Erdös-Straus Conjecture is Tenable

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Abstract

First, divide all integers ≥2 in 8 kinds, then, formulates each of 7 kinds therein into a sum of 3 unit fractions. For unsolved one kind, again divide it in 3 genera, then, formulates each of 2 genera therein into a sum of 3 unit fractions. For unsolved one genus, further divide it in 5 sorts, then, formulates each of 3 sorts therein into a sum of 3 unit fractions. For unsolved two sorts i.e. 4/(49+120c) and 4/(121+120c) where c≥0, the author has to depend on logical deduction to prove them.

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Contents

1. Introduction

Erdös-Straus conjecture relates to Egyptian fractions. In 1948, Paul Erdös conjectured that for any integer n≥2, there are invariably 4/n=1/x+1/y+ 1/z, where x, y and z are positive integers, [1].

Later, Ernst G. Straus further conjectured that the equation's solutions x, y and z satisfy x≠y, y≠z and z≠x, because there are convertible 1/2r+1/2r= 1/(r+1)+1/r(r+1) and 1/(2r+1)+1/(2r+1)=1/(r+1)+1/(r+1)(2r+1), where r≥1, [2].

Thus, the Erdös conjecture and the Straus conjecture are equivalent from each other, and they are called the Erdös-Straus conjecture collectively.

As a general rule, the Erdös-Straus conjecture states that for every integer n≥2, there are positive integers x, y and z, such that 4/n=1/x+1/y+1/z. Yet, the conjecture is yet both unproved and un-negated hitherto, [3].

2. Divide integers≥2 in 8 kinds and formulate 7 kinds therein

First, divide integers≥2 into 8 kinds, i.e. 8k+1, 8k+2, 8k+3, 8k+4, 8k+5, 8k+6, 8k+7 and 8k+8, where k≥0, and that arrange them as follows orderly:

K,	8k+1,	8k+2,	8k+3,	8k+4,	8k+5,	8k+6,	8k+7,	8k+8
0,	①,	2,	3,	4,	5,	6,	7,	8,
1,	9,	10,	11,	12,	13,	14,	15,	16,
2,	17,	18,	19,	20,	21,	22,	23,	24,
3,	25,	26,	27,	28,	29,	30,	31,	32,
…,	…,	…,	…,	…,	…,	…,	…,	…,

Excepting n=8k+1, formulate each of other 7 kinds as listed below:

(1) For n=8k+2, there are 4/(8k+2)=1/(4k+1)+1/(4k+2) +1/(4k+1)(4k+2);

(2) For n=8k+3, there are 4/(8k+3)=1/(2k+2)+1/(2k+1)(2k+2)+1/(2k+1)(2k+3);

(3) For n=8k+4, there are 4/(8k+4)=1/(2k+3)+1/(2k+2)(2k+3)+1/(2k+1)(2k+2);

(4) For n=8k+5, there are 4/(8k+5)=1/(2k+2)+1/(8k+5)(2k+2)+1/(8k+5)(k+1);

(5) For n=8k+6, there are 4/(8k+6)=1/(4k+3)+1/(4k+4)+1/(4k+3)(4k+4);

(6) For n=8k+7, there are 4/(8k+7)=1/(2k+3)+1/(2k+2)(2k+3)+1/(2k+2)(8k+7);

(7) For n=8k+8, there are 4/(8k+8)=1/(2k+4)+1/(2k+2)(2k+3)+1/(2k+3)(2k+4).

By this token, n as above 7 kinds of integers be suitable to the conjecture.

3. Divide the unsolved kind in 3 genera and formulate 2 genera therein

For n=8k+1 with k≥1, divide it by the modulus 3 into 3 genera to get (1) the remainder 0; (2) the remainder 1; (3) the remainder 2.

Excepting the genus (2), formulate each of other 2 genera as listed below:

(8) For n=8k+1 by the modulus 3 to the remainder 0, i.e. let k=3t+1 with t≥0, there are 4/(8k+1)=1/(8k+1)/3+1/(8k+2)+1/(8k+1)(8K+2) with k≥1, of course, (8k+1)/3 at here is an integer.

(9) For n=8k+1 by the modulus 3 to the remainder 2, i.e. let k=3t+2 with t≥0, there are 4/(8k+1)=1/(8k+2)/3+1/(8k+1)+1/(8k+1)(8k+2)/3 with k≥2, of course, (8k+2)/3 at here is an integer.

4. Divide the unsolved genus in 5 sorts and formulate 3 sorts therein

For the unsolved genus 8k+1 by the modulus 3 to the remainder 1, i.e. let k=3t with t≥1, divide it into 5 sorts, i.e. 25+120c, 49+120c, 73+120c, 97+120c and 121+120c, where c≥0. They are listed as the follows.

