On Wave Function Collapse in Quantum Mechanics in the case of a Spin system having three anticommuting operators: The reason because it is so difficult to realize a theory of collapse in quantum mechanics.

Elio Conte

School of Advanced International Studies on Applied Theoretical and Non Linear Methodologies of Physics, Bari, Italy.

Abstract: We use a two states quantum spin system S, and thus considering the particular case of three anticommuting elements e_1, e_2, e_3 and the measurement of e_3. We evidence that, during the wave collapse, we have a transition of standard commutation relation of the spin to new commutation relations and this occurs during the interaction of the S system with the macroscopic measurement system M. The reason to accept such view point is that it causes the destruction of the interferential factors and of the fermion creation and annihilation operators of the S system without recourse to further elaborations based on the use of Hamiltonians or other methods. By this formulation we propose a new method in attempting to solve the problem of wave function collapse. The concept of Observable , in use in standard quantum mechanics, is resolved in an abstract entity to which is connected a linear hermitean operator that signs mathematically the operation that we must perform on the wave function in order to obtain the potential and possible values of the observable. It does not commute with a number of other operators characterizing the system and the non commuting rules have a fundamental role in quantum mechanics .They have a logic that must be analyzed in each phase of the non measuring and the measuring processes. When we consider the dynamics of wave function collapse we must account that the observed observable becomes a number ,with proper unity of measurements ,during the measurement, thus the linear hermitean operator to which is connected before the measurement, disappears and in its place it appears a new operator that maintain the non commutativity with the other operators to which the old and disappeared operator was connected.

corresponding author : elio.conte@fastwebnet.it

key words: foundations of quantum mechanics, quantum collapse, commutation relations of Pauli matrices.

1. Introduction

In ninety years since its beginnings, quantum mechanics has had great functional and theoretical success leaving little reason to doubt its intrinsic validity. Nevertheless, we cannot ignore that some questions concerning the foundations of this theory remained unsolved, and a historic debates among scientists arose and deeply influenced the early development of the theory.

The first important question concerns the problem of the wave-function collapse by measurement.

Its solution would be of relevant significance because it would provide us with a *self-consistent* formulation of the theory, which presently depends on the von Neumann postulates that have been added from the outside to the body of the theory.

For a complete examination of the actual problems, as they are resolved to da, we refer the reader to the several reviews that may be found in pertinent literature[1,2,8,9,10,11,12,18,19,20].
Consider the measurement of a given observable *F* on a quantum-mechanical system S that is in a normalized superposition of states

$$\psi = \sum_i c_i \varphi_i \quad ; \quad c_i = (\varphi_i, \psi) \quad ; \quad \sum_i |c_i|^2 = 1 ; \tag{1}$$

where φ_i is a normalized eigenstate of F, relative to an eigenvalue λ_i, $F\varphi_i = \lambda_i \varphi_i$, $(\varphi_i, \varphi_j) = \delta_{ij}$.
The probability of finding the eigenvalue λ_i during the measurement is $|c_i|^2$, the corresponding eigenstate is φ_i and during the measurement the wave function ψ is subjected to the transition $\psi \to \varphi_i$ characterizing the completed collapse.
The density matrix approach as it was initiated by von Neumann is

$$\rho_S = |\psi\rangle\langle\psi| = \sum_i \sum_j c_i c_j^* |\varphi_i\rangle\langle\varphi_j| \to \rho_{S,F_{\lambda k}} = \sum_k |\varphi_k\rangle\langle\varphi_k| . \tag{2}$$

Usually, we consider a macroscopic measuring device M and we postulate that the states of M entangle with those of S

$$\rho = \rho_S \otimes \rho_M = \sum_i \sum_j c_i c_j^* |\varphi_i\rangle\langle\varphi_j| \otimes \rho_M \to \rho_{S,M_t} = \sum_k |c_k|^2 |\varphi_k\rangle\langle\varphi_k| \otimes \rho_{M(k)_t} \tag{3}$$

