp(j(\omega t + ky)) \hat{z} \quad (95)
 \end{aligned}$$

Among them

$$k = \frac{\omega}{c} = \omega \sqrt{\mu_0 \epsilon_0} \quad (96)$$

In the latter term, when  $R \rightarrow \infty$ , the rapid vibration is divergent, but the average value is zero. Therefore, it can be ignored.

$$\mathbf{e} = \frac{J_0 \eta_0}{2} \exp(j(\omega t + ky)) \hat{z} \quad (97)$$

$$\begin{aligned}
 \mathbf{b} = \nabla \times \mathbf{a}^{(-)} &= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & \mathbf{a}_z^{(+)} \end{vmatrix} \\
 &= \hat{x} \frac{\partial \mathbf{a}_z^{(-)}}{\partial y}
 \end{aligned}$$

$$\begin{aligned}
 &= -\frac{1}{jk} \frac{\mu_0}{2} J_0 \frac{\partial}{\partial y} [\exp(j(\omega t + ky)) - \exp(j(\omega t + k\sqrt{y^2 + R^2}))] \hat{x} \\
 &= -\frac{1}{jk} \frac{\mu_0}{2} J_0 [(+jk) \exp(j(\omega t + ky)) \\
 &\quad - (+jk) \exp(j(\omega t + k\sqrt{y^2 + R^2})) \frac{y}{\sqrt{y^2 + R^2}}] \hat{x} \\
 &= \frac{\mu_0}{2} J_0 [-\exp(j(\omega t + ky)) \\
 &\quad + \exp(j(\omega t + k\sqrt{y^2 + R^2})) \frac{y}{\sqrt{y^2 + R^2}}] \hat{x} \quad (98)
 \end{aligned}$$

When  $R \rightarrow \infty$  is considered, the latter term of magnetic field tends to zero, so

$$\mathbf{b} = -\frac{\mu_0 J_0}{2} \exp(j(\omega t + ky)) \hat{x} \quad (99)$$

Let's find the electric field from  $\mathbf{b}$ ,

$$\nabla \times \mathbf{h} = \frac{\partial}{\partial t} \epsilon_0 \mathbf{e} \quad (100)$$

$$+ j\mathbf{k} \times \mathbf{h} = j\omega \epsilon_0 \mathbf{e} \quad (101)$$

$$\frac{\mathbf{k}}{\omega} \times \mathbf{h} = \epsilon_0 \mathbf{e} \quad (102)$$

Hence the electric field,

$$\begin{aligned}
 \mathbf{e} &= \frac{1}{\epsilon_0} \frac{\mathbf{k}}{\omega} \times \mathbf{h} \\
 &= \frac{1}{\epsilon_0} \frac{1}{\omega} (k\hat{y}) \times \left(-\frac{1}{2} J_0 \exp(j(\omega t + ky)) \hat{x}\right) \\
 &= \frac{1}{\epsilon_0} \frac{k}{\omega} \frac{1}{2} J_0 \exp(j(\omega t + ky)) \hat{z}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} J_0 \frac{1}{\epsilon_0} \frac{k}{\omega} \exp(j(\omega t + ky)) \hat{z} \\
 &= \frac{1}{2} J_0 \eta_0 \exp(j(\omega t + ky)) \hat{z} \quad (103)
 \end{aligned}$$

Formula Eq.(100) is consistent with formula Eq.(91). For the advanced wave, the electric field and current direction are the same,

$$\mathbf{e}^{(-)}(y=0) \sim \mathbf{J}_0 \quad (104)$$

This is inconsistent with the retarded wave. The electric field of the retarded wave is in the opposite direction of the current,

$$\mathbf{e}^{(+)}(y=0) \sim -\mathbf{J}_0 \quad (105)$$

For the Poynting vector direction of the advanced wave,

$$\begin{aligned}
 \mathbf{S} &= \mathbf{e}^{(-)} \times (\mathbf{h}^{(-)})^* \\
 &= \left(\frac{1}{2} J_0 \sqrt{\frac{\mu_0}{\epsilon_0}} \exp(j(\omega t + ky)) \hat{z}\right) \times \left(-\frac{1}{2} J_0 \exp(j(\omega t + ky)) \hat{x}\right)^* \\
 &= \frac{1}{4} |J_0|^2 \eta_0 (-\hat{y}) \quad (106)
 \end{aligned}$$

The direction of Poynting vector is pointing to  $-\hat{y}$ . This is correct. Energy is transmitted to the plane-sheet current. In short, whether it is an advanced wave or a retarded wave, the direction of the magnetic field near the current boundary is always determined by the Ampere circuital law. The direction of the electric field, if it is a retarded wave, is opposite to the direction of the current. If it is an advanced wave, it is consistent with the direction of the current.