C,	25+120c,	49+120c,	73+120c,	97+120c,	121+120c,
0,	25,	49,	73,	97,	121,
1,	145,	169,	193,	217,	241,
2,	205,	289,	313,	337,	361,
…,	…,	…,	…,	…,	…,

Excepting n as 49+120c and 121+120c, formulate each of other 3 sorts below:

(10) For n=25+120c, there are 4/(25+120c)=1/(25+120c)+1/(50+240c)+1/(10+48c);

(11) For n=73+120c, there are 4/(73+120c)=1/(73+120c)(10+15c)+1/(20+30c)+ 1/(73+120c)(4+6c) ;

(12) For n=97+120c, there are 4/(97+120c)=1/(25+30c)+1/(97+120c)(50+60c)+ 1/(97+120c)(10+12c).

For each of listed above 12 equalities that express 4/n into a sum of 3 unit fractions, please, each of readers self to make a check respectively.

5. Proving the sort 4/(49+120c)=1/x+1/y+1/z by logical deduction

For a proof of the sort 4/49+120c, it means that when c is equal to each of positive integers plus 0, there are 4/(49+120c)=1/x+1/y+1/z .

4/(49+120c) can be substituted by infinitely more a sum of 2 fractions:

4/(49+120c)

= 1/(13+30c) + 3/(13+30c)(49+120c)

= 1/(14+30c) + 7/(14+30c)(49+120c)

= 1/(15+30c) + 11/(15+30c)(49+120c)

= 1/(16+30c) + 15/(16+30c)(49+120c)

…

= 1/(13+α+30c) + (3+4α)/(13+α+30c)(49+120c), where α⩾0 and c⩾0

…

As listed above, it is observed that we can first let $1/(13+\alpha+30c)=1/x$, after that, prove $(3+4\alpha)/(13+\alpha+30c)(49+120c)=1/y+1/z$.

Proof· When c=0, the fraction $4/(49+120c)$ is exactly $4/49$, then there are $4/49=1/14+1/99+1/(98\times99)$.

When c=1, the fraction $4/(49+120c)$ is exactly $4/169$, then there are $4/169=1/52+1/(2\times169)+1/(2^2\times169)$.

This shows that when c=0 and 1, $4/(49+120c)$ has been expressed into two of sum of 3 unit fractions.

Thereinafter, let us analyze and prove $(3+4\alpha)/(13+\alpha+30c)(49+120c)=1/y+1/z$ by c=2k and 2k+1, one after another, where $k\geq1$.

The numerator $3+4\alpha$ except for itself as an integer, others can be turned into the sum of two integers, i.e. $1+(2+4\alpha)$, $2+(1+4\alpha)$, $3+(4\alpha)$, $(1+\alpha)+(2+3\alpha)$, $(2+\alpha)+(1+3\alpha)$, $(3+\alpha)+3\alpha$, $(3+2\alpha)+2\alpha$ and $(2+2\alpha)+(1+2\alpha)$.

For the denominator $(13+\alpha+30c)(49+120c)$, actually, merely need to convert $13+\alpha+30c$, and that can continue to have $49+120c$.

For $13+\alpha+30c$ after evaluations of α, because begin with each constant i.e. 13, 14, 15…p…, there is $a\geqslant0$ in like wise, so $13+\alpha+30c$ can be converted to $p+a+30c$ where $a\geqslant0$, $c\geq0$, and $p\geq13$.

Such being the case, so let c=2k, then the fraction $(3+4\alpha)/(p+a+30c)$ is exactly $(3+4\alpha)/(p+a+60k)$; again let c=2k+1, then the fraction $(3+4\alpha)/(p+a+30c)$ is exactly $(3+4\alpha)/(p+a+30+60k)$, where $k\geq1$.

In fractions $(3+4\alpha)/(p+a+60k)$ and $(3+4\alpha)/(p+a+30+60k)$, the denominator p+a+60k can be any integer≥73, and the denominator p+a+30+60k can be any integer≥103. Also, for the numerator 3+4α, either it is an integer or the sum of two integers as listed above.

In any case, not only each numerator as listed above is smaller than a corresponding denominator p+a+60k or p+a+30+60k, but also p+a+60k and p+a+30+60k contain respectively integers of the whole multiple of 3+4α and either of two integers which divide 3+4α into.

Therefore, (3+4α)/(p+a+60k) can be expressed into a sum of two unit fractions, and (3+4α)/(p+a+30+60k) can be expressed into a sum of two unit fractions too, in which case c is equal to every integer >1.

If 3+4α serve as an integer, and from this get an unit fraction, then can multiply the denominator by 2 to make a sum of two identical unit fractions, again convert the sum into a sum of two each other's-distinct unit fractions by the formula 1/2r+1/2r=1/(r+1)+1/r(r+1).

Let a sum of two unit fractions which express (3+4α)/(p+a+60k) is written into 1/μ+1/ν, again let a sum of two unit fractions which express (3+4α)/(p+a+30+60k) is written into 1/φ+1/ψ.