If the system is not destroyed by the measurement, and if the interaction fits into the so called measurement of the first kind, then the quantum state after the measurement will be the eigenstate associated with the measurement outcome, or more generally (to include degenerancies), the normalized projection of the original state onto the eigensubspace associated with the outcome. This rule is known as the *projection postulate*. It originated with Dirac and von Neumann [17], and was later formalized in degenerate cases by Luders and Ludwig [14,15].
Consider S to be a quantum two states system. The complete phase-damping by using projection postulate gives

$$D(\rho) = |0\rangle\langle 0|\rho|0\rangle\langle 0| + |1\rangle\langle 1|\rho|1\rangle\langle 1| \tag{4}$$

Generally speaking, we have a set of mutually orthogonal projectors ($P_1, P_2, ..., P_N$) which complete to unity, $P_i P_j = \delta_{ij} P_j$, $\sum_i P_i = 1$, the result (i) is obtained with probability $p_i = \langle\psi|P_i|\psi\rangle$ and the state collapses to

$$\frac{1}{\sqrt{p_i}} P_i |\psi\rangle .$$

It is known that quantum mechanics has some peculiar features that are missing in the counterpart of classical physics. Two basic features are quantum interference and the collapse.
Starting with 2009 [5,6,7] our tentative approach was to use the Clifford algebra with the aim to construct a bare bone skeleton of quantum mechanics but giving collapse. We will deepen here some basic features but remaining fully in the aim of the foundations of quantum mechanics and thus without recurring to the Clifford algebra.

2. Theoretical Elaboration

Consider the measurement of e_3 spin z-component. We have three operators , e_1, e_2, and e_3 that satisfy the relation

$$e_j e_k + e_k e_j = 2\delta_{jk} \qquad \text{for } j,k,\lambda \in [1,2,3] \tag{5}$$

with the following basic relations

$e_\lambda^2 = 1$ (6)

and

$e_j e_k = -e_k e_j$ for $j \neq k$ (7)

In matrix form we have that

$$e_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; e_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{8}$$

These basic operators, e_i, with $i = 1,2,3$, satisfy the following relations

a) it exists the scalar square for each basic operator:

$e_1 e_1 = k_1$, $e_2 e_2 = k_2$, $e_3 e_3 = k_3$ with $k_i \in \Re$. (9)

In particular we have also the unit element, e_0, such that that

$e_0 e_0 = 1$, and $e_0 e_i = e_i e_0$

b) The basic elements e_i are anticommuting operators , that is to say

$e_1 e_2 = -e_2 e_1$, $e_2 e_3 = -e_3 e_2$, $e_3 e_1 = -e_1 e_3$. (10)

Theorem n.1.

Assuming the two postulates given in (a) and (b) with $k_i = 1$, the following commutation relations hold for such algebra :

$e_1 e_2 = -e_2 e_1 = ie_3$**;** $e_2 e_3 = -e_3 e_2 = ie_1$**;** $e_3 e_1 = -e_1 e_3 = ie_2$ **;** $i = e_1 e_2 e_3$**, (**$e_1^2 = e_2^2 = e_3^2 = 1$**)** (11)

Proof.

Consider the general multiplication of the three basic operators e_1, e_2, e_3, using scalar coefficients $\omega_k, \lambda_k, \gamma_k$ pertaining to some field:

$e_1 e_2 = \omega_1 e_1 + \omega_2 e_2 + \omega_3 e_3$; $e_2 e_3 = \lambda_1 e_1 + \lambda_2 e_2 + \lambda_3 e_3$;

$e_3 e_1 = \gamma_1 e_1 + \gamma_2 e_2 + \gamma_3 e_3$. (11a)

Let us introduce left and right alternation: for any (i,j), associativity exists $e_i e_i e_j = (e_i e_i) e_j$ and $e_i e_j e_j = e_i (e_j e_j)$ that is to say

$e_1 e_1 e_2 = (e_1 e_1) e_2$; $e_1 e_2 e_2 = e_1 (e_2 e_2)$; $e_2 e_2 e_3 = (e_2 e_2) e_3$; $e_2 e_3 e_3 = e_2 (e_3 e_3)$; $e_3 e_3 e_1 = (e_3 e_3) e_1$;
$e_3 e_1 e_1 = e_3 (e_1 e_1)$. (12)