### C. Energy flow of transformer

It is assumed that the primary coil is a plane-sheet current and the secondary coil is also a plane-sheet current. The secondary coil is on the right side of the primary coil. The two plane-sheet current are infinite large. From last section we obtained that, for the primary coil with retarded wave,

$$\mathbf{e}_1 = -\frac{1}{2} J_{10} \eta_0 \exp(j(\omega t - ky)) \hat{z} \quad (107)$$

$$\mathbf{h}_1 = -\frac{1}{2} J_{10} \exp(j(\omega t - ky)) \hat{x} \quad (108)$$

Suppose the secondary coil is close to the primary coil, the two coil are close together. For the secondary coil, we assume that the current is in the same direction as the induced electric field of the primary coil and points to the  $-\hat{z}$  direction. For the advanced wave on the left side of the plate, the direction of motion of the wave is to the right. Current direction of secondary coil  $\mathbf{J}_2$  and the electric field  $\mathbf{e}_1$  are consistent, so there are,

$$\mathbf{J}_2 = \mathbf{e}_1(y=0)\sigma$$

$\sigma$  is the conductivity of the secondary coil.  $\mathbf{e}_1$  is the electric field on the secondary coil produced by the primary coil. We assume the load resistance of the secondary coil is because of this conductivity.



$$\mathbf{e}_1(y=0) = -\frac{1}{2}J_{10}\eta_0\sigma \exp(j(\omega t))\hat{z}$$

Hence,

$$J_{20} = \frac{1}{2}J_{10}\eta_0\sigma$$

The electric field of the secondary coil can be calculated same as  $\mathbf{e}_1$  in Eq.(104),

$$\begin{aligned} \mathbf{e}_2 &= \frac{1}{2}J_{20}\eta_0 \exp(j(\omega t - ky))(-\hat{z}) \\ &= -\frac{1}{2}\left(\frac{1}{2}J_{10}\eta_0\sigma\right)\eta_0 \exp(j(\omega t - ky))\hat{z} \\ &= -\frac{1}{4}J_{10}\eta_0^2\sigma \exp(j(\omega t - ky))\hat{z} \end{aligned} \quad (109)$$

The amount of  $J_{20}$  is determined by the load resistance on the secondary coil. We do not give the specific value of this resistance, but only tell that the resistance is large enough, so the secondary coil is almost pure Resistive impedance. So this current is in the same direction and phase as the electric field  $\mathbf{e}_1$ . The above formula tell us the advanced wave of the secondary coil has the same direction with the electric field of the primary coil.

The direction of the magnetic field is determined by Amperé's circuital law,

$$\begin{aligned} \mathbf{h}_2 &= -\frac{1}{2}J_{20} \exp(j(\omega t - ky))\hat{x} \\ &\sim \frac{1}{2}\left(\frac{1}{2}J_{10}\eta_0\sigma\right) \exp(j(\omega t - ky))(-\hat{x}) \end{aligned} \quad (110)$$

It is considered that the advanced wave on the left side of the plate is the wave running to the right. Hence the mutual energy flow is,

$$\begin{aligned} \mathbf{S}_m &= \mathbf{e}_1 \times \mathbf{h}_2^* + \mathbf{e}_2^* \times \mathbf{h}_1 \\ &= \frac{1}{2}J_{10}\eta_0 \exp(j(\omega t - ky))\hat{z} \times \left(\frac{1}{4}(J_{10}\eta_0\sigma) \exp(j(\omega t - ky))\hat{x}\right)^* \\ &\quad + \frac{1}{4}(J_{10}\eta_0\sigma)\eta_0 \exp(j(\omega t - ky))\hat{z} \times \frac{1}{2}J_{10} \exp(j(\omega t - ky))\hat{x} \\ &= \left(\frac{1}{2}J_{10}\eta_0\hat{z}\right) \times \left(\frac{1}{4}(J_{10}\eta_0\sigma)\right)^* + \left(\frac{1}{4}(J_{10}\eta_0\sigma)\eta_0\hat{z}\right)^* \left(\frac{1}{2}J_{10}\hat{x}\right) \\ &= \frac{1}{4}J_{10}^2\eta_0^2\sigma\hat{y} \end{aligned} \quad (111)$$

