Proof of Twin Prime Conjecture

K.H.K. Geerasee Wijesuriya

Research Scientist in Physics and Astronomy

PhD student in Astrophysics, Belarusian National Technical University

BSc (Hons) in Physics and Mathematics University of Colombo, Sri Lanka
Doctorate Degree in Physics

geeraseew@gmail.com , geerasee1@gmail.com

August 2019

Author’s Biography

The author of this research paper is K.H.K. Geerasee Wijesuriya . And this proof of twin prime conjecture is completely K.H.K. Geerasee Wijesuriya's proof.

Geerasee is now 30 years old and she studied before at Faculty of Science, University of Colombo Sri Lanka. And she graduated with BSc (Hons) in Physics and Mathematics from the University of Colombo, Sri Lanka in 2014. And in March 2018, she completed her first Doctorate Degree in Physics with first class recognition. Now she is following her second PhD in Astrophysics with Belarusian National Technical University.

Geerasee has been invited by several Astronomy/Physics institutions and organizations world-wide, asking to get involve with them. Also, She has received several invitations from some private researchers around the world asking to contribute to their researches. She worked as Mathematics tutor/Instructor at Mathematics department, Faculty of Engineering, University of Moratuwa, Sri Lanka. Furthermore she has achieved several other scientific achievements already.

List of abbreviations

Faculty of Science, University of Colombo, Sri Lanka , Belarusian National Technical University Belarus

Acknowledgement

I would be thankful to my parents who gave me the strength to go forward with mathematics and Physics knowledge and achieve my scientific goals.

Keywords: prime; contradiction; greater than ; natural number

Abstract

A twin prime numbers are two prime numbers which have the difference of 2 exactly. In other words, twin primes is a pair of prime that has a prime gap of two. Sometimes the term twin prime is used for a pair of twin primes; an alternative name for this is prime twin or prime pair. Up to date there is no any valid proof/disproof for twin prime conjecture. Through this research paper, my attempt is to provide a valid proof for twin prime conjecture.

Literature Review

The question of whether there exist infinitely many twin primes has been one of the great open questions in number theory for many years. This is the content of the twin prime conjecture, which states that there are infinitely many primes p such that p + 2 is also prime. In 1849, de Polignac made the more general conjecture that for every natural number k, there are infinitely many primes p such that p + 2k is also prime. The case k = 1 of de Polignac's conjecture is the twin prime conjecture.

A stronger form of the twin prime conjecture, the Hardy–Littlewood conjecture (see below), postulates a distribution law for twin primes akin to the prime number theorem. On April 17, 2013, Yitang Zhang announced a proof that for some integer N that is less than 70 million, there are infinitely many pairs of primes that differ by N. Zhang's paper was accepted by Annals of Mathematics in early May 2013. Terence Tao subsequently proposed a Polymath Project collaborative effort to optimize Zhang's bound. As of April 14, 2014, one year after Zhang's announcement, the bound has been reduced to 246. Further, assuming the Elliott–Halberstam conjecture and its generalized form, the Polymath project wiki states that the bound has been reduced to 12 and 6, respectively. These improved bounds were discovered using a different approach that was simpler than Zhang's and was discovered independently by James Maynard and Terence Tao.

Assumption

Let's assume that there are finitely many twin prime numbers.

Therefore we proceed by considering that there are finitely many twin prime numbers. Then let the highest twin prime numbers are P_{n-1} and $(P_{n-1} + 2)$. Then for all prime numbers P_n greater than P_{n-1}, $(P_n + 2)$ is not a prime number.

Methodology

With this mathematical proof, I use the contradiction method to prove the twin prime conjecture.

Let P_n is an arbitrary prime number greater than P_{n-1} (because there are infinite number of prime numbers). Then according to our consideration, $(P_n + 2)$ is not a prime number. Since $P_n > 2$ and since P_n is a prime number and since P_n is an odd number, for all prime numbers P_i :

P_i $(< P_n / 2)$: $P_n / P_i = r_1$

Thus $P_n = P_i * r_1$..................(01)

Where r_1 is a rational number (which is not a natural number)

But according to our consideration, $(P_n + 2)$ is not a prime number. Also since P_n is a prime number greater than 2, $(P_n + 2)$ is an odd number.

Thus for some prime number P_1 $(< [(P_n + 2) / 2])$; $(P_n + 2) / P_1 = x_1$. Where we choose P_1 such that x_1 is a natural number. But since previously chose P_i is any arbitrary prime number less than $(P_n / 2)$; now we consider $P_1 = P_i$.

