On some possible mathematical connections between various equations concerning the Zeta Cosmology,  $\phi$ ,  $\zeta(2)$  and some parameters of Cosmology, String Theory and Particle Physics

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#### **Abstract**

In this paper we have described some possible mathematical connections between various equations concerning the Zeta Cosmology,  $\phi$ ,  $\zeta(2)$  and some parameters of Cosmology, String Theory and Particle Physics

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https://commons.wikimedia.org/wiki/File:AnatolyA.Karatsuba.jpg

$$\begin{array}{c}
1 & 729 \\
1^3 + 12^3 = 9^3 + 10^3
\end{array}$$

https://funwithfunctions.es/post/168921716439/ramanujan

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

For more information on the data entered for the development of the various equations, see the "Observations" section.

From:

A. A. Karatsuba, Cosmology and zeta, Sovrem. Probl. Mat., 2016, Issue 23, 17–23 - DOI: https://doi.org/10.4213/spm58

We have the following equation:

$$\frac{Z'(t)}{Z(t)} = \sum_{k=1}^{\infty} \frac{a(k)}{t - \gamma_k} + t \sum_{n=1}^{\infty} \frac{1}{(2n + 1/2)^2 + t^2} - \frac{t}{t^2 + 1/4},\tag{7}$$

and  $a(k) \ge 1$  (a(k) are multiplication of zeros). Now let  $0 < \gamma_k < \gamma_{k+1}, k \ge 1$ . Consider the interval  $\gamma_k < t < \gamma_{k+1}$ .

$$a(k) \ge 1 = 2$$
;  $\gamma_k = 3$ ;  $\gamma_{k+1} = 8$ ;  $\gamma_k \le t \le \gamma_{k+1}$ ;  $t = 4$ 

we obtain:

Sum 
$$(1/((2n+1/2)^2+t^2)-t/(t^2+1/4))$$
,  $n = 1$ ..infinity

#### **Input interpretation:**

$$\sum_{n=1}^{\infty} \left( \frac{1}{\left(2n + \frac{1}{2}\right)^2 + t^2} - \frac{t}{t^2 + \frac{1}{4}} \right)$$

**Result:** 

$$\sum_{n=1}^{\infty} \left( \frac{1}{\left(2n + \frac{1}{2}\right)^2 + t^2} - \frac{t}{t^2 + \frac{1}{4}} \right) \text{ converges when } \frac{t}{4t^2 + 1} = 0$$

**Convergence tests:** 

The ratio test is inconclusive.

Partial sum formula:

$$\begin{split} \sum_{n=1}^{m} \left( \frac{1}{\left(2\,n + \frac{1}{2}\right)^2 + t^2} - \frac{t}{t^2 + \frac{1}{4}} \right) &= \\ \frac{1}{4\,t\,\left(4\,t^2 + 1\right)}\,i\left(16\,i\,m\,t^2 - 4\,t^2\,\psi^{(0)}\!\left(-\frac{i\,t}{2} + m + \frac{5}{4}\right) + 4\,t^2\,\psi^{(0)}\!\left(\frac{i\,t}{2} + m + \frac{5}{4}\right) - \\ \psi^{(0)}\!\left(-\frac{i\,t}{2} + m + \frac{5}{4}\right) + \psi^{(0)}\!\left(\frac{i\,t}{2} + m + \frac{5}{4}\right) + 4\,t^2\,\psi^{(0)}\!\left(\frac{5}{4} - \frac{i\,t}{2}\right) - \\ 4\,t^2\,\psi^{(0)}\!\left(\frac{i\,t}{2} + \frac{5}{4}\right) + \psi^{(0)}\!\left(\frac{5}{4} - \frac{i\,t}{2}\right) - \psi^{(0)}\!\left(\frac{i\,t}{2} + \frac{5}{4}\right) \end{split}$$

For t = 4, we have:

4\* Sum 
$$(1/((2n+1/2)^2+4^2)-4/(4^2+1/4))$$
, n = 1..infinity

Input interpretation:

$$4\sum_{n=1}^{\infty} \left( \frac{1}{\left(2n + \frac{1}{2}\right)^2 + 4^2} - \frac{4}{4^2 + \frac{1}{4}} \right)$$

**Result:** 

õ

For  $n = 1..10^4$ , we obtain:

$$4*$$
 Sum  $(1/((2n+1/2)^2+4^2)-4/(4^2+1/4))$ ,  $n = 1..10^4$ 

4

**Input interpretation:** 

$$4\sum_{n=1}^{10^4} \left( \frac{1}{\left(2n + \frac{1}{2}\right)^2 + 4^2} - \frac{4}{4^2 + \frac{1}{4}} \right)$$

#### **Result:**

-9845.55

-9845.55

Without the number 4 that multiply the sum, we obtain:

Sum 
$$(1/((2n+1/2)^2+4^2)-4/(4^2+1/4))$$
,  $n = 1..10^4$ 

#### Sum:

$$\sum_{n=1}^{10000} \left( \frac{1}{\left(2n + \frac{1}{2}\right)^2 + 4^2} - \frac{4}{4^2 + \frac{1}{4}} \right) \approx -2461.387781356798833703649394125425728097$$

#### **Decimal approximation:**

-2461.38778135679883370364939412542572809664051034341200635...

-2461.3877813... result very near to the rest mass of charmed Xi baryon 2467.8

From which:

$$-(([Sum (1/((2n+1/2)^2+4^2)-4/(4^2+1/4)), n = 1..10^4]+521+199+11+3/2))$$

#### **Input interpretation:**

$$-\left(\sum_{n=1}^{10^4} \left(\frac{1}{\left(2n+\frac{1}{2}\right)^2+4^2}-\frac{4}{4^2+\frac{1}{4}}\right)+521+199+11+\frac{3}{2}\right)$$

#### **Result:**

1728.89

 $1728.89 \approx 1729$ 

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

 $((-(([Sum\ (1/((2n+1/2)^2+4^2)-4/(4^2+1/4)),\ n=1..10^4]+521+199+11+3/2))))^1/15$ 

**Input interpretation:** 

$$\frac{15}{15} - \left( \sum_{n=1}^{10^4} \left( \frac{1}{\left(2n + \frac{1}{2}\right)^2 + 4^2} - \frac{4}{4^2 + \frac{1}{4}} \right) + 521 + 199 + 11 + \frac{3}{2} \right)$$

#### **Result:**

1.64381

$$1.64381 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

and:

$$((-(([Sum (1/((2n+1/2)^2+4^2)-4/(4^2+1/4)), n = 1..10^4]+521+199+11+3/2))))^1/15-26/10^3$$

**Input interpretation:** 

$$\frac{15}{15} \left[ -\left( \sum_{n=1}^{10^4} \left( \frac{1}{\left( 2n + \frac{1}{2} \right)^2 + 4^2} - \frac{4}{4^2 + \frac{1}{4}} \right) + 521 + 199 + 11 + \frac{3}{2} \right) - \frac{26}{10^3} \right]$$

#### **Result:**

1.61781

1.61781 result that is a very good approximation to the value of the golden ratio 1.618033988749...

We have also:

Sum 
$$(2/(4-3k))$$
,  $k = 1..10^4$ 

#### Sum:

$$\sum_{k=1}^{10000} \frac{2}{4-3k} \approx -5.018994300915839239126849140954765737395$$

#### **Decimal approximation:**

-5.01899430091583923912684914095476573739480704855757567819...

-5.0189943...

Thence:

$$(((Sum (2/(4-3k)), k = 1..10^4))) + 4* Sum (1/((2n+1/2)^2+4^2)-4/(4^2+1/4)), n = 1..10^4$$

**Input interpretation:** 

$$\sum_{k=1}^{10^4} \frac{2}{4-3k} + 4 \sum_{n=1}^{10^4} \left( \frac{1}{\left(2n + \frac{1}{2}\right)^2 + 4^2} - \frac{4}{4^2 + \frac{1}{4}} \right)$$

#### **Result:**

-9850.57

-9850.57

From which:

$$\left[ -(322+123+7+(((Sum\ (2/(4-3k)),\ k=1..10^4)))+4*\ Sum\ (1/((2n+1/2)^2+4^2)-4/(4^2+1/4)),\ n=1..10^4) \right]$$

Where 7, 123 and 322 are Lucas numbers

**Input interpretation:** 

$$-\left(322+123+7+\sum_{k=1}^{10^4}\frac{2}{4-3\,k}+4\sum_{n=1}^{10^4}\left(\frac{1}{\left(2\,n+\frac{1}{2}\right)^2+4^2}-\frac{4}{4^2+\frac{1}{4}}\right)\right)$$

#### **Result:**

9398.57

9398.57 result practically equal to the rest mass of Bottom eta meson 9398

and also:

$$1/6[-((((Sum (2/(4-3k)), k = 1..10^4))) + 4* Sum (1/((2n+1/2)^2+4^2)-4/(4^2+1/4)), n = 1..10^4)]+89-3/2$$

Where 89 is a Fibonacci number

**Input interpretation:** 

$$\frac{1}{6} \left[ -\left( \sum_{k=1}^{10^4} \frac{2}{4-3k} + 4 \sum_{n=1}^{10^4} \left( \frac{1}{\left(2n + \frac{1}{2}\right)^2 + 4^2} - \frac{4}{4^2 + \frac{1}{4}} \right) \right] + 89 - \frac{3}{2} \right]$$

#### **Result:**

1729.26

1729.26

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From which:

$$((((1/6[-((((Sum (2/(4-3k)), k = 1..10^4))) + 4* Sum (1/((2n+1/2)^2+4^2)-4/(4^2+1/4)), n = 1..10^4)]+89-3/2))))^1/15$$

**Input interpretation:** 

$$\frac{1}{6} \left[ -\left( \sum_{k=1}^{10^4} \frac{2}{4-3k} + 4 \sum_{n=1}^{10^4} \left( \frac{1}{\left(2n + \frac{1}{2}\right)^2 + 4^2} - \frac{4}{4^2 + \frac{1}{4}} \right) \right] + 89 - \frac{3}{2} \right]$$

#### **Result:**

1.64383

$$1.64383 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

and:

$$((((((1/6[-((((Sum (2/(4-3k)), k = 1..10^4))) + 4* Sum (1/((2n+1/2)^2+4^2)-4/(4^2+1/4)), n = 1..10^4)] + 89-3/2))))^1/15)) - 26/10^3$$

**Input interpretation:** 

#### **Result:**

1.61783

1.61783 result that is a very good approximation to the value of the golden ratio 1.618033988749...