Now let us calculate the power offered by the current  $\mathbf{J}_1$

$$\begin{aligned} &-\mathbf{e}_2(y=0)^* \cdot \mathbf{J}_{10} \\ &\frac{1}{2}\left(\frac{1}{2}J_{10}\eta_0\sigma\right)\eta_0 J_{10} \\ &= \frac{1}{4}J_{10}^2\eta_0^2\sigma \end{aligned} \quad (112)$$

and

$$\mathbf{e}_1(y=0) \cdot \mathbf{J}_2^*$$

$$\begin{aligned} &= \left(-\frac{1}{2}J_{10}\eta_0\hat{z}\right) \cdot \frac{1}{2}J_{10}\eta_0\sigma(-\hat{z}) \\ &= \frac{1}{4}\eta_{10}^2J_{10}^2\sigma \end{aligned} \quad (113)$$

Thus, the mutual energy flow theorem is satisfied,

$$-\iint_{\sigma_1} \mathbf{e}_2^* \cdot \mathbf{J}_1 d\sigma = \iint_{\Gamma} (\mathbf{e}_1 \times \mathbf{h}_2^* + \mathbf{e}_2^* \times \mathbf{h}_1) \cdot \hat{n} d\Gamma = \iint_{\sigma_2} \mathbf{e}_1 \cdot \mathbf{J}_2^* d\sigma \quad (114)$$

or

$$-\mathbf{e}_2^* \cdot \mathbf{J}_1 = (\mathbf{e}_1 \times \mathbf{h}_2^* + \mathbf{e}_2^* \times \mathbf{h}_1) \cdot \hat{n} = \mathbf{e}_1 \cdot \mathbf{J}_2^* \quad (115)$$

Where  $\hat{n} = \hat{y}$ . The above verifies that the mutual energy flow theorem is strictly satisfied for this example.

On the other hand, Poynting vector can not explain the energy flow from the primary coil to the secondary coil of the transformer. For the primary coil, Poynting's theorem tells us that the primary coil has an energy flow to the left and right at the same time. For the secondary coil, there is also an energy flow into the coil from left and right at the same time. However, these energy flows obviously do not explain any problem. Since the conductivity on the secondary coil can be arbitrarily determined, the amount of energy flow received by the secondary coil can be completely inconsistent with the output energy flow of the primary coil. Therefore, Poynting's theorem is indeed worthless, especially for devices such as transformers. The energy flow represented by Poynting vector seems to be expressed as its energy flow only when there is perfect energy absorption around the primary coil (left and right sides).

#### D. Energy flow outside the transformer

We have already mentioned that for the flat plane-sheet current, a rightward wave and a leftward wave can be generated. For the wave to the right, the retarded wave is on the right side of the current and, the advanced wave is on the left side of the current. For the wave to the left, there is a retarded wave on the left side of the current and an advanced wave on the right side of the current. For the primary coil, because there is a secondary coil on its right side, a rightward wave is generated, and the leftward wave is an invalid wave. For the secondary coil, because there is a primary coil on its left to provide energy, it has an advanced wave on its left and a retarded wave on its right. Such a wave is also a wave to the right. Therefore, the waves of both coils are to the right. Therefore, the two waves can be synchronized.

However, the sign of the magnetic field is reversed when the wave passes through the plate. The two terms of mutual energy flow just offset after passing through the plate. Suppose the primary coil is at  $y=0$ . The secondary coil is at  $y=L$ .  $L$  can be large or very small (close to 0). Mutual energy flow

$$\mathbf{S}_m = \mathbf{e}_1 \times \mathbf{h}_2^* + \mathbf{e}_2^* \times \mathbf{h}_1 \quad (116)$$

$$= \begin{cases} 0 & -\infty < y < 0 \\ \frac{1}{4}\eta_0^2J_{10}^2\sigma & 0 \leq y \leq L \\ 0 & 0 < y < \infty \end{cases} \quad (117)$$

This shows that the mutual energy flow is generated on the primary coil and annihilated on the secondary coil. It conforms to the properties of photons. Therefore, the author has always adhered to the belief that mutual energy flow is photon. If we use Poynting energy flow, we can't describe the nature of energy flow generated and annihilated somewhere.

