An Essential History of Euclidean Geometry

Saburou Saitoh
Institute of Reproducing Kernels
saburou.saitoh@gmail.com

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Abstract: In this note, we would like to refer simply to the great history of Euclidean geometry and as a result we would like to state the great and essential development of Euclidean geometry by the new discovery of division by zero and division by zero calculus. We will be able to see the important and great new world of Euclidean geometry by Hiroshi Okumura.

Key Words: Euclidean geometry, non-Euclidean geometry, division by zero, division by zero calculus, Descartes, Descartes's circle theorem, Brahmagupta, point at infinity, Euclid's parallel postulate, horn torus, Wasan geometry, H. Okumura.

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1 Introduction

In this note, we would like to refer simply to the great history of Euclidean geometry and as a result we would like to state the essential and great development of Euclidean geometry by the new discovery of division by zero and division by zero calculus. We will be able to see the great new world of Euclidean geometry by Hiroshi Okumura in connection with Wasan.

2 Great events in Euclidean geometry

Look at the earth from the moon and imagine the history of mankind. From various points of view, we will have various thoughts. Imagine the history of Euclidean geometry here.

It reminds me of how Euclid built his Euclidean geometry, along with the discoveries of non-Euclidean geometry I read during my undergraduate school days. It was the Euclid's desire to build an eternally immortal geometry that is absolutely unwavering.

Euclidean geometry gave the basic spirit of all scholarly books and prospered as an eternal scholarship for over 2200 years and is still the basis of elementary mathematics. No, it can be said that it is the basis of mathematics, even now. So what happens when we see the whole of Euclidean geometry from the world of the moon? After all, the essential event is the emergence of non-Euclidean geometry pioneered by the three giants:

> Non-Euclidean geometry Source: Free encyclopedia "Wikipedia" In a letter on November 8, 1824, Carl Friedrich Gauss suggested that consistent geometry could be established under the assumption of acute angles, and there are certain constants, and the larger this is. He said it would approach normal geometry. Gauss's constant is-(1 / k) with respect to the curvature of space k in modern language. Gauss himself appears to have been convinced of the existence of non-Euclidean geometry, but has not made it public. Some speculate that he is "afraid of being involved in religious controversy." Establishment of non-Euclidean geometry [edit] Nikolai Ivanovich Lobachevsky constructed and showed the geometry named "imaginary geometry" in "The New Principles of Geometry and the Complete Theory of Parallel Lines" (1829). This was a geometry that included sharp-angle assumptions. Boyai Janos took over the work of his father, Boyai Farkas, and published "Spatial Theory" in 1832. In "Spatial Theory", we discussed the geometry assuming the parallel postulate (Σ) and the geometry assuming the negation of the parallel postulate (S). Furthermore, in 1835, "Proof of Impossible to Prove or Refute the 11th Euclid Axiom", it was proved that which of Σ and S actually holds is not determined by any logical reasoning.

In short, we could not prove Euclid's parallel postulate, and discovered the existence of geometry in which the parallel postulate did not hold. At the same time, this was a historical event that denied the existence of absolute geometry, showed that there were various mathematics in geometry, and essentially changed the view of mathematics. In that sense, those discoveries are considered to be the greatest event in Euclidean geometry.

3 Coordinate systems

The next thing to notice is the introduction of the coordinate system introduced by Descartes (1596-1650), the idea that points in a plane or space can be represented by a set of numbers. Therefore, figures are represented by equations, and figures are connected to the world of algebra and numbers, and geometry and algebra are connected. On these foundations, calculus, analysis, geometry, algebra, and mathematics on manifolds are considered to be developing. We would like to highly appreciate Descartes' feat here. In addition, Descartes obtained the beautiful theorem of Descartes's circle theorem ([2]) in his Euclidean geometry itself, and its development is dazzling. It can be said to be the most beautiful theorem in Euclidean geometry.

Isn't two major events in Euclidean geometry?