Using the (11) in the (12) it is obtained that

$$k_1 e_2 = \omega_1 k_1 + \omega_2 e_1 e_2 + \omega_3 e_1 e_3 ; \qquad k_2 e_1 = \omega_1 e_1 e_2 + \omega_2 k_2 + \omega_3 e_3 e_2 ;$$

$$k_2 e_3 = \lambda_1 e_2 e_1 + \lambda_2 k_2 + \lambda_3 e_2 e_3 ; \qquad k_3 e_2 = \lambda_1 e_1 e_3 + \lambda_2 e_2 e_3 + \lambda_3 k_3 ;$$

$$k_3 e_1 = \gamma_1 e_3 e_1 + \gamma_2 e_3 e_2 + \gamma_3 k_3 ; \qquad k_1 e_3 = \gamma_1 k_1 + \gamma_2 e_2 e_1 + \gamma_3 e_3 e_1 . \tag{13}$$

From the (13), using the assumption (b), we obtain that

$$\frac{\omega_1}{k_2} e_1 e_2 + \omega_2 - \frac{\omega_3}{k_2} e_2 e_3 = \frac{\gamma_1}{k_3} e_3 e_1 - \frac{\gamma_2}{k_3} e_2 e_3 + \gamma_3 ;$$

$$\omega_1 + \frac{\omega_2}{k_1} e_1 e_2 - \frac{\omega_3}{k_1} e_3 e_1 = -\frac{\lambda_1}{k_3} e_3 e_1 + \frac{\lambda_2}{k_3} e_2 e_3 + \lambda_3 ;$$

$$\gamma_1 - \frac{\gamma_2}{k_1} e_1 e_2 + \frac{\gamma_3}{k_1} e_3 e_1 = -\frac{\lambda_1}{k_2} e_1 e_2 + \lambda_2 + \frac{\lambda_3}{k_2} e_2 e_3 \tag{14}$$

We have that it must be

$$\omega_1 = \omega_2 = \lambda_2 = \lambda_3 = \gamma_1 = \gamma_3 = 0 \tag{15}$$

and

$$-\lambda_1 k_1 + \gamma_2 k_2 = 0 \qquad \gamma_2 k_2 - \omega_3 k_3 = 0 \qquad \lambda_1 k_1 - \omega_3 k_3 = 0 \tag{16}$$

The following set of solutions is given:

$$k_1 = -\gamma_2 \omega_3 , \; k_2 = -\lambda_1 \omega_3 \; , k_3 = -\lambda_1 \gamma_2 \tag{17}$$

that is to say

$$\omega_3 = \lambda_1 = \gamma_2 = i \tag{18}$$

In this manner, as a theorem, the existence of such operators is proven. The basic features are given in the following manner

$$e_1^2 = e_2^2 = e_3^2 = 1 ; e_1 e_2 = -e_2 e_1 = i e_3 ; \; e_2 e_3 = -e_3 e_2 = i e_1 ; e_3 e_1 = -e_1 e_3 = i e_2 ; i = e_1 e_2 e_3 \tag{19}$$

Note that the e_i ($i = 1,2,3$) have an intrinsic potentiality that is to say an ontic potentiality or equivalently an irreducible intrinsic indetermination. Since $e_i^2 = 1$ ($i = 1,2,3$), the numerical value +1 or the numerical value –1 are potentially possible . Such two alternatives (+1 and -1) both coexist ontologically and this potential possibility intrinsically travels in each possible formal application of this operators .

Consider now the following new operators

$$e_1^2 = e_2^2 = 1\text{; } \boldsymbol{i}^2 = -1\text{;} \tag{20}$$

$$e_1 e_2 = \boldsymbol{i}\ , \ e_2 e_1 = -\boldsymbol{i}\ , \quad e_2 \boldsymbol{i} = -e_1\ , \quad \boldsymbol{i} e_2 = e_1\ , \quad e_1 \boldsymbol{i} = e_2\ , \quad \boldsymbol{i} e_1 = -e_2 \tag{21}$$

and we will verify that the (21) holds if the result of the measurement has given the value +1 for e_3.

