# The anomalous magnetic moment: classical calculations

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Abstract: Critics of the *Zitterbewegung* model often ask what *predictions* come out of the model. The answer to this question is quite simple: in order to gain credibility, the model would need to explain the anomalous magnetic moment as measured in, for example, the Harvard single-electron cyclotron experiments . If it could do this, then it should be recognized as a valid and alternative interpretation of quantum mechanics. This paper explores the geometry of the *zbw* model in very much detail and argues it can be done. In this paper, we do the calculations assuming the naked *zbw* charge has *zero* rest mass, and we find an anomalous magnetic moment that's off by a factor of the order of  $1/\alpha$ . This is quite encouraging because the model has a flexible assumption (the rest mass of the naked zbw charge may have some value *close* to zero rather than exactly zero) which can be further tuned.

**Keywords**: *Zitterbewegung*, mass-energy equivalence, wavefunction interpretations, realist interpretation of quantum mechanics, anomalous magnetic moment, classical explanation of anomalous magnetic moment.

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# The anomalous magnetic moment: classical calculations

### Introduction

There are various varieties of the *Zitterbewegung* model. In our previous paper<sup>1</sup>, we presented the simplest of simple models that, in our humble opinion, is consistent with the interpretation. It is probably useful to repeat the basics. We took Einstein's mass-energy equivalence relation ( $E = m \cdot c^2$ ) and, interpreting *c* as the tangential velocity of the naked charge (the toroidal photon, as Burinskii refers to it<sup>2</sup>), substituted *c* for *a*· $\omega$ : the tangential velocity equals the radius times the angular frequency. We then can then use the Planck-Einstein relation ( $E = \hbar \cdot \omega$ ) to find the Compton radius:

$$a = \frac{c}{\omega} = \frac{c \cdot \hbar}{m \cdot c^2} = \frac{\hbar}{m \cdot c} = \frac{\lambda_c}{2\pi} \approx 0.386 \times 10^{-12} \text{ m}$$

The idea here is that one rotation – one *cycle* of the electron in its *Zitterbewegung* – packs the electron's energy ( $E = E = m \cdot c^2$ ) and – importantly – it also packs one unit of physical action (S = h). This idea may not be very familiar but it is quite simple: just re-write the Planck-Einstein relation as  $h = E \cdot f = E/T$ . The cycle time T = h/E is equal to:

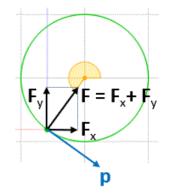
$$T = \frac{h}{E} \approx \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{8.187 \times 10^{-14} \text{ J}} \approx 0.8 \times 10^{-20} \text{ s}$$

Hence, this cycle time T is the time it takes for the *zbw* charge (or the naked charge, if you prefer that term) to go around the loop ( $\lambda_c$ ) at the extreme velocity we assume it has (v = c):

$$T = \lambda_c/c = (h/mc) \cdot (1/c) = h/E$$

Figure 1 illustrates the model. We have a centripetal force (F) holding our *zbw* charge (the naked charge, which has zero rest mass) in its circular orbit around some center.

Figure 1: The Zitterbewegung model of an electron



<sup>&</sup>lt;sup>1</sup> See: Jean Louis Van Belle, The electron as a harmonic electromagnetic oscillator, 1 June 2019, <u>http://vixra.org/abs/1905.0521</u>.

<sup>&</sup>lt;sup>2</sup> See Alexander Burinskii's 2008, 2016 and 2017 publications.

Because the naked charge goes around at the speed of light (or *almost* the speed of light, as we will argue later), it acquires some mass which we'll denote as  $m_{\gamma}$ . We use the  $\gamma$  subscript here because it is just like a photon, which also acquires relativistic mass because of its extreme velocity. The only thing is that our *zbw* charge also has electric charge (all of the charge of the electron, in fact), which a photon doesn't have, of course! The point is: the zbw charge will also have some non-zero momentum  $p = m_{\gamma}v = \gamma m_0 v = \gamma m_0 c$ , even if  $m_0$  (the rest mass of the naked charge) is zero.