Although we use the transformer as an example above, in fact, because  $L$  can be large, the primary coil plays the role of transmitting antenna and the secondary coil plays the role of receiving antenna. In the author's mutual energy theory, the primary coil of transformer, transmitting antenna and radiator charge are the source of electromagnetic wave or light; The secondary coil of the transformer, the receiving antenna and the absorber charge are the sinks of electromagnetic waves or light. Mutual energy theory is the theory of interaction between source and sink. Obey the law of conservation of energy, the principle of mutual energy and the theorem of mutual energy flow. Here, the primary coil of the transformer, the transmitting antenna and the radiator charge are essentially the same. Generate retarded wave and send energy. The secondary coil of the transformer, the receiving antenna and the absorber charge are the sinks of electromagnetic wave or light, and the essence is the same, generating advanced wave to absorb energy.

#### E. Problems

For the above transformer, the mutual energy flow represents the energy flow, which correctly describes the fact that the energy flow from the primary coil to the secondary coil. It also describes the fact that the energy flow is generated on the primary coil and annihilated on the secondary coil.

Self energy flow is also correct. For example, Poynting vector of primary coil  $\mathbf{e}_1 \times \mathbf{h}_1^*$  it describes the energy flow if there are secondary coils on both sides of the primary coil. The two secondary coils must can absorb all the energy flow of the primary coil send out.

Another example is the Poynting vector for the secondary coil  $\mathbf{e}_2 \times \mathbf{h}_2^*$  represents the energy flow when there are primary coils on both sides of the secondary coil. This energy flow is directed to the current of the secondary coil, so it absorbs energy.

However, it makes no sense to add the energy flow transmitted by self energy flow to the energy flow of mutual energy flow. The energy of the total energy flow is more than that of the current output. The author is an advocate of the theory of mutual energy. Of course, he supports that mutual energy flow is the real carrier of energy flow. Then self energy flow should not transfer energy flow.

Poynting energy cannot describe the nature of energy flow generated from one coil and annihilated on another coil.

The author has introduced the concept of time-reversal wave which is additional wave added to the top of classical electromagnetic field theory. The time-reversal wave can cancel the self-energy flow[8].

If we do not introduce the time-reversal wave, we can just say the electromagnetic field is a auxiliary function, hence, it does not carry energy. The energy flow is the mutual energy

flow. The above example shows the author's view of point is correct.

#### IV. INTERPRETATION OF QUANTUM MECHANICS

The author puts forward the mutual energy flow interpretation of quantum mechanics [8], which is very close to the transactional interpretation proposed by Cramer [3], [4].

This paper provides an example of electromagnetic field. From this example, we can further see the similarities and differences between the author's interpretation of mutual energy flow and transactional interpretation. The formula (60,114) can be regarded as that photons are generated on the source and annihilated on the sink. Annihilation is caused by the reversal of the direction of the magnetic field on the plane-sheet current.

In Cramer's transactional interpretation of quantum mechanics, on the right side of the light sink, the retarded wave emitted by the light sink and the retarded wave emitted by the light source have a phase difference of 180 degrees, so the waves are offset. On the left side of the light source, the advanced wave emitted by the light source and the advanced wave emitted by the light sink have a phase difference of 180 degrees, so the waves are offset. But the reason why there is a 180 degree phase difference cannot be given. In the author's mutual energy flow theory, two mutual energy flow components  $\mathcal{S}_{12} = \mathbf{e}_1 \times \mathbf{h}_2^*$  and  $\mathcal{S}_{21} = \mathbf{e}_2^* \times \mathbf{h}_1$  are superimposed. Between two plane-sheet currents, these two components can be superimposed. Outside the two plane-sheet currents, due to the reversal of the magnetic field, the two components change from addition to elimination. The reverse of the magnetic field direction naturally explains the cancellation of the two components.

In addition, the sink proposed by Cramer generates a confirmation wave, which is also answered by the electromagnetic field generated by the induced current of the plane-sheet current of the secondary coil. Therefore, the author's mutual energy theory is a concrete realization of Cramer's transactional interpretation. The interpretation of mutual energy flow can also be regarded as a further development of transactional interpretation.