 $1/89[-((((Sum (2/(4-3k)), k = 1..10^4))) + 4* Sum (1/((2n+1/2)^2+4^2)-4/(4^2+1/4)), n = 1..10^4)]+21+8$ 

Where 8 and 21 are Fibonacci numbers

**Input interpretation:** 

$$\frac{1}{89} \left( - \left( \sum_{k=1}^{10^4} \frac{2}{4 - 3 \, k} + 4 \sum_{n=1}^{10^4} \left( \frac{1}{\left(2 \, n + \frac{1}{2}\right)^2 + 4^2} - \frac{4}{4^2 + \frac{1}{4}} \right) \right) \right) + 21 + 8$$

#### **Result:**

139.681

139.681 result practically equal to the rest mass of Pion meson 139.57 MeV

$$1/89[-((((Sum (2/(4-3k)), k = 1..10^4))) + 4* Sum (1/((2n+1/2)^2+4^2)-4/(4^2+1/4)), n = 1..10^4)]+13+2$$

Where 2 and 13 are Fibonacci numbers

#### Input interpretation:

$$\frac{1}{89} \left( -\left( \sum_{k=1}^{10^4} \frac{2}{4-3k} + 4 \sum_{n=1}^{10^4} \left( \frac{1}{\left(2n + \frac{1}{2}\right)^2 + 4^2} - \frac{4}{4^2 + \frac{1}{4}} \right) \right) \right) + 13 + 2$$

#### **Result:**

125.681

125.681 result very near to the Higgs boson mass 125.18 GeV

Now, from:

$$K(t) = \sum_{n=1}^{\infty} \frac{1}{(2n+1/2)^2 + t^2} - 2t^2 \sum_{n=1}^{\infty} \frac{1}{((2n+1/2)^2 + t^2)^2} - \frac{1}{t^2 + 1/4} + \frac{2t^2}{(t^2 + 1/4)^2}$$

For t = 4, we have:

 $((Sum (1/((2n+1/2)^2+4^2)), n = 1..infinity)) - ((32 sum (1/((2n+1/2)^2+4^2)) - (1/(4^2+1/4)) + (32/(16+1/4)^2), n = 1..infinity))$ 

**Input interpretation:** 

$$\sum_{n=1}^{\infty} \frac{1}{\left(2n + \frac{1}{2}\right)^2 + 4^2} - 32 \sum_{n=1}^{\infty} \left( \frac{1}{\left(2n + \frac{1}{2}\right)^2 + 4^2} - \frac{1}{4^2 + \frac{1}{4}} + \frac{32}{\left(16 + \frac{1}{4}\right)^2} \right)$$

#### **Result:**

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While, for  $n = 1..10^4$ 

$$((Sum (1/((2n+1/2)^2+4^2)), n = 1..10^4)) - ((32 sum (1/((2n+1/2)^2+4^2)) - (1/(4^2+1/4)) + (32/(16+1/4)^2), n = 1..10^4))$$

**Input interpretation:** 

$$\sum_{n=1}^{10^4} \frac{1}{\left(2\,n+\frac{1}{2}\right)^2+4^2} -32\,\sum_{n=1}^{10^4} \left(\frac{1}{\left(2\,n+\frac{1}{2}\right)^2+4^2} -\frac{1}{4^2+\frac{1}{4}} +\frac{32}{\left(16+\frac{1}{4}\right)^2}\right)$$

#### **Result:**

-19091.1

-19091.1

From which:

$$[-(76+7+2+1/2*((((Sum (1/((2n+1/2)^2+4^2)), n = 1..10^4)) - ((32 sum (1/((2n+1/2)^2+4^2)) - (1/(4^2+1/4)) + (32/(16+1/4)^2), n = 1..10^4))))]$$

Input interpretation:

$$-\left(76+7+2+\frac{1}{2}\left(\sum_{n=1}^{10^4}\frac{1}{\left(2\,n+\frac{1}{2}\right)^2+4^2}-32\sum_{n=1}^{10^4}\left(\frac{1}{\left(2\,n+\frac{1}{2}\right)^2+4^2}-\frac{1}{4^2+\frac{1}{4}}+\frac{32}{\left(16+\frac{1}{4}\right)^2}\right)\right)\right)$$

#### **Result:**

9460.53

9460.53 result practically equal to the rest mass of Upsilon meson 9460.30

and:

$$\frac{1}{6}\left[-(76+7+2+1/2*((((Sum(1/((2n+1/2)^2+4^2)), n = 1..10^4)) - ((32 sum(1/((2n+1/2)^2+4^2)) - (1/(4^2+1/4)) + (32/(16+1/4)^2), n = 1..10^4))))\right] + 199-47$$

Input interpretation:

$$\frac{1}{6} \left( -\left[ 76 + 7 + 2 + \frac{1}{2} \left( \sum_{n=1}^{10^4} \frac{1}{\left( 2n + \frac{1}{2} \right)^2 + 4^2} - 32 \sum_{n=1}^{10^4} \left( \frac{1}{\left( 2n + \frac{1}{2} \right)^2 + 4^2} - \frac{1}{4^2 + \frac{1}{4}} + \frac{32}{\left( 16 + \frac{1}{4} \right)^2} \right) \right) \right) + 199 - 47$$

#### **Result:**

1728.76

 $1728.76 \approx 1729$ 

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$$(((1/6[-(76+7+2+1/2*((((Sum (1/((2n+1/2)^2+4^2)), n = 1..10^4)) - ((32 sum (1/((2n+1/2)^2+4^2)) - (1/(4^2+1/4)) + (32/(16+1/4)^2), n = 1..10^4))))] + 199-47)))^1/15$$

**Input interpretation:** 

$$\left(\frac{1}{6}\left[-\left(76+7+2+\frac{1}{2}\left(\sum_{n=1}^{10^4}\frac{1}{\left(2\,n+\frac{1}{2}\right)^2+4^2}-32\sum_{n=1}^{10^4}\left(\frac{1}{\left(2\,n+\frac{1}{2}\right)^2+4^2}-\frac{1}{4^2+\frac{1}{4}}+\frac{32}{\left(16+\frac{1}{4}\right)^2}\right)\right]\right)+199-47\right)^{4} - \frac{1}{4^2+\frac{1}{4}} + \frac{32}{\left(16+\frac{1}{4}\right)^2}\right)\right)\right) + 199-47\right)^{4} - \frac{1}{4^2+\frac{1}{4}} + \frac{32}{\left(16+\frac{1}{4}\right)^2}\right) + \frac{1}{4^2+\frac{1}{4}} + \frac{1}{4^2+\frac{1}{4}} + \frac{32}{\left(16+\frac{1}{4}\right)^2}\right) + \frac{1}{4^2+\frac{1}{4}} + \frac{1}{4^2+\frac{1}{4^2+\frac{1}{4}}} + \frac{1}{4^2+\frac{1}{4^2+\frac{1}{4}}} + \frac{1}{4^2+\frac{1}{4^2+\frac{1}{4}}} + \frac{1}{4^2+\frac{1}{4^2+\frac{1}{4}}} + \frac{1}{4^2+\frac{1}{4^2+\frac{1}{$$

#### **Result:**

1.6438

$$1.6438 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

and:

$$(((1/6[-(76+7+2+1/2*((((Sum (1/((2n+1/2)^2+4^2)), n = 1..10^4)) - ((32 sum (1/((2n+1/2)^2+4^2)) - (1/(4^2+1/4)) + (32/(16+1/4)^2), n = 1..10^4))))]+199-47)))^1/15-26/10^3$$

#### **Input interpretation:**

$$\left(\frac{1}{6}\left(-\left(76+7+2+\frac{1}{2}\left(\sum_{n=1}^{10^4}\frac{1}{\left(2\,n+\frac{1}{2}\right)^2+4^2}-32\sum_{n=1}^{10^4}\left(\frac{1}{\left(2\,n+\frac{1}{2}\right)^2+4^2}-\frac{1}{4^2+\frac{1}{4}}+\frac{32}{\left(16+\frac{1}{4}\right)^2}\right)\right)\right)\right)+199-47\right)^{4}\left(1/15\right)-\frac{26}{10^3}$$

#### **Result:**

1.6178

1.6178 result that is a very good approximation to the value of the golden ratio 1.618033988749...

Now, we have:

Moser Postulate. For  $\gamma_k < t < \gamma_{k+1}$  we set

$$R(t) = \alpha(t_0)|Z(t)|,\tag{18}$$

where

$$\alpha(t_0) = \frac{c}{|Z(t_0)|} \left\{ \beta \sum_{k=1}^{\infty} \frac{a(k)}{(t_0 - \gamma_k)^2} \right\}^{-1/2},\tag{19}$$

$$a(k) \ge 1 = 4; \quad \gamma_k = 3; \quad \gamma_{k+1} = 8; \quad \gamma_k \le t \le \gamma_{k+1}; \quad t = 5$$
  
$$a(k) = (5-3)^2 = 4$$

For: 
$$\gamma' = 3$$
;  $t_0 = 5$ ;  $\gamma'' = 8$ ;  $\omega(t_0) = 10^{-58}$ ,...

$$\gamma = 2$$
;  $t = 6$ ;  $A = 2.5$ ;  $Z(t) = 2$ 

 $\beta = 3/2$  is a constant,  $1 < \beta < 2$ ,

we obtain:

$$(3e+8)/2*((((3/2*sum(((k/(5-3)^2))),k = 1..10^43))))^-0.5$$

#### **Input interpretation:**

$$\frac{\frac{3 \times 10^{3}}{2}}{\left(\frac{3}{2} \sum_{k=1}^{10^{43}} \frac{k}{(5-3)^{2}}\right)^{0.5}}$$

#### **Result:**

 $3.4641 \times 10^{-35}$ 

 $3.4641 * 10^{-35}$ 

From which:

 $1/(1.2619+0.88137)*(((((3e+8)/2*((((3/2*sum(((k/(5-3)^2))),k=1..10^43))))^-$ 0.5))))

where 1.2619 and 0.88137 are two Hausdorff dimensions

#### **Input interpretation:**

Input interpretation: 
$$\frac{1}{1.2619 + 0.88137} \times \frac{\frac{\frac{3 \times 10^8}{2}}{2}}{\left(\frac{3}{2} \sum_{k=1}^{10^{43}} \frac{k}{(5-3)^2}\right)^{0.5}}$$

#### **Result:**

 $1.61627 \times 10^{-35}$ 

1.61627\*10<sup>-35</sup> result equal to the value of Planck length

From:

$$\gamma_{k+1} - \gamma_k \leqslant \frac{c_1}{\log\log\gamma_k} \,, \tag{24}$$

$$\varkappa p(t_0) \geqslant \frac{2-\beta}{c_1} \log \log \gamma_k - c_2 \frac{1}{\gamma_k},$$

$$E(t_0) \geqslant \frac{2(\beta-1)}{c_1} \log \log \gamma_k - c_3 \frac{1}{\gamma_k}.$$

$$E(t_0) = (2(3/2-1))/5* \ln \ln (3) - 3/6*1/3$$

**Input:** 

$$\left(\frac{1}{5}\left(2\left(\frac{3}{2}-1\right)\right)\right)\log(\log(3))-\frac{3}{6}\times\frac{1}{3}$$

log(x) is the natural logarithm

**Exact result:** 

$$\frac{1}{5}\log(\log(3)) - \frac{1}{6}$$

#### **Decimal approximation:**

 $-0.14785710114332686343179980024976786810959906074298296906\dots \\$ 

-0.1478571011...