Here we would like to confirm the following to touch on the third event.

4 Division by zero and division by zero calculus

Regarding what mathematics is in the first place, it is thought that the origin of mathematics lies in Euclidean geometry and four arithmetic operations. Four arithmetic laws were established in 628 AD with the introduction of ZERO by Brahmagupta in India in some strict sense - the history of ZERO is long and wide. See [7] for example. It is a figure and the law of arithmetic. Importantly and interestingly, the ideas of Euclid and Brahmagupta are unified with the introduction of the Cartesian coordinate system. Geometry, algebra, and analysis are considered to be developing on these foundations. Surprisingly, however, it is believed that there were defects in both foundations from the beginning. They are the ideas of the point at infinity and division by zero. In the perception of space, infinite distance lacked the perspective of what Euclid was doing. In the four arithmetic operations, it is the problem of dividing by zero. Brahmagupta himself defined $0/0 = 0$ from the beginning, but he didn't think of division by zero in general. - It seems he thought this was impossible. The history of division by zero is older, and in a physical sense, Aristotle has had a great influence on Western culture, stating that division by zero should not be considered and is impossible. Greek culture has a strong culture that hates zero, void, and nothing. On the other hand, India has a fairly deep idea. See [7] for example.

Therefore, the elucidation of division by zero by the concept of division by zero calculus had a wide range of influences on elementary mathematics, but even the supposedly immutable Euclidean geometry had a revolutionary renewal. For the division by zero and division by zero calculus with the impact to geometry, see [4, 5, 6].

For the horn torus model that shows the attaching of the zero point and the point at infinity, see [1]. That model should be used for the Riemann sphere model and it realizes the division by zero calculus as the extended complex plane with conformal structure.

5 Wasan

In the Edo period, mathematics was loved by the common people as a result of long-term peace, and a vast cultural heritage by many people is left as Wasan in Japan. There are still many Wasan enthusiasts and researchers who are studying those heritage sites. Hiroshi Okumura is one of them, especially Gunma, where has a strong tradition of Wasan, and he has been studying all the time in response to the cultural background. Many achievements show that he has been particularly focused on the study of Wasan geometry. These enormous geometries are naturally considered to be the world of Euclidean geometry, and Wasan has obtained many beautiful and deep results, however there are essential flaws in logic and description, and it has been neglected in the Western world. Under such circumstances, with the strong will of Professor Yoshimasa Michiwaki, our teacher, we discovered new mathematical results from Wasan and started research activities in mathematics that would be accepted by Europe and the United States as mathematics. Dr. Okumura has inherited such a spirit and has been steadily developing, launching an international journal specializing in Wasan with international colleagues, and conducting research activities with **Sangaku** as an international language. See [2] and

Statement of the Institute for Reproducing Kernels 588 (2020.11.30): Dr. Hiroshi Okumura's Great Contribution to Wasan Mathematics-Sequel to Statement 569-

Statement of the Institute for Reproducing Kernels 569 (2020.07.21): Dr. Hiroshi Okumura's great contribution to Wasan and the beautiful world of geometry - Euclidean geometry and new developments in Wasan geometry

By applying the division by zero method to them, he is getting more and more completely new results, as if he is divine. The results are concrete and marvelous, furthermore Descartes's circle theorem is beautifully unified, and he even discovers new phenomena there. See [2, 3].

6 The living king in Euclidean geometry

The quality of his research and the quantity of beautiful results will clearly show that he is one of the best in the history of Euclidean geometry in the history of the world. In conflict with the non-Euclidean parallel postulate, he discovered a new non-Euclidean geometry in a different way from the three giants, and he is beyond even Descartes essentially for his circle theorem. We understand. When we think about it like this, it is thought that the name of Dr. Hiroshi Okumura as a great mathematician in Japan deserves to be called the **king in Euclidean geometry**. It was conceived.

This is the evaluation of value and the creation of value. It is a discovery of value.

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