#### V. THE AUTHOR'S GUESS

The above two chapters describe the magnetic quasi-static magnetic field and the radiated electromagnetic field. For the magnetic quasi-static magnetic field, the self energy flow (the energy flow corresponding to the Poynting vector) is reactive power, so it does not transfer energy. Energy is always transferred by mutual energy flow. The mutual energy flow can be used to describe the energy flow from the primary coil to the secondary coil of the transformer.

Corresponding to radiated electromagnetic field. Both self-energy flow and mutual energy flow transfer energy. The self energy flow of the primary coil describes the energy flow if the surrounding is full of secondary coils which can absorb all energy flow sent out by the primary coil. The following is the author's guess.

(1a) the magnetic quasi-static magnetic field is correct. Therefore, when developing the radiated electromagnetic field theory, we must modify the electromagnetic field theory so that the self energy flow still maintains the radiated reactive power. The corresponding plane-sheet current is easy to realize, as long as we determine the electromagnetic field on the surface of the plate according to magnetic quasi-static magnetic field, and then let the electric field and magnetic field propagate according to plane wave, so that the electromagnetic field and magnetic field still meet the phase difference of 90 degrees. In other cases, such as dipole antenna, the author still doesn't know how to extend from magnetic quasi-static magnetic field to radiated electromagnetic field, and keep the phase difference between the newly defined the far field of electric field and the far field of magnetic field at 90 degrees.

(1b) the electromagnetic field calculated according to Maxwell's equations, and then the self energy flow of Poynting vector calculated is not self energy flow, but a kind of mutual energy flow, which represents a kind of mutual energy flow when the surrounding of the radiation source is full of absorbing materials. in other words,

$$\mathbf{e}_1 \times \mathbf{h}_1^* = \mathbf{E}_1 \times \mathbf{H}_2^*$$

or

$$\mathbf{e}_1 \times \mathbf{h}_1^* = \mathbf{E}_2 \times \mathbf{H}_1^*$$

Where  $\mathbf{e}_1, \mathbf{h}_1$  are the electromagnetic fields of the source obtained according to Maxwell's equations, and the far field of this electromagnetic field maintains the same phase.  $\mathbf{e}_1 \times \mathbf{h}_1^*$  is Poynting vector.  $\mathbf{E}_1, \mathbf{H}_1$  are a new definition of electromagnetic field that the author is looking for. This electromagnetic field maintains a 90 degree phase difference in the far field.  $\mathbf{E}_2, \mathbf{H}_2$  are the electromagnetic field generated by the sink covered around the source.  $\mathbf{E}_1 \times \mathbf{H}_2^*, \mathbf{E}_2 \times \mathbf{H}_1^*$  are parts of the mutual energy flow of the two fields. In this way, we actually reinterpret the meaning of the solution of Maxwell's equations. Maxwell's equations remains intact. In actual calculation, we only consider  $\mathbf{e}_1, \mathbf{h}_1$ , do not consider  $\mathbf{E}_1, \mathbf{H}_1, \mathbf{E}_2, \mathbf{H}_2$ . Here  $\mathbf{E}_1, \mathbf{H}_1, \mathbf{E}_2, \mathbf{H}_2$  are only useful in the interpretation of Maxwell's equations.

For (1a), we need to find  $\mathbf{E}_1, \mathbf{H}_1, \mathbf{E}_2, \mathbf{H}_2$ , for (1b), we give up looking for  $\mathbf{E}_1, \mathbf{H}_1, \mathbf{E}_2, \mathbf{H}_2$ , the author thinks their usefulness lies only in the reinterpretation of Maxwell's theory. This interpretation will also extend to quantum mechanics.

(2) It is assumed that the radiated electromagnetic field calculated according to Maxwell's theory is correct. Then due to the self energy flow radiation becomes redundant. The time reversal wave must be added to counteract the self energy flow. Ensure that the self energy flow does not transfer energy. This idea was put forward by the author in his 2017 paper on the interpretation of quantum mechanics by mutual energy flow [8].

However, method (2) needs to add a time reversal wave, and the modification of Maxwell's theory is quite large. Time reversal wave needs experimental verification.

However, the author of the first method (1a) has not found a general method. Describe all cases, such as the reception and transmission of dipole antenna. The theory will change a lot.