In 1/μ+1/ν and 1/φ+1/ψ, multiply every denominator by 49+120c, then get 1/μ +1/ν =1/y +1/z and 1/φ +1/ψ =1/y +1/z.

To sum up, it is not difficult to get 4/(49+120c)=1/(13+α+30c)+1/y+1/z.

Furthermore tidy up 1/(13+α+30c)+1/y+1/z, and uniform different letters which express same values, then get 4/(49+120c)=1/x+1/y+1/z.

6. Proving the sort 4/(121+120c)=1/x+1/y+1/z by logical deduction

For a proof of the sort 4/(121+120c), it means that when c is equal to each of positive integers plus 0, there are 4/(49+120c)=1/x+1/y+1/z.

4/(121+120c) can be substituted by infinitely more a sum of 2 fractions:

4/(121+120c)

= 1/(31+30c) + 3/(31+30c)(121+120c),

= 1/(32+30c) + 7/(32+30c)(121+120c),

= 1/(33+30c) + 11/(33+30c)(121+120c),

= 1/(34+30c) + 15/(34+30c)(121+120c),

…

= 1/(31+α+30c) + (3+4α)/(31+α+30c)(121+120c), where $\alpha\geq 0$ and $c\geq 0$.

…

As listed above, it is observed that we can first let 1/(31+α+30c)=1/x, after that, prove (3+4α)/(31+α+30c)(121+120c)=1/y +1/z.

Proof· When c=0, the fraction 4/(121+120c) is exactly 4/121, then there are $4/121=1/(3\times11)+1/(3\times11^2+1)+ 1/(3\times11^2)(3\times11^2+1)$;

When c=1, the fraction 4/(121+120c) is exactly 4/241, then there are $4/241=1/(3^2\times7)+1/(2\times3\times241)+1/(2\times3^2\times7\times241)$.

This shows that when c=0 and 1, 4/(121+120c) has been expressed into two of sum of 3 unit fractions.

Thereinafter, let us analyze and prove (3+4α)/(31+α+30c)(121+120c)= 1/y+1/z by c=2k and 2k+1, one after another, where $k\geq 1$.

The numerator 3+4α except for itself as an integer, others can be turned into the sum of two integers, i.e. 1+(2+4α), 2+(1+4α), 3+(4α), (1+α)+(2+3α), (2+α)+(1+3α), (3+α)+3α, (3+2α)+2α and (2+2α)+(1+2α).

For the denominator (31+α+30c)(121+120c), actually, merely need to convert 31+α+30c, and that can continue to have 121+120c.

For 31+α+30c after evaluations of α, because begin with each constant i.e. 31, 32, 33…q…, there is a ⩾ 0 in like wise, so 31+α+30c can be converted to q+a+30c where a ⩾ 0, c≥0, and q≥31.

Such being the case, so let c=2k, then the fraction (3+4α)/(q+a+30c) is exactly (3+4α)/(q+a+60k); again let c=2k+1, then the fraction (3+4α)/(q+a+30c) is exactly (3+4α)/(q+a+30+60k) , where k≥1.

In fractions (3+4α)/(q+a+60k) and (3+4α)/(q+a+30+60k), the denominator q+a+60k can be any integer≥91, and the denominator p+a+30+60k can be any integer≥121. Also for the numerator 3+4α, either it is an integer or the sum of two integers as listed above.

In any case, not only each numerator as listed above is smaller than a corresponding denominator q+a+60k or q+a+30+60k, but also q+a+60k and q+a+30+60k contain respectively integers of the whole multiple of 3+4α and either of two integers which divide 3+4α into.

Therefore, (3+4α)/(q+a+60k) can be expressed into a sum of two unit fractions, and (3+4α)/(q+a+30+60k) can be expressed into a sum of two unit fractions too, in which case c is equal to every integer >1.

If 3+4α serve as an integer, and from this get an unit fraction, then can multiply the denominator by 2 to make a sum of two identical unit fractions, again convert the sum into a sum of two each other's-distinct unit fractions by the formula $1/2r+1/2r=1/(r+1)+1/r(r+1)$.

Let a sum of two unit fractions which express $(3+4\alpha)/(q+a+60k)$ is written into $1/\beta+1/\xi$, again let a sum of two unit fractions which express $(3+4\alpha)/(p+a+30+60k)$ is written into $1/\eta+1/\delta$.

In $1/\beta+1/\xi$ and $1/\eta+1/\delta$, multiply every denominator by $121+120c$, then get $1/\beta+1/\xi=1/y+1/z$ and $1/\eta+1/\delta=1/y+1/z$.

To sum up, it is not difficult to get $4/(121+120c)=1/(31+\alpha+30c)+1/y+1/z$.

Furthermore tidy up $1/(31+\alpha+30c)+1/y+1/z$, and uniform different letters which express same values, then get $4/(121+120c)=1/x+1/y+1/z$.

Overall, the author has proved that the Erdös-Straus conjecture is tenable.

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