We have instead

$$e_1^2 = e_2^2 = 1;\ \boldsymbol{i}^2 = -1;$$

$$e_1 e_2 = -\boldsymbol{i}\ , \ e_2 e_1 = \boldsymbol{i}\ , \ e_2 \boldsymbol{i} = e_1, \boldsymbol{i} e_2 = -e_1, e_1 \boldsymbol{i} = -e_2,\ \boldsymbol{i} e_1 = e_2 \tag{22}$$

and we will verify that the (22) holds if the result of the measurement has given the value -1 for e_3

Theorem n.2 .

Assuming the relations given in (20) , having $k_1 = 1$, $k_2 = 1$, $k_3 = -1$, the following commutation rules hold :

$$e_1^2 = e_2^2 = 1\text{; } i^2 = -1\text{;}$$

$$e_1 e_2 = i\ , \ e_2 e_1 = -i\ , \ e_2 i = -e_1, i e_2 = e_1, e_1 i = e_2\ , \ i e_1 = -e_2 \tag{23}$$

Proof

To give proof, rewrite the (11a) in our case, and perform step by step the same calculations of the previous proof, we arrive to the solutions of the corresponding homogeneous algebraic system that in this new case are given in the following manner:

$$k_1 = -\gamma_2 \omega_3;\ k_2 = -\lambda_1 \omega_3; k_3 = -\lambda_1 \gamma_2 \tag{24}$$

where this time it must be $k_1 = k_2 = +1$ and $k_3 = -1$. It results

$$\lambda_1 = -1; \gamma_2 = -1;\ \omega_3 = +1 \tag{25}$$

and the proof is given.

The content of the theorem 2 is thus established. **When we attribute to e_3 the numerical value +1, we pass from the previous one relations**

$$e_1 e_2 = -e_2 e_1 = i e_3; e_2 e_3 = -e_3 e_2 = i e_1;\ e_3 e_1 = -e_1 e_3 = i e_2\ ; i = e_1 e_2 e_3, (e_1^2 = e_2^2 = e_3^2 = 1)$$

to the following new basic rules:

$$e_1^2 = e_2^2 = 1;\; i^2 = -1;$$

$$e_1 e_2 = i\;,\; e_2 e_1 = -i\,,\; e_2 i = -e_1\,, ie_2 = e_1\,, e_1 i = e_2\,,\; ie_1 = -e_2 \qquad (26)$$

When we attribute to e_3 the numerical value of -1 , we have the new fundamental relations

$$e_1^2 = e_2^2 = 1;\; i^2 = -1;$$

$$e_1 e_2 = -i\;,\; e_2 e_1 = i\,,\; e_2 i = e_1\,, ie_2 = -e_1\,, e_1 i = -e_2\,,\; ie_1 = e_2 \qquad (27)$$

To give proof , consider the solutions of the (24) that are given in this new case by

$$\lambda_1 = +1;\gamma_2 = +1;\; \omega_3 = -1 \qquad (28)$$

and the proof is given.

The content of the theorem n.2 is thus established. When we attribute to e_3 the numerical value –1, we pass to new commutation relations with the following new basic rules:

$$e_1^2 = e_2^2 = 1;\; i^2 = -1;$$

$$e_1 e_2 = -i\;,\; e_2 e_1 = i\,,\; e_2 i = e_1\,, ie_2 = -e_1\,, e_1 i = -e_2\,,\; ie_1 = e_2 \qquad (29)$$

In the case of previous measurement we have that the imaginary unit i has its mathematical representation by

e_1, e_2, e_3, by the following relation $i = e_1 e_2 e_3$. In the case of $e_3 \to +1$ measurement we have instead

$\boldsymbol{i} = e_1 e_2$, and , in the case of $e_3 \to -1$ measurement, we have $\boldsymbol{i} = -e_1 e_2$. In both cases ***i*** becomes an operator that completes the triplet with e_1 and e_2 while , before the measurement, it is the scalar $i = e_1 e_2 e_3$.