Then $(P_n + 2) = P_1 * x_1$ ....(02) and $P_n = P_1 * r_1$..................(01)

Let P_N is a prime number (greater than P_n). Then according to our assumption, $(P_N + 2)$ is not a prime number. Here P_N is a prime number such that $(P_N + 2)$ is dividing by prime number P_2. …………………(1.1)

Thus $(P_N + 2) = P_2 * x_2$ for some x_2 natural number. Because there are infinitely many prime numbers.

Since P_N is a prime number, for some r_2 (rational number which is not a natural number):

$P_N / r_2 = P_2$.

Thus $(P_N + 2) = P_2 * x_2$ ………………(03) and $P_N = r_2 * P_2$ ……………….(04)

x, x' are natural numbers and P_1 and P_2 are prime numbers.

Since P_N is a prime number , $(P_N – 2)$ is also not a prime number (Since $P_N – 2 > P_{n-1}$)

Then for some prime P_3 , $(P_N – 2) / P_3 = x_3$

$(P_N – 2) = P_3 * x_3$ ……………………..(05)

By (03) and (05): $P_3 * x_3 = (P_2 * x_2) - 4$…………………….(5.1)

By (04) and (05): $P_3 * x_3 = P_2 * r_2 - 2$ …………………..(06)

Since P_N and P_n are two prime numbers greater than 2, P_N and P_n both are odd numbers.

Thus $P_N - P_n = 2*l$ for some natural number l . Thus $P_N = P_n + 2*l$ ………………….(*)

By (*): $P_1 * r_1 + 2l = P_2 * r_2$. Thus by (06): $P_3 * x_3 = P_1 * r_1 + 2(l – 1)$……………(07)

By (5.1) and (07): $P_1 * r_1 + 2(l – 1) = (P_2 * x_2) - 4$

Thus $(P_2 * x_2) - 2* (l + 1) = P_1 * r_1$ ……………………….(08)

But we can put l as a natural number greater than 1. We can’t put l = 1. Because $P_N - P_n \neq 2$

Choose $l = (m.P_1 – 1)$ where m is a natural number. Then $(m.P_1 – 1)$ is a natural number.

Please <u>refer “proof “ below to see the proof of the existence of ‘*l*’ natural number</u> such that <u>$P_N = P_n + 2*l$</u> . i.e. there exists P_N (> P_n) such that for P_n prime; $P_N = P_n + 2*l$ Where we consider that $l = m.P_1 – 1$ …………..(9.0.0)

NOW BEFORE READING THE NEXT PART BELOW, PLEASE REFER ‘PROOF’.

Then:

For $l = m.P_1 - 1$, by (08): $(P_2 * x_2) - [2* m.P_1] = P_1 * r_1$ ………………………………(09) where m is a natural number. Also $(m.P_1 – 1)$ is a natural number.

But for the already chosen P_N (greater than P_{n-1}), there exists P_1 prime number such that $(P_N + 2) / P_1 = x_2$ …………………………(10) (According to the equation (12) in the "proof" mentioned below)

Because we chose P_N (i.e. $(P_N + 2)$) in that manner. Where P_N is a prime number.

Thus in (03) and in (09): $P_2 \equiv P_1$.

Thus by (09): $(P_1 * x_2) - [2.m.P_1] = P_1 * (x_2 - 2.m) = P_1 * r_1$

Thus $x_2 - [2* m] = r_1$. But (2*m) is a natural number. Thus $x_2 - [2* m]$ is a natural number. But r_1 is not a natural number……………..(10.0)

Thus by (10.0), there is a contradiction.

Therefore the only possibility is: our assumption is false.

Therefore there are infinitely many Twin Prime Numbers.

Proof

We know the equation related to "prime gap" as written below.

$P_N = 2 + \sum_{j=1}^{N-1} gj$ ……………..(i) *** refer the 2nd reference.

Let $g_j = a_j$ ……..(ii) for all $j < (N – 1)$. Where a_j is a natural number. Let $\Sigma a_j = A$ for $j < N – 1$.

But for all $\epsilon_{N-1} > 0$, there exists 'N - 2' natural number such that for all N -1 (> N – 2),

$g_{N-1} < P_{N-1} * \epsilon_{N-1}$

*** refer the 2nd reference below.

Then for some C_{N-1} positive number, $g_{N-1} = P_{N-1} * \epsilon_{N-1} - C_{N-1}$ for all $\epsilon_{N-1} > 0$

But $g_{N-1} = P_{N-1} * \epsilon_{N-1} - C_{N-1}$ for all $N – 1 > N - 2$

Choose $\epsilon_{N-1} = [(P_n + 2.m.P_1 - 4 - A) + C_{N-1}] / P_{N-1} > 0$. Then $g_{N-1} = P_{N-1} * \epsilon_{N-1} - C_{N-1} =$ $(P_n + 2.m.P_1 - A - 4)$. Here m is also giving $\epsilon_{N-1} > 0$.