#### Alternate form:

$$\frac{1}{30}$$
 (6 log(log(3)) – 5)

#### **Alternative representations:**

$$\frac{1}{5}\log(\log(3))\left(2\left(\frac{3}{2}-1\right)\right)-\frac{3}{3\times 6}=\frac{\log_e(\log(3))}{5}-\frac{1}{6}$$

$$\frac{1}{5} \log(\log(3)) \left( 2 \left( \frac{3}{2} - 1 \right) \right) - \frac{3}{3 \times 6} = \frac{1}{5} \log(a) \log_a(\log(3)) - \frac{1}{6}$$

$$\frac{1}{5} \log(\log(3)) \left( 2 \left( \frac{3}{2} - 1 \right) \right) - \frac{3}{3 \times 6} = -\frac{1}{5} \operatorname{Li}_1(1 - \log(3)) - \frac{1}{6}$$

#### Series representations:

$$\frac{1}{5}\log(\log(3))\left(2\left(\frac{3}{2}-1\right)\right) - \frac{3}{3\times 6} = -\frac{1}{6} - \frac{1}{5}\sum_{k=1}^{\infty} \frac{(1-\log(3))^k}{k}$$

$$\begin{split} \frac{1}{5} \log(\log(3)) \left( 2 \left( \frac{3}{2} - 1 \right) \right) - \frac{3}{3 \times 6} &= \\ &- \frac{1}{6} + \frac{2}{5} i \pi \left[ \frac{\arg(-x + \log(3))}{2 \pi} \right] + \frac{\log(x)}{5} - \frac{1}{5} \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(3))^k}{k} & \text{for } x < 0 \end{split}$$

$$\begin{split} \frac{1}{5} \log(\log(3)) \left( 2 \left( \frac{3}{2} - 1 \right) \right) - \frac{3}{3 \times 6} &= -\frac{1}{6} + \frac{1}{5} \left\lfloor \frac{\arg(\log(3) - z_0)}{2 \, \pi} \right\rfloor \log \left( \frac{1}{z_0} \right) + \\ & \frac{\log(z_0)}{5} + \frac{1}{5} \left\lfloor \frac{\arg(\log(3) - z_0)}{2 \, \pi} \right\rfloor \log(z_0) - \frac{1}{5} \sum_{k=1}^{\infty} \frac{(-1)^k \, (\log(3) - z_0)^k \, z_0^{-k}}{k} \end{split}$$

#### **Integral representations:**

$$\frac{1}{5}\log(\log(3))\left(2\left(\frac{3}{2}-1\right)\right) - \frac{3}{3\times 6} = -\frac{1}{6} + \frac{1}{5}\int_{1}^{\log(3)}\frac{1}{t}\ dt$$

$$\frac{1}{5} \log(\log(3)) \left( 2 \left( \frac{3}{2} - 1 \right) \right) - \frac{3}{3 \times 6} = -\frac{1}{6} - \frac{i}{10 \pi} \int_{-i + \gamma}^{i + \gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(3))^{-s}}{\Gamma(1-s)} ds$$
for  $-1 < \gamma < 0$ 

#### From which:

$$1/\text{sqrt}(24+11) (((((2(3/2-1))/5* \ln \ln (3) - 3/6*1/3))))^(64-24)$$

#### **Input:**

$$\frac{1}{\sqrt{24+11}} \left( \left( \frac{1}{5} \left( 2 \left( \frac{3}{2} - 1 \right) \right) \right) \log(\log(3)) - \frac{3}{6} \times \frac{1}{3} \right)^{64-24}$$

log(x) is the natural logarithm

#### **Exact result:**

$$\frac{\left(\frac{1}{5}\log(\log(3)) - \frac{1}{6}\right)^{40}}{\sqrt{35}}$$

#### **Decimal approximation:**

 $1.0511228278146048863376205587661688114012045968260604...\times10^{-34}$ 

 $1.051122827814...*10^{-34}$  result very near to the value of reduced Planck constant  $1.054571817*10^{-34}$ 

#### **Alternate forms:**

$$\frac{\left(\frac{1}{6} - \frac{1}{5}\log(\log(3))\right)^{40}}{\sqrt{35}}$$

 $(6 \log(\log(3)) - 5)^{40}$ 

#### Alternative representations:

$$\frac{\left(\frac{1}{5} \left(2 \left(\frac{3}{2}-1\right)\right) \log (\log (3))-\frac{3}{6 \times 3}\right)^{64-24}}{\sqrt{24+11}}=\frac{\left(\frac{\log_e (\log (3))}{5}-\frac{1}{6}\right)^{40}}{\sqrt{35}}$$

$$\frac{\left(\frac{1}{5}\left(2\left(\frac{3}{2}-1\right)\right)\log(\log(3))-\frac{3}{6\times 3}\right)^{64-24}}{\sqrt{24+11}}=\frac{\left(\frac{1}{5}\log(a)\log_a(\log(3))-\frac{1}{6}\right)^{40}}{\sqrt{35}}$$

$$\frac{\left(\frac{1}{5} \left(2 \left(\frac{3}{2}-1\right)\right) \log (\log (3))-\frac{3}{6 \times 3}\right)^{64-24}}{\sqrt{24+11}} = \frac{\left(-\frac{1}{5} \text{Li}_1 (1-\log (3))-\frac{1}{6}\right)^{40}}{\sqrt{35}}$$

#### Series representations:

$$\frac{\left(\frac{1}{5}\left(2\left(\frac{3}{2}-1\right)\right)\log(\log(3))-\frac{3}{6\times 3}\right)^{64-24}}{\sqrt{24+11}}=\frac{\left(\frac{1}{6}+\frac{1}{5}\sum_{k=1}^{\infty}\frac{(1-\log(3))^k}{k}\right)^{40}}{\sqrt{35}}$$

$$\begin{split} \frac{\left(\frac{1}{5}\left(2\left(\frac{3}{2}-1\right)\right)\log(\log(3))-\frac{3}{6\times 3}\right)^{64-24}}{\sqrt{24+11}} &= \\ \frac{\left(-\frac{1}{6}+\frac{1}{5}\left(2\,i\,\pi\left\lfloor\frac{\arg(-x+\log(3))}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,x^{-k}\,(-x+\log(3))^k}{k}\right)\right)^{40}}{\sqrt{35}} & \text{for } x<0 \end{split}$$

$$\begin{split} &\frac{\left(\frac{1}{5}\left(2\left(\frac{3}{2}-1\right)\right)\log(\log(3))-\frac{3}{6\times 3}\right)^{64-24}}{\sqrt{24+11}} = \\ &\frac{\left(-\frac{1}{6}+\frac{1}{5}\left(\log(z_0)+\left\lfloor\frac{\arg(\log(3)-z_0)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\log(3)-z_0\right)^kz_0^{-k}}{k}\right)\right)^{40}}{\sqrt{35}} \end{split}$$

#### **Integral representations:**

$$\frac{\left(\frac{1}{5} \left(2 \left(\frac{3}{2}-1\right)\right) \log (\log (3))-\frac{3}{6 \times 3}\right)^{64-24}}{\sqrt{24+11}} = \frac{\left(\frac{1}{6}-\frac{1}{5} \int_{1}^{\log (3)} \frac{1}{t} \ dt\right)^{40}}{\sqrt{35}}$$

$$\frac{\left(\frac{1}{5}\left(2\left(\frac{3}{2}-1\right)\right)\log(\log(3))-\frac{3}{6\times 3}\right)^{64-24}}{\sqrt{24+11}} = \frac{\left(-\frac{1}{6}-\frac{i}{10\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\,(-1+\log(3))^{-s}}{\Gamma(1-s)}\,ds\right)^{40}}{\sqrt{35}}$$
 for  $-1<\gamma<0$ 

From

$$\gamma_{k+1} - \gamma_k \leqslant c_1,$$

$$8-3 \ge 5$$
;  $8-3=5$ 

$$\begin{split} \varkappa p(t_0) \geqslant \frac{2-\beta}{c_1} - c_2 \frac{1}{\gamma_k} > 0, \\ E(t_0) \geqslant \frac{2(\beta-1)}{c_1} - c_3 \frac{1}{\gamma_k} > 0 \end{split}$$

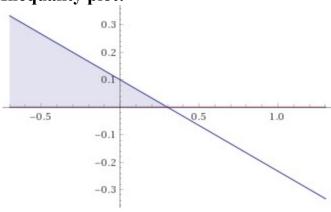
$$(2-3/2)/5 - x*1/3 > 0$$

**Input:** 
$$\frac{1}{5} \left( 2 - \frac{3}{2} \right) - x \times \frac{1}{3} > 0$$

#### **Exact result:**

$$\frac{1}{10} - \frac{x}{3} > 0$$

**Inequality plot:** 



**Alternate forms:** 

$$x<\frac{3}{10}$$

$$\frac{1}{30} (3 - 10 x) > 0$$

**Solution:** 

$$x < \frac{3}{10}$$

$$c_2 = 3/15$$

Thence, we obtain:

$$(2-3/2)/5 - 3/15*1/3$$

**Input:** 
$$\frac{1}{5} \left( 2 - \frac{3}{2} \right) - \frac{3}{15} \times \frac{1}{3}$$

**Exact result:** 

$$\frac{1}{30}$$

**Decimal approximation:** 

0.033333...

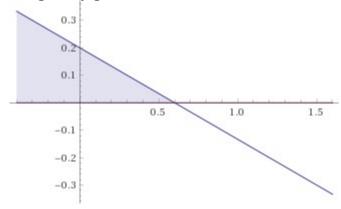
(2(3/2-1))/5-x\*1/3 > 0

Input: 
$$\frac{1}{5} \left( 2 \left( \frac{3}{2} - 1 \right) \right) - x \times \frac{1}{3} > 0$$

**Exact result:** 

$$\frac{1}{5} - \frac{x}{3} > 0$$

**Inequality plot:** 



**Alternate forms:** 

$$x < \frac{3}{5}$$

$$\frac{1}{15} (3 - 5 x) > 0$$

**Solution:** 

$$x < \frac{3}{5}$$

$$c_3 = 3/6$$

Thence, we obtain:

Input: 
$$\frac{1}{5} \left( 2 \left( \frac{3}{2} - 1 \right) \right) - \frac{3}{6} \times \frac{1}{3}$$

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#### **Exact result:**

 $\frac{1}{30}$ 

#### **Decimal approximation:**

0.033333...

From the two results, i.e. 5 and 1/30, we obtain, adding 0.979, that is the Regge slope of  $\Psi$  meson:

$$0.979 + [5*((((2-3/2)/5 - 3/15*1/3)))]^1/4$$

#### **Input:**

$$0.979 + \sqrt[4]{5\left(\frac{1}{5}\left(2 - \frac{3}{2}\right) - \frac{3}{15} \times \frac{1}{3}\right)}$$

#### **Result:**

1.617943104246272475855349305160556433665279688460039701954...