In a similar way, proofs may be obtained when we consider the cases attributing numerical values (± 1) to e_1 or to e_2.

Consider the previous two states of system S with its proper representation in Hilbert space.

The complex coefficients $c_i\,(i = 1{,}2)$ are the well known probability amplitudes for the considered quantum state

$$\psi = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \quad \text{and} \quad |c_1|^2 + |c_2|^2 = 1 \qquad (30)$$

For a pure state in quantum mechanics it is $\rho^2 = \rho$. We have a corresponding Clifford algebraic member that is given in the following manner

$$\rho_S = a + be_1 + ce_2 + de_3 \qquad (31)$$

with

$$a = \frac{|c_1|^2}{2} + \frac{|c_2|^2}{2}\ ,\ b = \frac{c_1^* c_2 + c_1 c_2^*}{2}\ ,\ c = \frac{i(c_1 c_2^* - c_1^* c_2)}{2}\ ,\quad d = \frac{|c_1|^2 - |c_2|^2}{2}$$

In our old scheme a theorem may be demonstrated in Clifford algebra [3,4]. It is that

$$\rho_S^2 = \rho_S \leftrightarrow\ a = \frac{1}{2}\ \text{ and }\ a^2 = b^2 + c^2 + d^2 \quad \text{ and }\ \mathrm{Tr}(\rho) = 1 \tag{32}$$

Let us write again the state of the two state spin z-component quantum system S with connected quantum observable $S_3 \to e_3$. We have

$$|\psi\rangle = c_1|\varphi_1\rangle + c_2|\varphi_2\rangle\ ,\ \varphi_1 = \begin{pmatrix}1\\0\end{pmatrix}\ ,\ \varphi_2 = \begin{pmatrix}0\\1\end{pmatrix} \tag{33}$$

and

$$|c_1|^2 + |c_2|^2 = 1$$

As we know, the density matrix of such system is easily written

$$\rho = a + be_1 + ce_2 + de_3 \tag{34}$$

with

$$a = \frac{|c_1|^2 + |c_2|^2}{2}\ ,\ b = \frac{c_1^* c_2 + c_1 c_2^*}{2}\ ,\ c = \frac{i(c_1 c_2^* - c_1^* c_2)}{2}\ ,\ d = \frac{|c_1|^2 - |c_2|^2}{2} \tag{35}$$

where in matrix notation, e_1 , e_2, and e_3 are the well known Pauli matrices

$$e_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\ ,\ e_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\ ,\ e_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{36}$$

The (31) and (34) coincide .

Write the (34) in the two forms.

$$\rho_S = \frac{1}{2}(|c_1|^2 + |c_2|^2) + \frac{1}{2}(c_1 c_2^*)(e_1 + e_2 i) + \frac{1}{2}(c_1^* c_2)(e_1 - ie_2) + \frac{1}{2}(|c_1|^2 - |c_2|^2)e_3 \tag{37}$$

and

$$\rho_S = \frac{1}{2}(|c_1|^2 + |c_2|^2) + \frac{1}{2}(c_1 c_2^*)(e_1 + ie_2) + \frac{1}{2}(c_1^* c_2)(e_1 - e_2 i) + \frac{1}{2}(|c_1|^2 - |c_2|^2)e_3 \tag{38}$$

The (37) and (38) contain the following interference terms.

$$\frac{1}{2}(c_1c_2^*)(e_1+e_2i)+\frac{1}{2}(c_1^*c_2)(e_1-ie_2) \tag{39}$$

and

$$\frac{1}{2}(c_1c_2^*)(e_1+ie_2)+\frac{1}{2}(c_1^*c_2)(e_1-e_2i) \tag{40}$$

We may write (38) in the following terms

$$\rho_S=\rho_{1S}+\rho_{S,\text{int.}} \tag{41}$$

where

$$\rho_{1S}=\rho_S=\frac{1}{2}(|c_1|^2+|c_2|^2)+\frac{1}{2}(|c_1|^2-|c_2|^2)e_3 \tag{42}$$

and

$$\rho_{S,\text{int}}=\frac{1}{2}(c_1c_2^*)(e_1+e_2i)+\frac{1}{2}(c_1^*c_2)(e_1-ie_2) \tag{43}$$

or equivalently

$$\rho_{S,\text{int}}=\frac{1}{2}(c_1c_2^*)(e_1+ie_2)+\frac{1}{2}(c_1^*c_2)(e_1-e_2i) \tag{44}$$

and respectively for the (37) and the (38).

