

2014 International Conference on Topology and its Applications, July 3-7, 2014, Nafpaktos, Greece

Selected papers of the 2014 International Conference on Topology and its Applications



Editors

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Preface

The 2014 International Conference on Topology and its Applications took place from July 3 to 7 in the 3rd High School of Nafpaktos, Greece. It covered all areas of Topology and its Applications (especially General Topology, Set-Theoretic Topology, Geometric Topology, Topological Groups, Dimension Theory, Dynamical Systems and Continua Theory, Computational Topology, History of Topology). This conference was attended by 235 participants from 44 countries and the program consisted by 147 talks.

The Organizing Committee consisted of S.D. Iliadis (University of Patras), D.N. Georgiou (University of Patras), I.E. Kougias (Technological Educational Institute of Western Greece), A.C. Megaritis (Technological Educational Institute of Western Greece), and I. Boules (Mayor of the city of Nafpaktos).

The Organizing Committee is very much indebted to the City of Nafpaktos for its hospitality and for its excellent support during the conference.

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This volume is a special volume under the title: "Selected papers of the 2014 International Conference on Topology and its Applications" which will be edited by the organizers (D.N. Georgiou, S.D. Iliadis, I.E. Kougias, and A.C. Megaritis) and published by the University of Patras. We thank the authors for their submissions.

Editors

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mI-open sets and quasi-mI-open sets in terms of minimal ideal topological spaces

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Abstract

The purpose of this paper is to introduce a new type of open sets called mI-open sets and quasi-mI-open sets in minimal ideal topological spaces and investigate the relation between minimal structure space and minimal ideal structure spaces. Basic properties and characterizations related to these sets are given.

Key words: Minimal ideals, minimal local functions, topological ideals, minimal ideal structure. *1991 MSC:* 54A05, 54C10, 54B05.

1. Introduction

An ideal [8] I on a nonempty set X is a nonempty collection of subsets of Xwhich satisfies (i) $A \in I$ and $B \subset A$ implies $B \in I$ and (ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. Given a topological space (X, τ) with an ideal I on X and if P(X) is the set of all subsets of X, a set operator $(.)^* : P(X) \to P(X)$, called a local function [7] of A with respect to τ and I is defined as follows: for $A \subset X$, $A^*(I, \tau) = \{x \in X : U \cap A \notin I \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I, \tau)$, called the *-topology, finer than τ is defined by $cl^*(A) = A \cup A^*(I, \tau)$ [16]. When there is no chance for confusion, we will simply write A^* for $A^*(I, \tau)$ and τ^* for $\tau^*(I, \tau)$. If I is an ideal on X, then the space (X, τ, I) is called an ideal space. A subset A of an ideal space is said to be *-dense in itself [5] (resp. *-closed [7]) if $A \subset A^*$ (resp. $A^* \subset A$). By a space (X, τ) , we always mean

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a topological space (X, τ) with no separation properties assumed. If $A \subset X$, then cl(A) and int(A) respectively, denote the closure and interior of A in (X, τ) and $int^*(A)$ will denote the interior of A in (X, τ^*) . The notion of Iopen sets was introduced by Jankovic et al. [6], further it was investigated by Abd El-Monsef [1].

2. Preliminaries

The notion of minimal structures and minimal spaces as a generalization of topology and topological spaces were introduced in [9, 10]. Some other results about minimal spaces can be seen in [2, 3, 14]. Also, generalized topologies which are the other generalization of topology were defined by Csaszar [4]. Noiri and Popa [11] obtained the definitions and characterizations of separation axioms by using the concept of minimal structures.

Let (X, \mathcal{M}) be a minimal space and $\mathcal{U}_m(x) = \{U_m : x \in U_m, U_m \in \mathcal{M}\}$ be the family of *m*-open sets which contain *x*.

A family $\mathcal{M} \subset P(X)$ is said to be a minimal structure on X if $\phi, X \in \mathcal{M}$. In this case (X, \mathcal{M}) is called a minimal space [9].