At this time, the author's next task. For (1b), give up finding a new definition of electromagnetic field and use it only as an interpretation. Imagine the existence of an electromagnetic field. The advantage is that we don't have to look for the time reversal wave like method (2).

## VI. CONCLUSION

The author has put forward the electromagnetic field mutual energy theory for 7 years, but it has remained in theory without any practical examples to support it. The author recently studied the electromagnetic field of the plane-sheet current. The author found that the vector potential of plane-sheet current can be obtained by integration with analytical method. Therefore, the electric field and magnetic field can be obtained from the vector potential.

The two plane-sheet currents can be regarded as a transformer, a transmitting antenna and a receiving antenna. The energy flow on the transmitting antenna and receiving antenna can also be regarded as photons. This example can be used to study the problem of energy flow and photons from the source of electromagnetic wave to the sink of electromagnetic wave.

The author studied the magnetic quasi-static magnetic field. In this case, the mutual energy flow from primary coil to secondary coil is active power. The self energy flow of primary coil and secondary coil is reactive power. Therefore, the energy flow can only be transmitted by mutual energy flow. Corresponding to the classical electromagnetic theory, there is no theory of mutual energy flow, so it is impossible to explain the energy flow from the primary coil to the secondary coil of the transformer. This paper achieves this goal.

The energy flow problem of the transformer has been solved. The author does not think that the radiated electromagnetic field is a seamless generalization of the magnetic quasi-static magnetic field. And think of them as two completely different systems. For radiated electromagnetic field, both mutual energy flow and self energy flow are active power. The author believes that energy flow is transmitted by mutual energy flow. Therefore, it is necessary to establish two time reversal waves for the source and sink respectively. The time reversal waves are used to counteract the self-energy flows. Of course, there is also a case that we have to appropriately modify Maxwell's electromagnetic theory so that for the radiated electromagnetic field, the self energy flow transfers reactive power as in the case of magnetic quasi-static field. Mutual energy flow transfers active power. In short, Maxwell's classical electromagnetic theory is not a complete theory and needs further modification.

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**Coauthor** Same again for the co-author, but without photo

## REFERENCES

- [1] Wheeler. J. A. and Feynman. R. P. *Rev. Mod. Phys.*, 17:157, 1945.
- [2] Wheeler. J. A. and Feynman. R. P. *Rev. Mod. Phys.*, 21:425, 1949.
- [3] John Cramer. The transactional interpretation of quantum mechanics. *Reviews of Modern Physics*, 58:647–688, 1986.
- [4] John Cramer. An overview of the transactional interpretation. *International Journal of Theoretical Physics*, 27:227, 1988.
- [5] Adrianus T. de Hoop. Time-domain reciprocity theorems for electromagnetic fields in dispersive media. *Radio Science*, 22(7):1171–1178, December 1987.
- [6] A. D. Fokker. *Zeitschrift für Physik*, 58:386, 1929.
- [7] Shuang ren Zhao. The application of mutual energy theorem in expansion of radiation fields in spherical waves. *ACTA Electronica Sinica, P.R. of China*, 15(3):88–93, 1987.
- [8] Shuang ren Zhao. A new interpretation of quantum physics: Mutual energy flow interpretation. *American Journal of Modern Physics and Application*, 4(3):12–23, 2017.
- [9] V.H. Rumsey. A short way of solving advanced problems in electromagnetic fields and other linear systems. *IEEE Transactions on antennas and Propagation*, 11(1):73–86, January 1963.
- [10] K. Schwarzschild. *Nachr. ges. Wiss. Göttingen*, pages 128,132, 1903.
- [11] Lawrence M. Stephenson. The relevance of advanced potential solutions of maxwell's equations for special and general relativity. *Physics Essays*, 13(1), 2000.
- [12] H. Tetrode. *Zeitschrift für Physik*, 10:137, 1922.
- [13] W. J. Welch. Reciprocity theorems for electromagnetic fields whose time dependence is arbitrary. *IRE trans. On Antennas and Propagation*, 8(1):68–73, January 1960.
- [14] Shuangren Zhao. The application of mutual energy formula in expansion of plane waves. *Journal of Electronics, P. R. China*, 11(2):204–208, March 1989.
- [15] Shuangren Zhao. The simplification of formulas of electromagnetic fields by using mutual energy formula. *Journal of Electronics, P.R. of China*, 11(1):73–77, January 1989.