The mechanism that induces the collapse of the wave function is now evident. During the interaction of the system S with the macroscopic apparatus M the previous interference terms are destroyed. It never can happen until we assume that in the $(S+M)$ interaction and during such coupling $(S+M)$, the system undergoes an operator transition. If , probabilistically speaking, the macroscopic instrument reads $S_3=+\frac{\hbar}{2}$, it means that the (37) has prevailed . If instead the macroscopic instrument reads $S_3=-\frac{\hbar}{2}$, it means that the (38) has prevailed.

In the first case the basic commutation rules that hold are those given in (26) ,

$$e_1e_2=\boldsymbol{i}\ ,\ e_2e_1=-\boldsymbol{i}\ , \tag{45}$$

$$e_2\boldsymbol{i}=-e_1\,,\ \boldsymbol{i}e_2=e_1\,,\ e_1\boldsymbol{i}=e_2\,,\ \boldsymbol{i}e_1=-e_2 \tag{46}$$

The density matrix becomes

$$\rho_{S,+1}=\rho_{1S}+\rho_{S,\text{int.}} \tag{47}$$

with

$$\rho_{S,\mathrm{int}} = \frac{1}{2}(c_1 c_2^*)(e_1 + e_2\boldsymbol{i}) + \frac{1}{2}(c_1^* c_2)(e_1 - \boldsymbol{i}e_2) = 0 \tag{48}$$

In the second case the basic commutation rules that hold are those given in (29),

$$e_1 e_2 = -\boldsymbol{i} \text{ , } e_2 e_1 = \boldsymbol{i} \text{ , } e_2\boldsymbol{i} = e_1 \text{ , } \boldsymbol{i}e_2 = -e_1 \text{ , } e_1\boldsymbol{i} = -e_2 \text{ , } ie_1 = e_2 \tag{49}$$

The density matrix becomes

$$\rho_{S,-1} = \rho_{1S} + \rho_{S,\mathrm{int}}. \tag{50}$$

with

$$\rho_{S,\mathrm{int}} = \frac{1}{2}(c_1 c_2^*)(e_1 + \boldsymbol{i}e_2) + \frac{1}{2}(c_1^* c_2)(e_1 - e_2\boldsymbol{i}) = 0 \tag{51}$$

The macroscopic apparatus has the task to differentiate $\rho_{S,+1}$ from $\rho_{S,-1}$ destroying interference.

There is another important feature in such mechanism. The basic matrix density expression, written previously in equivalent manner in the (37) and (38), contains two algebraic elements that in quantum mechanics relate the Fermion annihilation and creation operators. In fact they are explicitly expressed in such basic matrix density expression

$$\rho_S = \frac{1}{2}(|c_1|^2 + |c_2|^2) + \frac{1}{2}(c_1 c_2^*)(e_1 + e_2 i) + \frac{1}{2}(c_1^* c_2)(e_1 - ie_2) + \frac{1}{2}(|c_1|^2 - |c_2|^2)e_3 \tag{52}$$

They act before of the interaction of S with M. When the system S interacts with M, the new commutation relations, the (45) or the (49), act and they completely cancel the presence of the algebraic terms corresponding to the two fermion creation and annihilation operators. Quantum collapse requires the cancellation of such two operators and it happens during the transition from previous measurement to during the measurement. This is of course at the basis of the mechanism of the ($S+M$) interaction.