1.617943104246.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

From:

## **Some definite integrals** – *Srinivasa Ramanujan* Messenger of Mathematics, XLIV, 1915, 10 – 18

2. In a similar manner we can prove that

$$\begin{split} \int\limits_0^\infty \left(\frac{1+x^2/b^2}{1+x^2/a^2}\right) \left(\frac{1+x^2/(b+1)^2}{1+x^2/(a+1)^2}\right) \left(\frac{1+x^2/(b+2)^2}{1+x^2/(a+2)^2}\right) \cdots \cos mx \ dx \\ &= \frac{\pi \Gamma(2a) \{\Gamma(b)\}^2}{\{\Gamma(a)\}^2 \Gamma(b+a) \Gamma(b-a)} \left\{e^{-am} - \frac{2a}{1!} \frac{b-a-1}{b+a} e^{-(a+2)m} \right. \\ &\left. + \frac{2a(2a+1)}{2!} \frac{(b-a-1)(b-a-2)}{(b+a)(b+a+1)} e^{-(a+2)m} - \cdots \right\}, \end{split}$$

where m is positive and 0 < a < b. When  $0 < a < b - \frac{1}{2}$ , the integral and the series remain convergent for m = 0, and we obtain the formulæ

$$\int_{0}^{\infty} \left( \frac{1 + x^{2}/b^{2}}{1 + x^{2}/a^{2}} \right) \left( \frac{1 + x^{2}/(b+1)^{2}}{1 + x^{2}/(a+1)^{2}} \right) \left( \frac{1 + x^{2}/(b+2)^{2}}{1 + x^{2}/(a+2)^{2}} \right) \cdots dx$$

$$= \frac{1}{2} \sqrt{\pi} \frac{\Gamma(a + \frac{1}{2})\Gamma(b)\Gamma(b - a - \frac{1}{2})}{\Gamma(a)\Gamma(b - \frac{1}{2})\Gamma(b - a)}, \tag{3}$$

$$\int_{0}^{\infty} \left| \frac{\Gamma(a+ix)}{\Gamma(b+ix)} \right|^{2} dx = \frac{1}{2} \sqrt{\pi} \frac{\Gamma(a)\Gamma(a+\frac{1}{2})\Gamma(b-a-\frac{1}{2})}{\Gamma(b-\frac{1}{2})\Gamma(b)\Gamma(b-a)}.$$
 (4)

If  $a_1, a_2, a_3, \ldots, a_n$  be n positive numbers in arithmetical progression, then

$$\int_{0}^{\infty} \frac{\mathrm{d}x}{(a_1^2 + x^2)(a_2^2 + x^2)(a_3^2 + x^2) \cdots (a_n^2 + x^2)}$$

is a particular case of the above integral, and its value can be written down at once. Thus, for example, by putting  $a - \frac{11}{10}$  and  $b - \frac{61}{10}$ , we obtain

$$\int_{0}^{\infty} \frac{dx}{(x^2 + 11^2)(x^2 + 21^2)(x^2 + 31^2)(x^2 + 41^2)(x^2 + 51^2)}$$

$$= \frac{5\pi}{12 \cdot 13 \cdot 16 \cdot 17 \cdot 18 \cdot 22 \cdot 23 \cdot 24 \cdot 31 \cdot 32 \cdot 41}.$$

From:

Stringhe, Brane e (Super)Gravità - Augusto Sagnotti - Scuola Normale Superiore e INFN, Piazza dei Cavalieri 7, 56126 Pisa - Ithaca: Viaggio nella Scienza XII, 2018 • Stringhe, Brane e (Super)Gravità

We have the Virasoro-Shapiro Amplitude:

$$\mathcal{A}_{SV} = \frac{\Gamma\left(-1 - \frac{\alpha's}{4}\right)\Gamma\left(-1 - \frac{\alpha't}{4}\right)\Gamma\left(-1 - \frac{\alpha'u}{4}\right)}{\Gamma\left(2 + \frac{\alpha's}{4}\right)\Gamma\left(2 + \frac{\alpha't}{4}\right)\Gamma\left(2 + \frac{\alpha'u}{4}\right)}$$
$$= \frac{1}{\pi} \int d^2z \ |z|^{-4 - \frac{\alpha's}{2}} \ |1 - z|^{-4 - \frac{\alpha't}{2}}, (22)$$

or:

$$\frac{1}{\pi} \int d^2z \ |z|^{-4 - \frac{\alpha' s}{2}} \ |1 - z|^{-4 - \frac{\alpha' t}{2}}$$

$$= \frac{\Gamma\left(-1 - \frac{\alpha' s}{4}\right)\Gamma\left(-1 - \frac{\alpha' t}{4}\right)\Gamma\left(-1 - \frac{\alpha' u}{4}\right)}{\Gamma\left(2 + \frac{\alpha' s}{4}\right)\Gamma\left(2 + \frac{\alpha' u}{4}\right)}$$

We observe that this fundamental equation of the String Theory can be related with the following Ramanujan definite integral:

$$\int_{0}^{\infty} \left( \frac{1 + x^{2}/b^{2}}{1 + x^{2}/a^{2}} \right) \left( \frac{1 + x^{2}/(b+1)^{2}}{1 + x^{2}/(a+1)^{2}} \right) \left( \frac{1 + x^{2}/(b+2)^{2}}{1 + x^{2}/(a+2)^{2}} \right) \cdots dx$$

$$= \frac{1}{2} \sqrt{\pi} \frac{\Gamma(a + \frac{1}{2})\Gamma(b)\Gamma(b - a - \frac{1}{2})}{\Gamma(a)\Gamma(b - \frac{1}{2})\Gamma(b - a)}, \tag{3}$$

From which Ramanujan obtain:

$$\int_{0}^{\infty} \frac{dx}{(x^2 + 11^2)(x^2 + 21^2)(x^2 + 31^2)(x^2 + 41^2)(x^2 + 51^2)}$$
$$= \frac{5\pi}{12 \cdot 13 \cdot 16 \cdot 17 \cdot 18 \cdot 22 \cdot 23 \cdot 24 \cdot 31 \cdot 32 \cdot 41}.$$

Thence, we have that:

#### **Input:**

$$\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41}$$

#### **Result:**

#### **Decimal approximation:**

 $4.1638644406096341301956938554345688115959384341554306...\times10^{-14}$ 

$$4.16386444...*10^{-14}$$

### Property: $5\pi$

$$\frac{5\pi}{377244828499968}$$
 is a transcendental number

#### **Alternative representations:**

$$\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41} = \frac{900^{\circ}}{377244828499968}$$

$$\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41} = -\frac{5 i \log(-1)}{377244828499968}$$

$$\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41} = \frac{5 \cos^{-1}(-1)}{377244828499968}$$

#### **Series representations:**

$$\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41} = \frac{5\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}{94311207124992}$$

$$\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41} = \sum_{k=0}^{\infty} -\frac{(-1)^k 5^{-2k} \times 239^{-1-2k} \left(5^{1+2k} - 4 \times 239^{1+2k}\right)}{94311207124992 (1+2k)}$$

$$\frac{5\,\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41} = \frac{5\,\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^{k}\left(\frac{1}{1+2\,k} + \frac{2}{1+4\,k} + \frac{1}{3+4\,k}\right)}{377\,244\,828\,499\,968}$$

#### **Integral representations:**

$$\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41} = \frac{5}{94311207124992} \int_{0}^{1} \sqrt{1-t^{2}} dt$$

$$\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41} = \frac{5}{188622414249984} \int_{0}^{1} \frac{1}{\sqrt{1-t^{2}}} dt$$

$$\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41} = \frac{5\pi}{188622414249984} \int_{0}^{\infty} \frac{1}{1+t^{2}} dt$$

From which, we obtain:

Input:

Input: 
$$\frac{1}{2} \times \frac{1}{11} \sqrt{\left(5 \times \frac{\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}\right)^{5}}$$

#### **Exact result:**

$$25\sqrt{\frac{5}{71065423}}\pi^{5/2}$$

7213594830011932973832511539904512

#### **Decimal approximation:**

 $1.6081211894275943801226538056532738547262218478263011...\times 10^{-35}$ 

1.608121189...\*10<sup>-35</sup> result near to the value of Planck length

#### **Property:**

$$25\,\sqrt{\frac{5}{71065\,423}}\ \pi^{5/2}$$

7213594830011932973832511539904512 is a transcendental number

#### **Series representations:**

$$\frac{\sqrt{\left(\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41}\right)^{5}}}{11\times2} = \frac{1}{22} \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^{k} \left(-1 + \left(3125\pi^{5}\right)\right)$$

7 640 406 661 906 976 455 483 771 506 854 356 813 441 754 382  $^{\circ}$ . 270 548 448 843 286 446 046 445 568) $^k \left(-\frac{1}{2}\right)_k$ 

$$\frac{\sqrt{\left(\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41}\right)^{5}}}{11\times2} = \frac{1}{22}\sqrt{z_{0}}\sum_{k=0}^{\infty}\frac{1}{k!}\left(-1\right)^{k}\left(-\frac{1}{2}\right)_{k}\left(\left(3125\,\pi^{5}\right)/325\right)^{2}$$

7 640 406 661 906 976 455 483 771 506 854 356 813 441 754 382  $\cdot$ . 270 548 448 843 286 446 046 445 568  $-z_0$ )<sup>k</sup>

$$\mathbf{z_0^{-k}} \ \text{for} \left( \text{not} \left( z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0 \right) \right)$$

$$\frac{\sqrt{\left(\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41}\right)^{5}}}{11\times2} = -\frac{1}{44\sqrt{\pi}} \left( \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \left(-1 + \left(3125\pi^{5}\right)\right) \right)$$

7 640 406 661 906 976 455 483 771 506 854 356 813 441 754 382  $^{\circ}$ . 270 548 448 843 286 446 046 445 568) $^{-s}$ 

$$\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)$$

and:

where 2.02 and 1.5236 are two Hausdorff dimensions

#### **Input interpretation:**

$$\frac{1}{2.02+1.5236} \sqrt{\left(1729 \times \frac{1}{10} \times 5 \times \frac{\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}\right)^6}$$

#### **Result:**

 $1.05300... \times 10^{-34}$ 

1.053...\*10<sup>-34</sup> result very near to the value of reduced Planck constant

#### **Series representations:**

$$\frac{\sqrt{\left(\frac{1729\times5\,\pi}{10\,(12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41)}\right)^6}}{2.02+1.5236} = \\ 0.282199 \sum_{k=0}^{\infty} \frac{1}{k!} \left(-1\right)^k \left(-1 + \left(5\,534\,900\,853\,769\,\pi^6\right)\right) / \\ 38\,217\,267\,278\,202\,029\,677\,353\,560\,825\,993\,427\,026\,445\,627\,822 \\ 581\,702\,872\,198\,591\,971\,330\,828\,312\,200\,085\,504\right)^k$$

$$\left(-\frac{1}{2}\right)_{k}$$

$$\frac{\sqrt{\left(\frac{1729\times5\pi}{10(12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41)}\right)^{6}}}{2.02+1.5236} = 0.282199 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^{k} \left(-\frac{1}{2}\right)_{k} \left(\left(5534900\,853\,769\,\pi^{6}\right)\right) / \\
38\,217\,267\,278\,202\,029\,677\,353\,560\,825\,993\,427\,026\,445\,627 \times \\
822\,581\,702\,872\,198\,591\,971\,330\,828\,312\,200\,085\,504 - \\
z_{0}\right)^{k} z_{0}^{-k} \text{ for } \left(\text{not } \left(z_{0} \in \mathbb{R} \text{ and } -\infty < z_{0} \le 0\right)\right)$$

$$\begin{split} \frac{\sqrt{\left(\frac{1729\times5\,\pi}{10\,(12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41)}\right)^6}}{2.02+1.5236} = \\ -\frac{1}{\sqrt{\pi}}\,0.141099 \sum_{j=0}^{\infty} \mathrm{Res}_{s=-j} \left(-1+\left(5\,534\,900\,853\,769\,\pi^6\right)\right/\\ & 38\,217\,267\,278\,202\,029\,677\,353\,560\,825\,993\,427\,026\,445\,627\,822 \times \\ & 581\,702\,872\,198\,591\,971\,330\,828\,312\,200\,085\,504\right)^{-s} \\ \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s) \end{split}$$