Definition 2.1. (See [12]) Let (X, \mathcal{M}) be a minimal space with an ideal I on X and $(.)_m^*$ be a set operator from P(X) to P(X) (P(X) is the set of all subsets of X). For a subset $A \subseteq X$,

$$A_m^*(I, \mathcal{M}) = \{ x \in X : U_m \cap A \notin I : \text{ for every } U_m \in \mathcal{U}_m(x) \}$$

is called the minimal local function of A with respect to I and M. We will simply write A_m^* for $A_m^*(I, \mathcal{M})$.

Theorem 2.2. (See [12]) Let (X, \mathcal{M}) be a minimal space with I, I' ideals on X and A, B be subsets of X. Then

 $\begin{array}{ll} (\mathrm{i}) & A \subset B \Rightarrow A_m^* \subset B_m^*, \\ (\mathrm{ii}) & I \subset I' \Rightarrow A_m^*(I') \subset A_m^*(I), \\ (\mathrm{iii}) & A_m^* = m\text{-}cl(A_m^*) \subset m\text{-}cl(A), \\ (\mathrm{iv}) & A_m^* \cup B_m^* \subset (A \cup B)_m^*, \\ (\mathrm{v}) & (A_m^*)_m^* \subset A_m^*. \end{array}$

Remark 2.3. (See [12]) If (X, \mathcal{M}) has property [I], then $A_m^* \cup B_m^* = (A \cup B)_m^*$.

Definition 2.4. (See [12]) Let (X, \mathcal{M}) be a minimal space with an ideal Ion X. The set operator $m\text{-}cl^*$ is called a minimal *-closure and is defined as $m\text{-}cl^*(A) = A \cup A_m^*$ for $A \subset X$. We will denote by $\mathcal{M}^*(I, \mathcal{M})$ the minimal structure generated by $m\text{-}cl^*$, that is, $\mathcal{M}^*(I, \mathcal{M}) = \{U \subset X : m\text{-}cl^*(X - U) = X - U\}$. $\mathcal{M}^*(I, \mathcal{M})$ is called *-minimal structure which is finer than \mathcal{M} . The elements of $\mathcal{M}^*(I, \mathcal{M})$ are called minimal *-open (briefly, m*-open) and the complement of an m*-open set is called minimal *-closed (briefly, m*-closed). Throughout the paper we simply write \mathcal{M}^* for $\mathcal{M}^*(I, \mathcal{M})$. If I is an ideal on X, then (X, \mathcal{M}, I) is called an ideal minimal space.

Definition 2.5. A subset A of an m-space (X, \mathcal{M}) is called

- (i) an *m*-preopen set (see [15]) if $A \subseteq m$ -int(m-cl(A)) and a *m*-preclosed set if m-cl(m-int $(A)) \subseteq A$,
- (ii) an *m*-semiopen set (see [15]) if $A \subseteq m\text{-}cl(m\text{-}int(A))$ and a *m*-semiclosed set if $m\text{-}int(m\text{-}cl(A)) \subseteq A$,
- (iii) m- α -open set (see [13]) if $A \subseteq m$ -int(m-cl(m-int(A))) and an m- α -closed set if m-cl(m-int(m- $cl(A))) \subseteq A$.

The *m*-pre closure (resp. *m*-semi closure, *m*- α -closure) of a subset *A* of an m-space (X, \mathcal{M}) is the intersection of all *m*-pre closed (resp. *m*-semi closed, *m*- α -closed) sets that contain *A* and is denoted by *m*-*pcl*(*A*) (resp. *m*-*scl*(*A*), *m*- α *cl*(*A*)).

Definition 2.6. (see [14]) (i) A minimal structure (X, \mathcal{M}) has the property $[\mathcal{U}]$ if the arbitrary union of *m*-open sets is *m*-open.

(ii) A minimal structure (X, \mathcal{M}) has the property $[\mathcal{I}]$ if any finite intersection of *m*-open sets is *m*-open.

3. On minimal-*I*-open sets and minimal-*I*-closed sets

Definition 3.1. A subset A of a minimal ideal structure space (X, \mathcal{M}, I) is said to be minimal-*I*-open (briefly, *mI*-open) if $A \subseteq m\text{-}int(A_m^*)$. We denote $mIO(X, \mathcal{M}) = \{A \subseteq X : A \subseteq m\text{-}int(A_m^*)\}$ or simply we write mIO for $mIO(X, \mathcal{M})$ when there is no chance for confusion.