3. Conclusion

We have given indication of the mechanism of quantum collapse in quantum mechanics for a quantum system having only three anticommuting elements. The central approach is that during the interaction of the given quantum system with the macroscopic apparatus, we have a transition from the basic and standard commutation relations among the well known Pauli matrices to new commutation relations. This must be a basic feature of quantum collapse and this is the basic reason because it is so difficult to construct a real theory of wave function collapse. In this case the linear hermitean operator connected to the given Observable disappears because the Observable becomes a truly physical quantity in its proper unity of measurement but in its place a new operator appears that does not commute with the old operators of the system and not commuting with the operator that has disappeared. We have reached this result by using the Clifford algebra in an old paper and we reach the same result now , in this paper, using only the algebra of the linear hermitean operators.

References

[1]Araki, H. (1980): "A remark on Machida-Namiki theory of measurement". *Prog. Theor. Phys*. **64,** 719-730.

[2]Araki, H. (1986)**:** "A remark on Machida-Namiki theory of measurement". In: Gorini, V., Frigerio, A. (eds.) *Fundamental Aspects of Quantum Theory*. Plenum, New York.

[3]Conte, E. (1994): "An example of wave function collapse". *Phys. Essays* **6**, 4-10

[4]Conte, E. (2001): "Biquaternion quantum mechanics". Pitagora press, Italy.

[5]Conte, E. (2009): "Quantum Theory, Reconsideration of Foundations-5", AIP Conference Proceedings, 14-19 June 2009, A proof of von Neumann Postulates in quantum mechanics, American Institute of Physics, 201-206.

[6]Conte, E. (2010): "A demonstration of von Neumann postulates in quantum measurements using two theorems of Clifford algebra". *International Journal of Theoretical Physics* **49**, 587-614.

[7]Conte, E. (2011): "An investigation on the basic conceptual foundations of quantum mechanics by using the Clifford algebra". *Advanced Studies in Theoretical Physics* **5**, 485-544.

[8]D'Espagnat, B. (1976): "Conceptual Foundations of Quantum Mechanics". 2nd ed. Benjamin, Elmsford.

[9]Fine, A.I. (1973): "The two problems of quantum measurement". In: Suppes, P., Henkin, L., Joja, A., Moisil, G.C. (eds.) *Logic, Methodology and Philosophy of Science* IV. North-Holland, Amsterdam.

[10]Ghiradi, G.C., Rimini, C.A., Weber, T. (1986): "Unified dynamics for microscopic and macroscopic systems". *Phys. Rev.* **D34**, 470-491.

[11]Ghiradi, G.C., Rimini, C.A., Weber, T. (1987): "Unified dynamics for microscopic and macroscopic systems". *Phys. Rev.* **D36**, 3287-3289.

[12]Ghirardi, G.C., Grassi, R., Pearle, P. (1990): "Describing the macroscopic world: Closing the circle within the dynamical reduction program". *Found. Phys.* **20**, 1271-1316.

[13]Ilamed, Y., Salingaros, N. (1981): "On the algebra of three anticommuting elements". *J. Math. Phys.* **22** 2091-2095.

[14]Ludwig, G. (1964): "Versuch einer axiomatischen Grundlegung der Quantenmechanik und allgemeinerer physikalischer Theorien". *Zeitschrift für Physik* **181**, 233-260.

[15]Ludwig, G. (1970): "Deutung des Begriffs Physikalische Theorie" und Axiomatische Grundlegung der Hilbert raumstruktur der Quantenmechanik durch Hauptsätze des Messens. Springer, Berlin.

[16]Sussmann, G. (1957): "An analysis of measurement". In: Korner, S. (ed.) Observation and Interpretation in the *Philosophy of Physics*. Dover, New York.

[17]Von Neumann, J. (1932): "Die Mathematische Grundlagen der Quantenmechanik". Springer, Berlin.
Wheeler, J.A., Zurek, W.H. (1983): (eds.): "Quantum Theory and Measurement". Princeton University Press, Princeton.

[18]Zurek, H. (1981): "Pointer basis of quantum apparatus: Into what mixture does the wave packet collapse?". *Phys. Rev.* **D24**, 1516-1525.

[19]Zurek, H. (1982): "Environment-induced superselection rules". *Phys. Rev*. **D26**, 1862-1880.

[20]Zurek, H. (1991): "Decoherence and the transition from quantum to classical". *Phys. Today* **44**, 36-44.