We obtain also, performing the 64<sup>th</sup> root, the following result:

$$(((1/((((5Pi)/(12*13*16*17*18*22*23*24*31*32*41))))))^1/64$$

#### **Input:**

$$64\sqrt[]{\frac{1}{\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41}}}$$

#### **Exact result:**

$$\sqrt[4]{2}$$
  $\sqrt[16]{3}$   $\sqrt[64]{\frac{71\,065\,423}{5\,\pi}}$ 

#### **Decimal approximation:**

 $1.618342225845842761400413569275379945228266832333393483623\dots$ 

1.61834222584.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

#### **Property:**

$$\sqrt[4]{2}$$
  $\sqrt[16]{3}$   $\sqrt[64]{\frac{71\ 065\ 423}{5\ \pi}}$  is a transcendental number

### All 64th roots of 377244828499968/(5 $\pi$ ):

$$\sqrt[4]{2}$$
  $\sqrt[16]{3}$   $64$   $\sqrt[6]{71065423}$   $\sqrt[6]{5}$   $\pi$   $e^0 \approx 1.61834$  (real, principal root)

$$\sqrt[4]{2} \ \sqrt[16]{3} \ 6\sqrt[4]{\frac{71\ 065\ 423}{5\ \pi}} \ e^{(i\ \pi)/32} \approx 1.61055 + 0.15863\ i$$

$$\sqrt[4]{2} \ \sqrt[16]{3} \ 64 \sqrt[4]{\frac{71\ 065\ 423}{5\ \pi}} \ e^{(i\ \pi)/16} \approx 1.58725 + 0.31572\ i$$

$$\sqrt[4]{2} \sqrt[16]{3} \sqrt[64]{\frac{71\,065\,423}{5\,\pi}} \ e^{(3\,i\,\pi)/32} \approx 1.54866 + 0.4698\,i$$

$$\sqrt[4]{2} \sqrt[16]{3} \sqrt[64]{\frac{71065423}{5\pi}} e^{(i\pi)/8} \approx 1.4952 + 0.6193 i$$

#### **Alternative representations:**

$$\sqrt[64]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} = 64\sqrt[4]{\frac{1}{\frac{900^{\circ}}{377244828499968}}}$$

$$\sqrt[54]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} = \sqrt[64]{\frac{1}{\frac{5\cos^{-1}(-1)}{377244828499968}}}$$

#### **Series representations:**

$$_{64}\sqrt[]{\frac{1}{\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41}}} = 2^{7/32}\sqrt[16]{3}\sqrt[64]{\frac{71065423}{5}}\sqrt[64]{\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}}$$

$$\frac{1}{5\pi} = \frac{1}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41} = \frac{4\sqrt{2} \cdot 16\sqrt{3} \cdot 64\sqrt{\frac{71065423}{5}} \cdot 64\sqrt{\frac{1}{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^{k} \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}}$$

$$64\sqrt{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}} = \frac{2^{7/32} \cdot 16\sqrt{3} \cdot 64\sqrt{\frac{71065423}{5}} \cdot 64\sqrt{\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{1+k} \cdot 1195^{-1-2k} \left(5^{1+2k} - 4 \times 239^{1+2k}\right)}{1+2k}}}$$

#### **Integral representations:**

$$\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}} = 2^{\frac{15}{64}} \int_{0}^{16} \frac{1}{1 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}{5} = 2^{\frac{15}{64}} \int_{0}^{16} \frac{1}{1 \times 12 \times 13 \times 16 \times 17 \times 18} dt$$

$$\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}} = 2^{7/32} \frac{16\sqrt{3}}{5} \frac{64}{5} \frac{1}{\int_{0}^{1} \sqrt{1 - t^{2}} dt}$$

$$\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}} = 2^{15/64} \frac{1}{\sqrt{3}} \frac{5\pi}{64} \frac{1}{\sqrt{1-t^2}} \frac{1}{dt}$$

and:

2 log base

**Input interpretation:** 

$$2 \log_{1.6183422258458} \left( \frac{1}{\frac{5 \, \pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}} \right) - 3 + \frac{1}{2}$$

 $log_b(x)$  is the base- b logarithm

#### **Result:**

125.50000000001...

125.5... result very near to the Higgs boson mass 125.18 GeV

## **Rational approximation:** $\frac{251}{2} = 125 + \frac{1}{2}$

$$\frac{251}{2} = 125 + \frac{1}{2}$$

#### **Alternative representation:**

$$2 \log_{1.61834222584580000} \left( \frac{1}{\frac{5 \pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}} \right) - 3 + \frac{1}{2} = \frac{2 \log \left( \frac{1}{\frac{5 \pi}{377244828499968}} \right)}{\log(1.61834222584580000)}$$

#### **Series representation:**

2 log base

$$1.6183422258458(((1/((((5Pi)/(12*13*16*17*18*22*23*24*31*32*41)))))))+11+1/2$$

**Input interpretation:** 

$$2 \log_{1.6183422258458} \left( \frac{1}{\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41}} \right) + 11 + \frac{1}{2}$$

 $log_b(x)$  is the base- b logarithm

#### **Result:**

139.50000000001...

139.5... result practically equal to the rest mass of Pion meson 139.57 MeV

# Rational approximation: $\frac{279}{2} = 139 + \frac{1}{2}$

$$\frac{279}{2} = 139 + \frac{1}{2}$$

#### **Alternative representation:**

$$2 \log_{1.61834222584580000} \left( \frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}} \right) + 11 + \frac{1}{2} = \frac{23}{2} + \frac{2 \log \left( \frac{1}{\frac{5\pi}{377244828499968}} \right)}{\log(1.61834222584580000)}$$

#### **Series representation:**

$$\begin{split} 2\log_{1.61834222584580000} & \left( \frac{1}{\frac{5\,\pi}{12\,\times\,13\,\times\,16\,\times\,17\,\times\,18\,\times\,22\,\times\,23\,\times\,24\,\times\,31\,\times\,32\,\times\,41}} \right) + 11 + \frac{1}{2} = \\ & 11.50000000000000000000 + 4.2344548316497359 \log \left( \frac{377\,244\,828\,499\,968}{5\,\pi} \right) - \\ & 2.00000000000000000000 \log \left( \frac{377\,244\,828\,499\,968}{5\,\pi} \right) \\ & \sum_{k=0}^{\infty} 0.61834222584580000^k \, G(k) \\ & for \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k}\,k}{2\,(1+k)\,(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j}\,G(-j+k)}{1+j} \right) \end{split}$$

#### 27\* log base

#### **Input interpretation:**

$$27 \log_{1.6183422258458} \left( \frac{1}{\frac{5 \pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}} \right) + 1$$

 $\log_b(x)$  is the base- b logarithm

#### **Result:**

1729.0000000001...

1729

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

#### With regard 27 (From Wikipedia):

"The fundamental group of the complex form, compact real form, or any algebraic version of  $E_6$  is the cyclic group  $\mathbb{Z}/3\mathbb{Z}$ , and its outer automorphism group is the cyclic group  $\mathbb{Z}/2\mathbb{Z}$ . Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics,  $E_6$  plays a role in some grand unified theories".

#### **Alternative representation:**

$$27 \log_{1.61834222584580000} \left( \frac{1}{\frac{5 \pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}} \right) + 1 = \\ \frac{27 \log \left( \frac{1}{\frac{5 \pi}{377244828499968}} \right)}{\log(1.61834222584580000)}$$

#### **Series representation:**

$$((((27* log base)$$

 $1.6183422258458(((1/((((5Pi)/(12*13*16*17*18*22*23*24*31*32*41)))))))+1))))^{1}/(15)$ 

**Input interpretation:** 

$$\frac{1}{15} 27 \log_{1.6183422258458} \left( \frac{1}{\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41}} \right) + 1$$

 $log_b(x)$  is the base- b logarithm

#### **Result:**

1.64381522874873...

$$1.64381522874873...$$
  $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934...$ 

We have also:

$$(((1/((((5Pi)/(12*13*16*17*18*22*23*24*31*32*41))))))^{1/4}+55+13+5)$$

#### **Input:**

$$\sqrt[4]{\frac{1}{\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41}}} + 55 + 13 + 5$$

#### **Exact result:**

$$73 + 48 \sqrt[4]{\frac{71065423}{5\pi}}$$

#### **Decimal approximation:**

2286.736136652429861813916739810094456515474704559676280466...

2286.73613665... result practically equal to the rest mass of charmed Lambda baryon 2286.46

#### **Property:**

$$73 + 48 \sqrt[4]{\frac{71065423}{5\pi}}$$
 is a transcendental number

#### **Alternate form:**

$$\frac{1}{5} \left( 365 + 48 \times 5^{3/4} \sqrt[4]{\frac{71\,065\,423}{\pi}} \right)$$

#### **Alternative representations:**

$$\sqrt[4]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} + 55 + 13 + 5 = 73 + \sqrt[4]{\frac{1}{\frac{900^{\circ}}{377244828499968}}}$$

$$\sqrt[4]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} + 55 + 13 + 5 = 73 + \sqrt[4]{-\frac{1}{\frac{5i \log(-1)}{377244828499968}}}$$

$$\sqrt[4]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} + 55 + 13 + 5 = 73 + \sqrt[4]{\frac{1}{\frac{5\cos^{-1}(-1)}{377244828499968}}}$$

#### **Series representations:**

$$\sqrt[4]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} + 55 + 13 + 5 = 73 + 24\sqrt{2}\sqrt[4]{\frac{71065423}{5}\sqrt[4]{\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}}}$$

$$\frac{4}{\sqrt{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} + 55 + 13 + 5 = \frac{1}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41} + \frac{1}{\sqrt{\frac{1}{\sum_{k=0}^{\infty} -\frac{4(-1)^{k} \cdot 1195^{-1} - 2 \cdot k}{1 + 2 \cdot k}}}} = \frac{1}{1 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}} + 55 + 13 + 5 = \frac{1}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}} + \frac{1}{\sqrt{\frac{1}{\sum_{k=0}^{\infty} (-\frac{1}{4})^{k}} (\frac{2}{1 + 4 \cdot k} + \frac{2}{2 + 4 \cdot k} + \frac{1}{3 + 4 \cdot k})}}}}$$

#### **Integral representations:**

$$\sqrt[4]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} + 55 + 13 + 5 = 73 + 24\sqrt{2}\sqrt[4]{\frac{71065423}{5}\sqrt[4]{\frac{1}{\int_{0}^{1}\sqrt{1 - t^{2}}} dt}}}$$

$$\sqrt[4]{\frac{1}{\frac{5\pi}{12\times13\times16\times17\times18\times22\times23\times24\times31\times32\times41}}} + 55 + 13 + 5 = 73 + 24\times2^{3/4} \sqrt[4]{\frac{71065423}{5}} \sqrt[4]{\frac{1}{\int_0^\infty \frac{1}{14t^2} dt}}}$$

$$\sqrt[4]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} + 55 + 13 + 5 = 73 + 24 \times 2^{3/4} \sqrt[4]{\frac{71065423}{5}} \sqrt[4]{\frac{1}{\sqrt[4]{1-t^2}}} dt$$

#### We note that:

#### **Input:**

$$\frac{1}{5} \sqrt[3]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}}$$

#### **Exact result:**

$$\frac{96}{5} \sqrt[3]{\frac{426392538}{5\pi}}$$

#### **Decimal approximation:**

5770.292144369049110643513897658006721467676355485615735181...