Thus by (i): $P_N = 2 + P_n + 2.\ m.P_1 - A - 4 + A = P_n + 2.m.P_1 - 2 = P_n + 2.\ (m.P_1 - 1)$

Therefore there exists a natural number l ($= m.P_1 - 1$) such that $P_N = P_n + 2.\ (m.P_{1} - 1)$…….(11)

Now let's prove that for some P_1 prime number there are many P_L primes such that ($P_L + 2$) is divisible by P_1.

We know the equation related to "prime gap" as written below. Let **P_L is a prime number greater than P_n .** Then

$P_L = 2 + \sum_{j=1}^{M-1} hj$ ……………..(ii) *** refer the 2nd reference.

Let $h_j = b_j$ ……..(ii) for all $j < (M - 1)$. Where b_j is a natural number. Let $\Sigma\ b_j = X$ for $j < M - 1$.

But for all $\epsilon_{M-1} > 0$, there exists 'M - 2' natural number such that for all M -1 (> M – 2),

$G_{M-1} < P_{M-1} * \epsilon_{M-1}$ *** refer the 2nd reference below.

Then for some C_{M-1} positive number, $g_{M-1} = P_{M-1} * \epsilon_{M-1} - C_{M-1}$ for all $\epsilon_{M-1} > 0$

But $g_{M-1} = P_{M-1} * \epsilon_{M-1} - C_{M-1}$ for all $M - 1 > M - 2$

Choose $\epsilon_{M-1} = [(P_n + 2.m'.P_1 - 2 - X) + C_{M-1}] / P_{M-1} > 0$. Where m' is a natural number. And m' also gives $\epsilon_{M-1} > 0$.

Then $g_{M-1} = P_{M-1} * \epsilon_{M-1} - C_{M-1} = (P_n + 2.m'.P_1 - X - 2)$.

Thus by (ii): $P_L = 2 + P_n + 2.\ m'.P_1 - X - 2 + X = P_n + 2.m'.P_1 = P_n + 2.\ m'.P_1$

Then for that prime number P_L ; $P_L = P_n + 2.\ m'.P_1$

Then $(P_L + 2) = (P_n + 2) + 2.\ m'.P_1$. But $(P_n + 2) = P_1.\ x_1$. Thus $(P_L + 2) = (P_1 .\ x_1) + 2.\ m'.P_1$

Thus ($P_L + 2$) is divisible by P_1 (because x_1 and m' are natural numbers). Since P_L is a prime number greater than P_n , there are prime numbers P_L (which are greater than P_n) such that (P_L +2) is divisible by P_1 prime number.

Then in the proof we apply $P_N \equiv P_L$. Thus there exists P_N (greater than P_n) such that ($P_N + 2$) is divisible by P_1. Thus we can apply $P_2 \equiv P_1$ (which P_1 divides $P_N + 2$) in the methodology ….(12)

Discussion

We assumed initially that there are finitely many twin primes. After proceeding with that, I ended up with a contradiction. But to get the contradiction, I used that P_n and P_N as primes numbers greater than P_{n-1} . Also to get the contradiction, I used the facts that (P_n + 2) and (P_N + 2) as non-primes. And also I have used that x_1 , x_2 as natural numbers (since P_n + 2 and P_N + 2 are not prime numbers). Therefore to get the contradiction, I have used the facts got from our assumption. Then the only possibility is our assumption is false.

Results

Therefore I have used our assumption to get a contradiction finally as showed in (10.0). Therefore it is possible to conclude that our assumption is false.

Thus there are infinitely many twin prime numbers.

Appendix

Prime number: A natural number which divides by 1 and itself only.

Twin Prime Numbers: Two prime numbers which have the difference exactly 2.

We denote 'i' th prime gap $g_i = P_{i+1} - P_i$

Then according to the 2^{nd} reference; Prime number $P_N = 2 + \sum_{j=1}^{N-1} gj$

Also by 2^{nd} reference: for all $\epsilon > 0$, there is a natural number 'n' such that for all N -1 > n;

$g_{N-1} < P_{N-1} . \epsilon$

References

1. Zhang, Yitang (2014). "Bounded gaps between primes". Annals of Mathematics. **179** (3): 1121–1174.

2. https://en.wikipedia.org/wiki/Prime_gap

3. Terry Tao, Small and Large Gaps between the Primes

4. Maynard, James (2015), "Small gaps between primes", Annals of Mathematics, Second Series, **181** (1): 383–413

5. Tchudakoff, N. G. (1936). "On the difference between two neighboring prime numbers". Math. Sb. **1**: 799–814.

6. **I**ngham, A. E. (1937). "On the difference between consecutive primes". Quarterly Journal of Mathematics. Oxford Series. **8** (1): 255–266.