In this article the author wants to study the convergence of some series

Natural numbers are the set of numbers $\mathbb{N}$={1,2,3,4,5,...}

Definition 0.1. An extension of the natural numbers are the transfinite ordinal numbers. In set theory, ordinals are constructed as set, such that every ordinal is the set of all smaller ordinals than it:
0={}
1={0}
2={0,1}
3={0,1,2}
...
ω = {0,1,2,3,4,5,. . . }

ω is defined as the lowest transfinite ordinal number.
An ordinal lower than ω is a finite number, otherwise exist a transfinite number lower than ω and it is impossible because ω is the lowest transfinite number.

Definition 0.2. A succession $(a_n)_{n\in\mathbb{N}}$ is a map defined on the naturals such that at each natural associates a term a_n

Definition 0.3. A serie $\sum_{n=1}^{\omega} a_n$ is $\lim_{k\to\omega}\sum_{n=1}^{k} a_n = a_1 + a_2 + a_3 + ... + a_k = \alpha_k$

Definition 0.4. A serie is said convergent if the limit of succession of partial serie is lower than ω, otherwise it is said divergent.

Theorem 0.5. *All series with no-negative terms lower than serie* $\sum_{n=1}^{\omega} 1$ *are convergent*

Dimostrazione. The succession of partial sum of the serie 1+1+1+1+... is $\alpha_1 = 1$, $\alpha_2 = 2$, $\alpha_3 = 3$, ... $\alpha_n = n$ so the $lim_{n\to\omega}\alpha_n = lim_{n\to\omega} n = \omega$. The serie $\sum_{n=1}^{\omega} b_n < \sum_{n=1}^{\omega} 1$ so exist a n such that $b_n < 1$ for n big enough $\beta_n < \alpha_n$ so $\lim_{n\to\omega}\beta_n < lim_{n\to\omega}\alpha_n = \omega$ so $\sum_{n=1}^{\omega} b_n$ is convergent □

Example 0.1. It is ζ the Riemann Zeta defined $\zeta(s) = \sum_{n=1}^{\omega}\frac{1}{n^s}$ $\zeta(0) = \sum_{n=1}^{\omega} 1$
$\zeta(0) > \zeta(1) > \zeta(2) > ...$
so for $s \in \mathbb{R}, s > 0$ $\zeta(s)$ is convergent.