Remark 3.2. It is clear that *mI*-open and m-open are independent concepts. (Example 3.3.).

Example 3.3. (i) Let $X = \{a, b, c, d\}$. Define the m-structure on X as follows: $\mathcal{M} = \{X, \phi, \{c\}, \{d\}\}$ and the ideal $I = \phi, \{a\}$. For $A = \{a, c, d\}$, we have $A_m^* = X$ and $m\text{-int}(A_m^*) = X \supset A$. This shows that $A \in mIO$ but $A \notin \mathcal{M}$.

(*ii*) Let $X = \{a, b, c, d\}$. Define the m-structure on X as follows: $\mathcal{M} = \{X, \phi, \{a\}, \{b\}, \}$

 $\{a,c\},\{b,c\}\}$ and the ideal $I = \phi,\{b\},\{c\},\{b,c\}$. For $A = \{a,c\}$, we have $A_m^* = \{a,d\}$ and $m\text{-}int(A_m^*) = \{a\} \not\supseteq A$. This shows that $A \in \mathcal{M}$ but $A \notin mIO(X)$.

Theorem 3.4. Every mI-open set is a minimal pre open set. Also, mI-openness and m-semiopenness are independent concepts.

Proof. Let A be mI-open set. $A \subseteq m$ -int $(A_m^*) \Rightarrow A \subseteq m$ -int(m-cl(A)). Since $A_m^* \subset m$ -cl(A) by Theorem 2.2 (iii). Therefore A is minimal pre open and follows from the Example 3.5.

The converse of the above theorem need not be true in general, as shown by the following example.

Example 3.5. (i) Let $X = \{a, b, c\}$. Define the m-structure on X as follows: $\mathcal{M} = \{\phi, X, \{a\}, \{b\}\}$ and the ideal $I = \phi, \{a\}$. For $A = \{a\}$, we have $A_m^* = \phi$ and $m\text{-int}(A_m^*) = \phi \not\supseteq A$. But $m\text{-cl}(A) = \{a, c\} \Rightarrow m\text{-int}(m\text{-cl}(A)) = \{a\} \supseteq A$ and $m\text{-int}(A) = \{a\} \Rightarrow m\text{-cl}(m\text{-int}(A)) = \{a, c\} \supseteq A$. This shows that $A \in mPO$ and $A \in mSO$ but $A \notin mIO$.

(*ii*) Let $X = \{a, b, c, d\}$. Define the m-structure on X as follows: $\mathcal{M} = \{\phi, X, \{c\}, \{d\}\}$ and the ideal $I = \phi, \{a\}$. For $A = \{a, c, d\}$, we have $A_m^* = X$ and $m\text{-}int(A_m^*) = X \supseteq A$. But $m\text{-}int(A) = \{c\} \Rightarrow m\text{-}cl(m\text{-}int(A)) = \{a, b, c\} \not\supseteq A$. This shows that $A \in mIO$ and $A \notin mSO$.

Theorem 3.6. Arbitrary union of mI-open sets is also mI-open.

Proof. Let (X, \mathcal{M}, I) be any space and $W_i \in mIO(X, \mathcal{M})$ for $i \in \nabla$, this means that for each $i \in \nabla$, $W_i \subset (m\text{-}int((W_i)_m^*) \text{ and so } \cup_i W_i \subset \cup_i (m\text{-}int((W_i)_m^*) \subseteq m\text{-}int(\cup_i W_i)_m^*$. Hence $\cup_i W_i \in mIO(X, \mathcal{M})$.

Remark 3.7. The intersection of two mI-open sets need not be mI-open as is illustrated by the following example.

Example 3.8. Let $X = \{a, b, c, d\}$. Define the m-structure on X as follows: $\mathcal{M} = \{\phi, X, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$ and the ideal $I = \phi$. Then $\{a, c\}, \{b, c\} \in mIO$, but $\{a, c\} \cap \{b, c\} = \{c\} \notin mIO$.