Now, the angular momentum of the electron is equal to  $\hbar/2$  or some value very close to it.<sup>3</sup> We also know that angular momentum should be equal to the length of the lever arm (*a*) and the momentum p =  $m_{\gamma}c$ , so p is equal to p = L/a. It is useful to note that this formula – just like the others – is relativistically correct, so one should not cry wolf here. Hence, we get the following result:

1. 
$$L = \hbar/2 \Leftrightarrow p = L/a = (\hbar/2)/a = (\hbar/2) \cdot mc/\hbar = mc/2$$

$$\Rightarrow$$
 m<sub>y</sub>*c* = m*c*/2  $\Leftrightarrow$  m<sub>y</sub> = m/2

This is the grand result we expected to find: the *effective* mass of the pointlike charge – as it whizzes around the center of the two-dimensional oscillation that makes up our electron – is half of the (rest) mass of the electron. We interpreted this result in terms of a mathematical *equivalence* between the rotational motion and a two-dimensional oscillation—one perpendicular to (and, therefore, independent from) the other, each packing half of the total energy of the electron:

$$E_x = E_x = m_{\gamma} \cdot a^2 \cdot \omega^2 = m_{\gamma} \cdot c^2 = m \cdot a^2 \cdot \omega^2 / 2 = m \cdot c^2 / 2$$

Notation can be confusing here.  $E_x$  and  $E_y$  are often used to refer to the x- and y-component of the electric field vector (*E*), but that is not the case here:  $E_x$  and  $E_y$  is the *energy* (E) associated with the oscillation in the x- and y-direction respectively. To be precise, our model analyzes the electron pretty much like a perpetual current in a superconducting ring, as illustrated in Figure 2. Hence, the field is magnetic, rather than electric (in this particular reference frame, that is).

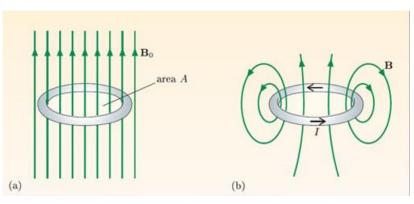


Figure 2: A perpetual current in a superconducting ring<sup>4</sup>

<sup>&</sup>lt;sup>3</sup> The anomalous magnetic moment or – to be precise – the anomalous g-ratio suggest angular momentum or magnetic moment, or both, are slightly off.

<sup>&</sup>lt;sup>4</sup> Source: Open University, Superconductivity, <u>https://www.open.edu/openlearn/science-maths-technology/engineering-and-technology/engineering/superconductivity/content-section-2.2#</u>.

This explains Hestenes' interpretation of the *zbw* model of an electron, which is equivalent to the oscillator model, and which he summarizes as follows:

"The electron is nature's most fundamental superconducting current loop. Electron spin designates the orientation of the loop in space. The electron loop is a superconducting LC circuit. The mass of the electron is the energy in the electron's electromagnetic field."<sup>5</sup>

You may think this interpretation has a problem because we do not have any *real* material ring or wire in free space to hold and our guide our charge. However, the more advanced calculations of Hestenes (1990, 2008) and Burinskii (2008, 2016 and 2017) show that the scale and the magnitude of the force and the other variables don't require any wiring: Nature has tuned this LC circuit perfectly well.

We explored many interesting properties and implications of this model in the mentioned paper (we mentioned, most notably, that this allows for a realist interpretation of the wavefunction) but we won't repeat these here. We will want to focus on the intriguing possibility that the rest mass of our toroidal photon (the naked charge) may be *almost* zero rather than zero, and that its velocity may be *almost* the speed of light, but not exactly.

Before we do so, we would just like to make one small point on the energy density inside the loop. We do so because we said little or nothing about that in our previous analysis. Let us use the *metaphor* of that superconducting ring to say a few words about it here. Figure 2(a) above shows a uniform magnetic field going through that ring made of superconducting material. The idea then is that we then cool the ring below the critical temperature and switch off the field. Lenz's law – Faraday's law of induction, really – then tells us the *change* in the magnetic field (so that's us flipping the *kill* switch, basically) will induce an electromotive force. Hence, we get an *induced* electric current, and its direction and magnitude will be such that the magnetic flux it generates will compensate for the flux change: the induced current in the superconducting circuit will just maintain the flux through the ring at the same value. However, while the flux will the same, you should note that the field looks different now: in Figure 2 (b) we have a *inform* magnetic field within the ring – the field in our apparatus, really – while in Figure 2 (b) we have a field that's produced by the current flowing in the ring now. The new field gives us the same flux, but the field density is now much larger close to the ring, and the field density at the center is rather weak, even if the total flux has the same value.