5770.292144.....

#### **Property:**

$$\frac{96}{5} \sqrt[3]{\frac{426392538}{5\pi}}$$
 is a transcendental number

#### **Alternative representations:**

$$\frac{1}{5} \sqrt[3]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} = \frac{1}{5} \sqrt[3]{\frac{1}{\frac{900^{\circ}}{377244828499968}}}$$

$$\frac{1}{5} \sqrt[3]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} = \frac{1}{5} \sqrt[3]{-\frac{1}{\frac{5i \log(-1)}{377244828499968}}}$$

$$\frac{1}{5} \sqrt[3]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} = \frac{1}{5} \sqrt[3]{\frac{1}{\frac{5\cos^{-1}(-1)}{377244828499968}}}$$

# **Series representations:**

$$\frac{1}{5} \sqrt[3]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} = \frac{48}{5} \times 2^{2/3} \sqrt[3]{\frac{213196269}{5}} \sqrt[3]{\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}}$$

$$\frac{1}{5} \sqrt[3]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} = \frac{96}{5} \sqrt[3]{\frac{426 \ 392 \ 538}{5}} \sqrt[3]{\frac{1}{\sum_{k=0}^{\infty} -\frac{4 \ (-1)^k \ 1195^{-1-2} \ k \left(5^{1+2} \ k-4 \times 239^{1+2} \ k\right)}{1+2 \ k}}}$$

$$\frac{1}{5}\sqrt[3]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} = \frac{96}{5}\sqrt[3]{\frac{426 392 538}{5}}\sqrt[3]{\frac{1}{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^{k} \left(\frac{1}{1+2 k} + \frac{2}{1+4 k} + \frac{1}{3+4 k}\right)}}$$

# **Integral representations:**

$$\frac{1}{5} \sqrt[3]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} = \frac{96}{5} \sqrt[3]{\frac{213 \ 196 \ 269}{5}} \sqrt[3]{\frac{1}{\int_0^\infty \frac{1}{1+t^2} \ dt}}$$

$$\frac{1}{5} \sqrt[3]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} = \frac{96}{5} \sqrt[3]{\frac{213 \ 196 \ 269}{5}} \sqrt[3]{\frac{1}{\sqrt[3]{\frac{1}{\sqrt{1-t^2}}}} \ dt}$$

$$\frac{1}{5} \sqrt[3]{\frac{1}{\frac{5\pi}{12 \times 13 \times 16 \times 17 \times 18 \times 22 \times 23 \times 24 \times 31 \times 32 \times 41}}} = \frac{96}{5} \sqrt[3]{\frac{213 \ 196 \ 269}{5}} \sqrt[3]{\frac{1}{\int_0^\infty \frac{\sin(t)}{t} \ dt}}$$

From the ratio between proton and electron, multiplied by  $\pi$ , we obtain:

(938.27204621 / 0.5109989500015)\*Pi

## **Input interpretation:**

 $\frac{938.27204621}{0.5109989500015} \pi$ 

#### **Result:**

5768.4434918...

5768.4434918...

# **Alternative representations:**

$$\frac{\pi\,938.272046210000}{0.51099895000150000} = \frac{168\,888.968317800\,^{\circ}}{0.51099895000150000}$$
 
$$\frac{\pi\,938.272046210000}{0.51099895000150000} = -\frac{938.272046210000\,i\,\log(-1)}{0.51099895000150000}$$
 
$$\frac{\pi\,938.272046210000}{0.51099895000150000} = \frac{938.272046210000\,\cos^{-1}(-1)}{0.51099895000150000}$$

# **Series representations:**

$$\frac{\pi\,938.272046210000}{0.51099895000150000} = 7344.61036530307 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2\,k}$$

$$\frac{\pi\,938.272046210000}{0.51099895000150000} = -3672.30518265153 + 3672.30518265153 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2\,k}{k}}$$

$$\frac{\pi\,938.272046210000}{0.51099895000150000} = 1836.15259132577 \sum_{k=0}^{\infty} \frac{2^{-k}\,(-6+50\,k)}{{3\,k \choose k}}$$

# **Integral representations:**

$$\frac{\pi\,938.272046210000}{0.51099895000150000} = 3672.30518265153 \int_0^\infty \frac{1}{1+t^2}\,dt$$

$$\frac{\pi\,938.272046210000}{0.51099895000150000} = 7344.61036530307 \int_0^1 \sqrt{1-t^2} \ dt$$

$$\frac{\pi\,938.272046210000}{0.51099895000150000} = 3672.30518265153 \int_0^\infty \frac{\sin(t)}{t}\,dt$$

We have two results very near: 5770.292144..... and 5768.4434

Adding 18 (that is a Lucas number) to these results, we obtain: 5788.29214 and 5786.4434 values very near to the rest mass of bottom Xi baryon 5787.8

From:

## Friedmann Equation in Codimension-two Braneworlds

*Chen Fang* - Master of Science - Department of Physics - McGill University - Montreal, Quebec, Canada - September 1, 2007

$$\frac{d \ln G}{d \delta \tau_{3}} = -\frac{k}{\left(\overline{P}^{3} - 1\right)^{2} \left(1 - k \overline{\tau}_{3}\right)} \left(\frac{3}{2} \frac{\overline{P}^{6} \left(4 \left(\alpha + 1\right) \overline{P}^{5} - 9 \alpha + 1\right)}{2 \left(\alpha - 5\right) \overline{P}^{5} + 3 \alpha + 5} + \frac{3}{2} - P^{3} (P^{3} - 1)\right)$$
(2.39)

For

$$\alpha = 5$$
.

$$k_6 = 1$$
 unit

$$k = k_6^2/(2\pi)$$
.

 $k\bar{\tau}_3$  must be in [0,1)

$$P \sim 10^{16} \ (P = 10^{16})$$

we obtain:

$$1/(2\text{Pi}) * 1/(((10^48-1)^2*(1-0.5))) * [(3/2*(((((10^96(4(5+1)*10^80-9*5+1))))) / (((2(5-5)*10^80+3*5+5)))))+3/2-10^48(10^48-1)]$$

## **Input:**

$$\frac{1}{2 \pi} \times \frac{1}{\left(10^{48} - 1\right)^2 (1 - 0.5)} \left(\frac{3}{2} \times \frac{10^{96} \left(4 (5 + 1) \times 10^{80} - 9 \times 5 + 1\right)}{2 (5 - 5) \times 10^{80} + 3 \times 5 + 5} + \frac{3}{2} - 10^{48} \left(10^{48} - 1\right)\right) + \frac{3}{2} + \frac{3}{2$$

#### **Result:**

$$5.72958... \times 10^{79}$$

# **Alternative representations:**

$$\frac{\frac{3 \left(10^{96} \left(4 \left(5+1\right) 10^{80}-9 \times 5+1\right)\right)}{2 \left(2 \left(5-5\right) 10^{80}+3 \times 5+5\right)}+\frac{3}{2}-10^{48} \left(10^{48}-1\right)}{\frac{\left(\left(10^{48}-1\right)^2 \left(1-0.5\right)\right) \left(2 \, \pi\right)}{\left(\left(10^{48}-1\right)^2 10^{48}+\frac{3 \left(-44+24 \times 10^{80}\right) 10^{96}}{2 \times 20}}}{\frac{3}{2}-\left(-1+10^{48}\right) 10^{48}+\frac{3 \left(-44+24 \times 10^{80}\right) 10^{96}}{2 \times 20}}{\left(360 \, ^{\circ}\right) \left(0.5 \left(-1+10^{48}\right)^2\right)}}$$

$$\frac{\frac{3\left(10^{96}\left(4\left(5+1\right)10^{80}-9\times5+1\right)\right)}{2\left(2\left(5-5\right)10^{80}+3\times5+5\right)}+\frac{3}{2}-10^{48}\left(10^{48}-1\right)}{\frac{\left(\left(10^{48}-1\right)^{2}\left(1-0.5\right)\right)\left(2\pi\right)}{\left(\left(10^{48}-1\right)^{2}\left(1-0.5\right)\right)\left(2\pi\right)}}=\\-\frac{\frac{3}{2}-\left(-1+10^{48}\right)10^{48}+\frac{3\left(-44+24\times10^{80}\right)10^{96}}{2\times20}}{\left(2i\log(-1)\right)\left(0.5\left(-1+10^{48}\right)^{2}\right)}$$

$$\frac{\frac{3 \left(10^{96} \left(4 \left(5+1\right) 10^{80}-9 \times 5+1\right)\right)}{2 \left(2 \left(5-5\right) 10^{80}+3 \times 5+5\right)}+\frac{3}{2}-10^{48} \left(10^{48}-1\right)}{\left(\left(10^{48}-1\right)^2 \left(1-0.5\right)\right) \left(2 \, \pi\right)} \\ = \\ \frac{\frac{3}{2}-\left(-1+10^{48}\right) 10^{48}+\frac{3 \left(-44+24 \times 10^{80}\right) 10^{96}}{2 \times 20}}{\left(2 \cos ^{-1}(-1)\right) \left(0.5 \left(-1+10^{48}\right)^2\right)}$$

# **Series representations:**

$$\frac{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{2(2(5-5)10^{80}+3\times5+5)} + \frac{3}{2} - 10^{48}(10^{48} - 1)}{((10^{48} - 1)^2(1 - 0.5))(2\pi)} = \frac{4.5 \times 10^{79}}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$\frac{\frac{3 \left(10^{96} \left(4 \left(5+1\right) 10^{80} - 9 \times 5+1\right)\right)}{2 \left(2 \left(5-5\right) 10^{80} + 3 \times 5+5\right)} + \frac{3}{2} - 10^{48} \left(10^{48} - 1\right)}{\left(\left(10^{48} - 1\right)^2 \left(1 - 0.5\right)\right) \left(2 \, \pi\right)} = \frac{9. \times 10^{79}}{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2 \, k}{k}}}$$

$$\frac{\frac{3 \left(10^{96} \left(4 \left(5+1\right) 10^{80} - 9 \times 5+1\right)\right)}{2 \left(2 \left(5-5\right) 10^{80} + 3 \times 5+5\right)} + \frac{3}{2} - 10^{48} \left(10^{48} - 1\right)}{\left(\left(10^{48} - 1\right)^2 \left(1 - 0.5\right)\right) \left(2 \, \pi\right)} = \frac{1.8 \times 10^{80}}{\sum_{k=0}^{\infty} \frac{2^{-k} \left(-6 + 50 \, k\right)}{\binom{3 \, k}{k}}}$$