Theorem 3.9. For a space and $A \subseteq X$, we have:

- (i) If $I = \phi$, then $A_m^* = m cl(A)$, and hence each of mI-open set and minimal pre open sets coincide.
- (ii) If I = P(X), then $A_m^* = \phi$ and hence A is mI-open iff $A = \phi$.

Theorem 3.10. For any *mI*-open set *A* of a minimal structure space (X, \mathcal{M}, I) , we have $A_m^* = (m \text{-}int(A_m^*))_m^*$.

Definition 3.11. A subset A of a minimal ideal structure space (X, \mathcal{M}, I) is said to be minimal-*I*-closed (briefly, *mI*-closed) if its complement is *mI*-open.

Theorem 3.12. For $A \subseteq (X, \mathcal{M}, I)$ we have $((int(A))_m^*)^c \neq (int(A^c))_m^*$ in general (Example 3.13.) where A^c denotes the complement of A.

Example 3.13. Let $X = \{a, b, c, d\}$. Define the m-structure on X as follows: $\mathcal{M} = \{\phi, U, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$ and the ideal $I = \phi$. Then it is clear that if $A = \{a, c\}$, then $((int(A))_m^*)^c = \{b\}$, but $(int(A^c))_m^* = \{b, d\}$.

Theorem 3.14. If $A \subseteq (X, \mathcal{M}, I)$ is minimal-*I*-closed, then $A \supset (int(A))_m^*$.

Proof. Follows from the definition of mI-closed sets and Theorem 2.2.(iii).

Corollary 3.15. (i) The union of mI-closed set and m-closed set is mI-closed. (ii) The union of mI-closed set and an $m\alpha$ -closed set is m-pre closed.

Theorem 3.16. If $A \subseteq (X, \mathcal{M}, I)$ is *mI*-open and *m*-semi closed, then $A = int(A_m^*)$.

Proof. Follows from Theorem 2.2.(iii). ■

Theorem 3.17. Let If (X, \mathcal{M}, I) be a minimal ideal structure spaces and $A \in X$. Then the following are equivalent.

- (i) A is mI-open.
- (ii) $A \subset A_m^*$ and m-scl(A) = m-int(m-cl(A)).
- (iii) $A \subset A_m^*$ and A is *m*-pre open.

Proof. $A \in mIO(X)$ if and only if $A \subset A_m^*$ and $A \subset int(A_m^*)$ if and only if $A \subset A_m^*$ and $A \subset m\text{-}int(m\text{-}cl(A))$, since m-cl(A) = A. if and only if $A \subset A_m^*$ and $A \cup m\text{-}int(m\text{-}cl(A)) = m\text{-}int(m\text{-}cl(A))$ if and only if $A \subset A_m^*$ and m-scl(A) = m-int(m-cl(A)). Therefore, (i) and (ii) are equivalent. It is clear that (i) and (iii) are equivalent.

Theorem 3.19. For a subset $A \subseteq (X, \mathcal{M}, I)$, we have:

- (i) If A is \mathcal{M}^* -closed and $A \in mIO(X)$, then m-int(A) = m-int (A_m^*) .
- (ii) A is \mathcal{M}^* -closed iff A is m-open and mI-closed.
- (iii) If A is n*-perfect, then $A = m \operatorname{int}(A_m^*)$, for every $A \in mIO(X, \mathcal{M})$.

Proof. Obvious.

4. Quasi-*m1*-open sets

Definition 4.1. A subset A of a minimal ideal space (X, \mathcal{M}, I) is quasi-mIopen (briefly, q-mI-open) if $A \subseteq m$ -cl(m-int (A_m^*)).

Theorem 4.2. Every mI-open set is q-mI-open. Also, q-mI-openness and m-semiopenness (resp., preopenness) are independent concepts (by, Examples 4.3.).

The family of all q-mI-open sets is denoted by $QmIO(X, \mathcal{M})$.

The connections between q-mI-openness with some other corresponding types have been given throughout the following implication.



The above relationships can not be reversible as the next examples illustrate.