Why is this important? It is important because we will probably want to know, at some point in the analysis, where the (field) energy is actually located. Why? Our  $m_{\gamma} = m/2$  formula establishes an equivalence between:

- 1. The moment of inertia of a point mass  $m_{\gamma}$  at a distance r = a from the axis of rotation:  $I = m_{\gamma} \cdot a^2$ .
- 2. The moment of inertia of a *disk* with radius *r* and mass *m*:  $I = m \cdot a^2/2$ .

Hence, we must show that – somehow – the energy (or  $mass^6$ ) effective mass of the electron will be spread over the disk. *If* we assume its energy – and, therefore, its mass – is spread uniformly over the whole disk<sup>7</sup>, *then* we can use the 1/2 form factor for the moment of inertia (*I*). Hence, we conceptually

<sup>&</sup>lt;sup>5</sup> Email from Dr. David Hestenes to the author dated 17 March 2019.

<sup>&</sup>lt;sup>6</sup> Einstein's mass-energy equivalence relation – written as  $E/m = c^2$  here – tells us that energy and mass are linearly proportional, and that the constant of proportionality is equal to  $c^2$ .

<sup>&</sup>lt;sup>7</sup> This is a very essential point. It is also very deep and philosophical. We say the energy is in the motion, but it's also in the oscillation. It is difficult to capture this in a mathematical formula. In fact, we think this is the key paradox in the model.

distinguish the moment of inertia of the pointlike charge ( $I_{\gamma}$ ) and the moment of inertia of our electron ( $I_{\gamma}$ ), and we write:

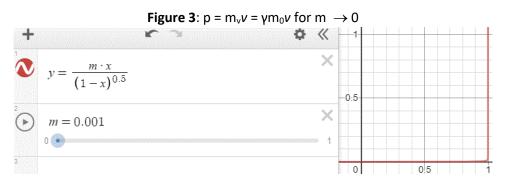
(1) 
$$\mathcal{L} = I_{\gamma} \cdot \omega = m_{\gamma}a^2 \cdot \frac{c}{a} = \frac{m}{2} \cdot \frac{\hbar^2}{m^2 c^2} \cdot \frac{mc^2}{\hbar} = \frac{\hbar}{2}$$
  
(2)  $\mathcal{L} = I_{e} \cdot \omega = \frac{ma^2}{2} \cdot \frac{c}{a} = \frac{m}{2} \cdot \frac{\hbar^2}{m^2 c^2} \cdot \frac{mc^2}{\hbar} = \frac{\hbar}{2}$ 

You may think this is rather obvious, but it isn't. It is a very deep and philosophical point. The energy is in the motion, but there is also energy in the magnetic field and we should, therefore, show how the magnetic energy is spread uniformly over the whole disk to validate the second of the two equations above. We haven't had the time to delve in this matter. The magnetic field becomes *weaker* as *r* goes to 0, and we know the energy *density* is proportional to the *square* of the magnetic field. Hence, if the magnetic field drops off as we move from the current ring to the center, we'd expect energy and, therefore, mass densities to *decrease* exponentially. This is a paradox which, hopefully, will not be too difficult to solve. We hope it's not a spoiler!

Let's move to the main topic of this paper.

#### The rest mass of the *zbw* charge

The Zitterbewegung model of an electron – or most flavors of that model, at least – assume the rest mass of the pointlike charge is zero. So why would we assume it would actually have some very tiny mass. The reason is the following: the  $p = m_v v = \gamma m_0 v = \gamma m_0 c$  involves the product of zero ( $m_0$ ) and infinity ( $\gamma$  for v = c). Such product doesn't make sense – not mathematically, and not physically. To illustrate the issue, we used an online graphing tool (desmos.com) to illustrate what happens with the p =  $m_v v = \gamma m_0 v = \gamma m_0 v$  function for m = 0.001 and v/c ranging between 0 and 1.