# **Integral representations:**

$$\frac{\frac{3 \left(10^{96} \left(4 \left(5+1\right) 10^{80}-9 \times 5+1\right)\right)}{2 \left(2 \left(5-5\right) 10^{80}+3 \times 5+5\right)}+\frac{3}{2}-10^{48} \left(10^{48}-1\right)}{\left(\left(10^{48}-1\right)^2 \left(1-0.5\right)\right) \left(2 \, \pi\right)}=\frac{9.\times 10^{79}}{\int_0^\infty \frac{1}{1+t^2} \, dt}$$

$$\frac{\frac{3 \left(10^{96} \left(4 \left(5+1\right) 10^{80}-9 \times 5+1\right)\right)}{2 \left(2 \left(5-5\right) 10^{80}+3 \times 5+5\right)}+\frac{3}{2}-10^{48} \left(10^{48}-1\right)}{\left(\left(10^{48}-1\right)^2 \left(1-0.5\right)\right) \left(2 \, \pi\right)}=\frac{4.5 \times 10^{79}}{\int_0^1 \sqrt{1-t^2} \ dt}$$

$$\frac{\frac{3 \left(10^{96} \left(4 \left(5+1\right) 10^{80}-9 \times 5+1\right)\right)}{2 \left(2 \left(5-5\right) 10^{80}+3 \times 5+5\right)}+\frac{3}{2}-10^{48} \left(10^{48}-1\right)}{\left(\left(10^{48}-1\right)^2 \left(1-0.5\right)\right) \left(2 \, \pi\right)}=\frac{9.\times 10^{79}}{\int_0^\infty \frac{\sin(t)}{t} \, dt}$$

### We have that:

For the special case  $\alpha = 5$ , the above approximations are not valid, and the time variation of G is relatively large if the radial size of the extra dimension P is much greater than unity. We get

$$\frac{d\ln G}{d\delta\tau_3} \cong -\frac{9}{5}\overline{P}^5 \frac{k}{(1-k\bar{\tau}_3)} \tag{2.43}$$

The constraint on  $\bar{\tau}_3$  becomes more stringent in that case, by a factor of  $\overline{P}^5$ . For the interesting value of  $P \sim 10^{16}$  which is required to tackle the hierarchy problem,

 $\frac{k}{(1-k\bar{\tau}_3)} \lesssim 10^{-36}/~(\text{GeV}^4)$  gives  $M_6 > 10^9~\text{GeV}$ . From the spatial constraint we get  $M_6 \geq M_p$ . Alternatively one can have  $P \sim 1$  and keep the constraint (2.42). In no case is there any constraint on  $\bar{\tau}_3$ , except that it cannot be too close to  $1/k_6$ .

## Thence, from:

$$\frac{1}{2\,\pi} \times \frac{1}{\left(10^{48}-1\right)^2\,(1-0.5)} \left(\frac{3}{2} \times \frac{10^{96} \left(4\,(5+1) \times 10^{80}-9 \times 5+1\right)}{2\,(5-5) \times 10^{80}+3 \times 5+5} +\frac{3}{2}-10^{48} \left(10^{48}-1\right)\right)$$

5.72958... × 10<sup>79</sup>

For

$$M_6 \geq 10^9 \; \mathrm{GeV}$$

we obtain:

$$1/2 \ sqrt(((10^9 * 1/(2Pi) * 1/(((10^48-1)^2*(1-0.5))) * \\ [(3/2*((((((10^96(4(5+1)*10^80-9*5+1))))) / (((2(5-5)*10^80+3*5+5)))))) + 3/2-10^48(10^48-1)])))$$

**Input:** 

$$\frac{1}{2} \sqrt{\left(10^9 \times \frac{1}{2\pi} \times \frac{1}{\left(10^{48} - 1\right)^2 (1 - 0.5)} + \frac{3}{2} \times \frac{10^{96} \left(4 (5 + 1) \times 10^{80} - 9 \times 5 + 1\right)}{2 (5 - 5) \times 10^{80} + 3 \times 5 + 5} + \frac{3}{2} - 10^{48} \left(10^{48} - 1\right)\right)}$$

## **Result:**

 $1.19683... \times 10^{44}$ 

 $1.19683...*10^{44} \approx 1.2 * 10^{44}$  value very near to the result of the following formula

$$\frac{d\ln G}{d\delta\tau_3} \lesssim 1.2 \times 10^{44} / \text{ GeV}^4 \tag{2.42}$$

$$= 1.2 * 10^{44}$$

## **Series representations:**

$$\frac{1}{2} \sqrt{\frac{10^{9} \left(\frac{3 \left(10^{96} \left(4 \left(5+1\right) 10^{80} - 9 \times 5+1\right)\right)}{2 \left(2 \left(5-5\right) 10^{80} + 3 \times 5+5\right)} + \frac{3}{2} - 10^{48} \left(10^{48} - 1\right)\right)}{\left(2 \pi\right) \left(\left(10^{48} - 1\right)^{2} \left(1 - 0.5\right)\right)}} = \frac{1}{2} \sqrt{-1 + \frac{1.8 \times 10^{89}}{\pi}} \sum_{k=0}^{\infty} \left(-1 + \frac{1.8 \times 10^{89}}{\pi}\right)^{-k} \left(\frac{1}{2} \atop k\right)}$$

$$\frac{1}{2} \sqrt{\frac{10^{9} \left(\frac{3 \left(10^{96} \left(4 \left(5+1\right) 10^{80} - 9 \times 5+1\right)\right)}{2 \left(2 \left(5-5\right) 10^{80} + 3 \times 5+5\right)} + \frac{3}{2} - 10^{48} \left(10^{48} - 1\right)\right)}{\left(2 \pi\right) \left(\left(10^{48} - 1\right)^{2} \left(1 - 0.5\right)\right)}}{\frac{1}{2} \sqrt{-1 + \frac{1.8 \times 10^{89}}{\pi} \sum_{k=0}^{\infty} \frac{\left(-1\right)^{k} \left(-1 + \frac{1.8 \times 10^{89}}{\pi}\right)^{-k} \left(-\frac{1}{2}\right)_{k}}{k!}}}$$

$$\begin{split} \frac{1}{2} \sqrt{\frac{10^9 \left(\frac{3 \left(10^{96} \left(4 \left(5+1\right) 10^{80} - 9 \times 5+1\right)\right)}{2 \left(2 \left(5-5\right) 10^{80} + 3 \times 5+5\right)} + \frac{3}{2} - 10^{48} \left(10^{48} - 1\right)\right)}}{(2 \pi) \left(\left(10^{48} - 1\right)^2 \left(1 - 0.5\right)\right)}} = \\ \frac{1}{2} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1.8 \times 10^{89}}{\pi} - z_0\right)^k z_0^{-k}}{k!}} \quad \text{for (not } \left(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0\right)\right)} \end{split}$$

From the previous Ramanujan expression

$$\int_{0}^{\infty} \frac{dx}{(x^2 + 11^2)(x^2 + 21^2)(x^2 + 31^2)(x^2 + 41^2)(x^2 + 51^2)}$$
$$= \frac{5\pi}{12 \cdot 13 \cdot 16 \cdot 17 \cdot 18 \cdot 22 \cdot 23 \cdot 24 \cdot 31 \cdot 32 \cdot 41}.$$

 $4.1638644406096341301956938554345688115959384341554306... \times 10^{-14}$ 

$$4.16386444...*10^{-14}$$

we have the following mathematical connection:

$$377*1/((((1/(2Pi)*1/(((10^48-1)^2*(1-0.5)))*[(3/2*((((((10^96(4(5+1)*10^80-9*5+1)))))/(((2(5-5)*10^80+3*5+5))))))*[(3/2*(((((10^96(4(5+1)*10^80-9*5+1)))))/(((2(5-5)*10^80+3*5+5))))))+3/2-10^48(10^48-1)]))))^1/5$$

## **Input:**

$$\frac{1}{5\sqrt{\frac{1}{2\pi} \times \frac{1}{\left(10^{48}-1\right)^2(1-0.5)} \left(\frac{3}{2} \times \frac{10^{96} \left(4 (5+1) \times 10^{80} - 9 \times 5 + 1\right)}{2 (5-5) \times 10^{80} + 3 \times 5 + 5} + \frac{3}{2} - 10^{48} \left(10^{48} - 1\right)\right)}}$$

#### **Result:**

$$4.214216... \times 10^{-14}$$

$$4.214216...*10^{-14}$$

## **Alternative representations:**

$$\frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{2(2(5-5)10^{80}+3\times5+5)}} + \frac{3}{2}-10^{48}(10^{48}-1)}} = \frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2\pi)((10^{48}-1)^2(1-0.5))}} + \frac{3}{2}-10^{48}(10^{48}-1)}} = \frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{2(2(5-5)10^{80}+3\times5+5)}} + \frac{3}{2}-10^{48}(10^{48}-1)}}} = \frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2\pi)((10^{48}-1)^2(1-0.5))}} + \frac{3}{2}-10^{48}(10^{48}-1)}} = \frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2(5-5)10^{80}+3\times5+5)}} + \frac{3}{2}-10^{48}(10^{48}-1)}}} = \frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2(5-5)10^{80}+3\times5+5)}}}} = \frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2(5-5)10^{80}+3\times5+5)}}} = \frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2(5-5)10^{80}+3\times5+5)}}} = \frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2(5-5)10^{80}+3\times5+5)}}} = \frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2(5-5)10^{80}+3\times5+5)}}} = \frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2(5-5)10^{80}+3\times5+5)}}} = \frac{377}{\sqrt[5]{\frac{3$$

# **Series representations:**

$$\frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{2(2(5-5)10^{80}+3\times5+5)} + \frac{3}{2}-10^{48}(10^{48}-1)}}} = \frac{4.42282\times10^{-14}}{\sqrt[5]{\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}}}}$$

$$\frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2\pi)((10^{48}-1)^2(1-0.5))} + \frac{3}{2}-10^{48}(10^{48}-1)}}} = \frac{3.85028\times10^{-14}}{\sqrt[5]{\frac{1}{-1+\sum_{k=1}^{\infty}\frac{2^k}{(2k)}}}}}$$

$$\frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2\pi)((10^{48}-1)^2(1-0.5))} + \frac{3}{2}-10^{48}(10^{48}-1)}}} = \frac{3.35187\times10^{-14}}{\sqrt[5]{\frac{2}{k+2}\sum_{k=1}^{\infty}\frac{\sin(kx)}{k}}}}$$
for  $(x \in \mathbb{R} \text{ and } x > 0)$ 