Example 4.3. (i) Let $X = \{a, b, c\}$. Define the m-structure on X as follows: $\mathcal{M} = \{\phi, X, \{a\}, \{b\}\}$ and the ideal $I = \phi, \{a\}$. For $A = \{a\}$, we have $A_m^* = \phi$ and $m\text{-}int(A_m^*) = \phi \Rightarrow m\text{-}cl(m\text{-}int(A_m^*)) = \phi \not\supseteq A$. But $m\text{-}cl(A) = \{a, c, d\} \Rightarrow m\text{-}int(m\text{-}cl(A)) = \{a\} \supseteq A$ and $m\text{-}int(A) = \{a\} \Rightarrow m\text{-}cl(m\text{-}int(A)) = \{a, c, d\} \supseteq A$. This shows that $A \in mPO$ and $A \in mSO$ but $A \notin QmIO$.

(*ii*) Let $X = \{a, b, c, d\}$. Define the m-structure on X as follows: $\mathcal{M} = \{\phi, X, \{c\}, \{d\}\}$ and the ideal $I = \phi, \{a\}$. For $A = \{a, c, d\}$, we have $A_m^* = X$ and $m\text{-}cl(m\text{-}int(A_m^*)) = X \supseteq A$. But $m\text{-}int(A) = \{c\} \Rightarrow m\text{-}cl(m\text{-}int(A)) = \{a, b, c\} \not\supseteq A$. This shows that $A \in QmIO$ and $A \notin mSO$.

(*iii*) Let $X = \{a, b, c, d\}$. Define the m-structure on X as follows: $\mathcal{M} = \{\phi, X, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$ and the ideal $I = \phi, \{b\}, \{c\}, \{b, c\}$. For $A = \{a, d\}$, we have $A_m^* = \{a, d\}$ and $m\text{-}cl(m\text{-}int(A_m^*)) = \{a, d\} \supseteq A$. But $m\text{-}cl(A) = \{a, d\} \Rightarrow m\text{-}int(m\text{-}cl(A)) = \{a\} \not\supseteq A$. This shows that $A \in QmIO$ but $A \notin mPO$ and $A \notin mIO$.

Theorem 4.4. Arbitrary union of quasi-mI-open sets is also quasi-mI-open.

Proof. Let (X, \mathcal{M}, I) be any space and $W_i \in QmIO(X, \mathcal{M})$ for $i \in \nabla$, this means that for each $i \in \nabla$, $W_i \subset m\text{-}cl(m\text{-}int((W_i)_m^*))$ and so $\cup_i \subset \nabla m\text{-}cl(m\text{-}int((W_i)_m^*)) \subseteq m\text{-}cl(m\text{-}int(\cup_i W_i)_m^*)$. Hence $\cup_i W_i \in QmIO(X, \mathcal{M})$.

Remark 4.5. A finite intersection of quasi-mI-open sets need not in general quasi-mI-open, as Example 4.6. shows.

Example 4.6. In Example 4.3. We deduce that the two sets $\{a, c\}$ and $\{b, c\}$ are quasi-mI-open while their intersection does not.

The above remark, turns our attention to establish the following result.

Proposition 4.7. The following statements holds:

- (i) For $(X, \mathcal{M}, P(X))$ then $QmIO(X, \mathcal{M}) = mIO(X, \mathcal{M})$.
- (ii) For any (X, \mathcal{M}, I) each q-mI-open (resp. m-semi open) which it is m*-closed (resp. m*-dense-in-itself) is m-semi open (resp. q-mI-open).

Since m*-dense-in-itself and m*-closeness together of any $W \in X$ in (X, \mathcal{M}, I) equivalent with the m*-perfect property of W in the same space. Then the two classes $QmIO(X, \mathcal{M})$ and $MSO(X, \mathcal{M})$ are coincides with each other, if each member of term is m*-perfect or both of m*-dense-in-itself and m*-closed. In other words, the two parts of statement (ii) previously equivalent with: $QmIO(X, \mathcal{M}) = MSO(X\mathcal{M})$ if $W \in X$ is m*-perfect for any (X, \mathcal{M}, I) .

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