## **Integral representations:**

$$\frac{377}{\sqrt[5]{\frac{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{2(2(5-5)10^{80}+3\times5+5)} + \frac{3}{2}-10^{48}(10^{48}-1)}}}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2\pi)((10^{48}-1)^2(1-0.5))}}} = \frac{3.85028\times10^{-14}}{\sqrt[5]{\frac{1}{0^{\infty}\frac{1}{1+t^2}}dt}}$$

$$\frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2\pi)((10^{48}-1)^2(1-0.5))}} + \frac{3}{2}-10^{48}(10^{48}-1)}}{\sqrt[5]{\frac{1}{0^{1}}\sqrt{1-t^2}dt}} = \frac{4.42282\times10^{-14}}{\sqrt[5]{\frac{1}{0^{1}}\sqrt{1-t^2}dt}}}$$

$$\frac{377}{\sqrt[5]{\frac{3(10^{96}(4(5+1)10^{80}-9\times5+1))}{(2\pi)((10^{48}-1)^2(1-0.5))}} + \frac{3}{2}-10^{48}(10^{48}-1)}}{\sqrt[5]{\frac{1}{0^{\infty}\frac{\sin(t)}{t}dt}}}} = \frac{3.85028\times10^{-14}}{\sqrt[5]{\frac{1}{0^{\infty}\frac{\sin(t)}{t}dt}}}$$

# From the following expression

$$\begin{split} \frac{1}{2} \ \sqrt{\left(10^9 \times \frac{1}{2 \, \pi} \times \frac{1}{\left(10^{48} - 1\right)^2 \, (1 - 0.5)} \right.} \\ \left. \left(\frac{3}{2} \times \frac{10^{96} \left(4 \, (5 + 1) \times 10^{80} - 9 \times 5 + 1\right)}{2 \, (5 - 5) \times 10^{80} + 3 \times 5 + 5} + \frac{3}{2} - 10^{48} \left(10^{48} - 1\right)\right) \right) \end{split}$$

we have also:

$$(1597+8)^3/(((((1/2 \text{ sqrt}(((10^9 * 1/(2Pi) * 1/(((10^48-1)^2*(1-0.5))) * ((3/2*(((((((10^96(4(5+1)*10^80-9*5+1))))) / (((2(5-5)*10^80+3*5+5)))))))+3/2-10^48(10^48-1)]))))))))$$

### **Input:**

$$\frac{\left(1597+8\right)^{3}}{\frac{1}{2}\,\sqrt{10^{9}\times\frac{1}{2\,\pi}\times\frac{1}{\left(10^{48}-1\right)^{2}\left(1-0.5\right)}\left(\frac{3}{2}\times\frac{10^{96}\left(4\,(5+1)\times10^{80}-9\times5+1\right)}{2\,(5-5)\times10^{80}+3\times5+5}+\frac{3}{2}-10^{48}\left(10^{48}-1\right)\right)}}$$

### **Result:**

$$3.45457... \times 10^{-35}$$

## **Series representations:**

$$\frac{1}{\frac{1}{2}}\sqrt{\frac{10^{9}\left(\frac{3\left(10^{96}\left(4\left(5+1\right)10^{80}-9\times5+1\right)\right)}{2\left(2\left(5-5\right)10^{80}+3\times5+5\right)}+\frac{3}{2}-10^{48}\left(10^{48}-1\right)\right)}{8269\,040\,250}}}{\frac{1}{2}\sqrt{\frac{10^{9}\left(\frac{3\left(10^{96}\left(4\left(5+1\right)10^{80}-9\times5+1\right)\right)}{\pi}\sum_{k=0}^{\infty}\left(-1+\frac{1.8\times10^{89}}{\pi}\right)^{-k}\left(\frac{1}{2}\right)}{2\left(2\left(5-5\right)10^{80}+3\times5+5\right)}}}}{\frac{1}{2}\sqrt{\frac{10^{9}\left(\frac{3\left(10^{96}\left(4\left(5+1\right)10^{80}-9\times5+1\right)\right)}{2\left(2\left(5-5\right)10^{80}+3\times5+5\right)}+\frac{3}{2}-10^{48}\left(10^{48}-1\right)\right)}{8269\,040\,250}}}{\frac{1}{2}\sqrt{\frac{10^{9}\left(\frac{3\left(10^{96}\left(4\left(5+1\right)10^{80}-9\times5+1\right)\right)}{2\left(2\left(5-5\right)10^{80}+3\times5+5\right)}+\frac{3}{2}-10^{48}\left(10^{48}-1\right)\right)}{k!}}}{\frac{1}{2}\sqrt{\frac{10^{9}\left(\frac{3\left(10^{96}\left(4\left(5+1\right)10^{80}-9\times5+1\right)\right)}{\pi}\sum_{k=0}^{\infty}\frac{\left(-1\right)^{k}\left(-1+\frac{1.8\times10^{89}}{\pi}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}}}}}{\frac{1}{2}\sqrt{\frac{10^{9}\left(\frac{3\left(10^{96}\left(4\left(5+1\right)10^{80}-9\times5+1\right)\right)}{2\left(2\left(5-5\right)10^{80}+3\times5+5\right)}+\frac{3}{2}-10^{48}\left(10^{48}-1\right)\right)}{\left(2\pi\right)\left(\left(10^{48}-1\right)^{2}\left(1-0.5\right)\right)}}}}{\frac{8269\,040\,250}{\sqrt{z_{0}}\sum_{k=0}^{\infty}\frac{\left(-1\right)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{1.8\times10^{89}}{\pi}-z_{0}\right)^{k}z_{0}^{k}}{k!}}}}}}}}{\sqrt{z_{0}\sum_{k=0}^{\infty}\frac{\left(-1\right)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{1.8\times10^{89}}{\pi}-z_{0}\right)^{k}z_{0}^{k}}{k!}}}}}}}}}}}}}}}}}}$$

That we can to connect with the previous analyzed expression:

$$\alpha(t_0) = \frac{c}{|Z(t_0)|} \left\{ \beta \sum_{k=1}^{\infty} \frac{a(k)}{(t_0 - \gamma_k)^2} \right\}^{-1/2}, \tag{19}$$

$$\frac{\frac{3 \times 10^8}{2}}{\left(\frac{3}{2} \sum_{k=1}^{10^{43}} \frac{k}{(5-3)^2}\right)^{0.5}}$$

$$3.4641 \times 10^{-35}$$
  
 $3.4641 \times 10^{-35}$ 

## Now, we have that:

Thus the radion mass can be expressed as:

$$m^{2} = \frac{5(\alpha - 5)}{l^{2}r^{3}} - 2(\alpha + 1)\tilde{H}^{2} + \frac{15(\alpha - 5)}{8l^{2}r^{2}}\frac{X'}{X} - \frac{3}{4}(\alpha - 1)\tilde{H}^{2}\left(\frac{rX'}{X}\right)\Big|_{r=P}$$
(3.54)

which is consistent with the result in [118] when  $\tilde{H} = \frac{H^2}{(1+H^2\ell^2)^{2/5}} = 0$ . For  $\alpha = 5$  the radion mass is almost zero.

In the region  $r \to \infty$  the order of X'/X is  $r^{-4}$ , so we ignore the last two terms and get

$$m^2 = \frac{5(\alpha - 5)}{l^2 P^3} - 2(\alpha + 1)\tilde{H}^2 \tag{3.55}$$

This expression is accurate as long as the 4-brane is far from the 3-brane, i.e.  $P \gg 1$ .

For:

$$H^2 = (8\pi G/3)\rho$$

$$\rho = 1$$

$$(8Pi*6.67408e-11)/3 = 5.59126 \times 10^{-10} = H^{2}$$

$$H = 0.0000236459$$

$$P = 10^{16}$$

$$\ell = 1.61623 \times 10^{-35}$$

and  $\alpha = 6$ , we obtain from:

$$m^2 = \frac{5(\alpha - 5)}{l^2 P^3} - 2(\alpha + 1)\tilde{H}^2$$

5/(((1.61623\*10^-35)^2\*10^48))-2\*7\*5.59126\*10^-10

Input interpretation: 
$$\frac{5}{(1.61623\times 10^{-35})^2\times 10^{48}} - 2\times 7\times 5.59126\times 10^{-10}$$

#### **Result:**

 $1.9140958287135981309828737080021090268684242013979973... \times 10^{22}$  $1.9140958287...*10^{22} = m^2$ 

From which, performing the square root:

# **Input interpretation:**

$$\sqrt{\frac{5}{(1.61623 \times 10^{-35})^2 \times 10^{48}} - 2 \times 7 \times 5.59126 \times 10^{-10}}$$

#### **Result:**

$$1.38351... \times 10^{11}$$
  
 $1.38351... \times 10^{11} = m$ 

From the initial expression, we obtain also:

where 728 and 129 are Ramanujan taxicab numbers

Input interpretation: 
$$\frac{5}{2} \times \frac{1}{728} \left(\frac{1}{1729}\right)^3 \times \frac{1}{\frac{5}{(1.61623 \times 10^{-35})^2 \times 10^{48}} - 2 \times 7 \times 5.59126 \times 10^{-10}}$$

# **Result:**

 $3.4710428311707317835996704935653236789904060653187674...\times10^{-35}\\ 3.47104283117...*10^{-35}$ 

result that can be related with the previous analyzed expression:

$$\alpha(t_0) = \frac{c}{|Z(t_0)|} \left\{ \beta \sum_{k=1}^{\infty} \frac{a(k)}{(t_0 - \gamma_k)^2} \right\}^{-1/2},\tag{19}$$

$$\frac{\frac{3 \times 10^8}{2}}{\left(\frac{3}{2} \sum_{k=1}^{10^{43}} \frac{k}{(5-3)^2}\right)^{0.5}}$$

$$3.4641 \times 10^{-35}$$
  
 $3.4641 \times 10^{-35}$ 

#### **Observations**

#### From:

https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn\_RpOSvJ1QxWsVLBcJ6KVgd\_Af\_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by  $5^3 = 125$  units, saying that the corresponding p(n)'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

# From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field  $\phi$  and a Dirac field  $\psi$ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for T=0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

*Note that:* 

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$64g_{22}^{24} = e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots,$$
  

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots,$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982...$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and  $4096 = 64^2$ 

(Modular equations and approximations to  $\pi$  - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted  $F_n$ , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio. The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803......

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is  $\varphi$ , the golden ratio. [1] That is, a golden spiral gets wider (or further from its origin) by a factor of  $\varphi$  for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies [3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball  $\mathbf{f_0}(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross-Zagier theorem. The number 1728 is one less than the Hardy-Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to  $\zeta(2) = \frac{\pi^2}{6} = 1.644934$ ...

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

### References

A. A. Karatsuba, Cosmology and zeta, Sovrem. Probl. Mat., 2016, Issue 23, 17–23 - DOI: https://doi.org/10.4213/spm58

**Some definite integrals** – *Srinivasa Ramanujan* Messenger of Mathematics, XLIV, 1915, 10 – 18

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# Friedmann Equation in Codimension-two Braneworlds

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