It is quite enlightening: p is (very close to) zero for v/c going from 0 to 1 but then becomes infinity at v/c = 1 itself. This is, obviously, not a regular function: we don't have a unique value for it at v/c = 1. What can we say about this? We think a particle that has some momentum should have some non-zero rest mass. Let us go through the math.

At first sight, the  $m_{\gamma} = \gamma m_0 = m/2$  is just like an  $x \cdot y = k$  relation: we have two variables ( $\gamma$  and  $m_0$ ), and their product is some constant (m/2), so they are inversely proportional to each other. However, the relationship is, obviously, much more complicated. To be precise, the variables are not  $m_0$  and  $\gamma$  but  $\gamma$  and v, or v/c. In fact, if we think of  $\beta = v/c$  as the variable, we may want to think of the other variable as some ratio between 0 and 1 too, so we can write it as  $m_0/m$  and re-write the equation accordingly.

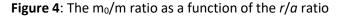
However, that doesn't help all that much. Let us try something else: if v is not equal to c, then it's actually the *radius* of that circular orbit that's going to change:  $v = r \cdot \omega = r \cdot E/\hbar = r \cdot E/\hbar = r \cdot m \cdot c^2/\hbar$ . Hence, we can write the relation as:

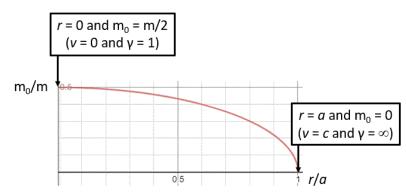
$$\frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \frac{r^2 \cdot m^2 \cdot c^4}{c^2 \cdot \hbar^2}}} = \frac{m_0}{\sqrt{1 - \frac{r^2}{a^2}}} = \frac{m}{2}$$

That's interesting because we can rewrite this as:

$$\frac{\mathbf{m}_0}{\sqrt{1-\frac{r^2}{a^2}}} = \frac{\mathbf{m}}{2} \Leftrightarrow 2 \cdot \left(\frac{\mathbf{m}_0}{\mathbf{m}}\right) = \sqrt{1-\left(\frac{r}{a}\right)^2}$$

This is a function that makes us think of the  $y^2 = 1 - x^2$  relation for a circle except for the 1/2 factor, but then we should note that the m<sub>0</sub>/m ratio will effectively vary between 0 and 1/2, as opposed to r/a, which will – just like the x in the  $x^2 + y^2 = 1$  relation – vary between 0 and 1. We get the following graph:





However, this nice graph still doesn't give us a good *second* fundamental relation that would solve the problem: what's the *actual*  $m_0/m$  ratio? Is it zero ( $m_0 = 0$ ), 1/2 (( $m_0 = m/2$ ) or some value in-between?

We will let this matter rest for a while (this sounds a bit funny in this context) and first explore why it would depend on the r/a ratio.

### The dependence of the anomalous magnetic moment on the *zbw* radius

It is easy to show why the anomalous magnetic moment would depend on the *Zitterbewegung* radius. If we denote this radius by r (which may or may not be equal to  $a = \hbar/mc$ ), then the formula for the angular momentum becomes:

$$\mathbf{L} = I_{\mathbf{e}} \cdot \boldsymbol{\omega} = \frac{\mathbf{m}r^2}{2} \cdot \frac{v}{r} = I_{\gamma} \cdot \boldsymbol{\omega} = \mathbf{m}_{\gamma}r^2 \cdot \frac{v}{r} = \mathbf{m}_{\gamma} \cdot r \cdot v = \frac{\mathbf{m} \cdot r \cdot v}{2}$$

The m is, once again, the *rest* mass of the electron<sup>8</sup>, so the formula is just the one we mentioned already. However, we substituted *c* for *v* and *a* for *r*. The idea here is that the angular frequency  $\omega$  remains the same ( $\omega = E/\hbar = v/r$ ) because the rest mass (or rest energy) of the electron is what it is and, therefore, the radius *r* and *v* may be different from *a* and *c* but they are still related through the tangential velocity formula:  $v = r \cdot \omega = r \cdot E/\hbar = r \cdot m \cdot c^2/\hbar$ . Note that  $I_e$  and  $I_v$  denote the moment of inertia of the electron and the *zbw* charge respectively.

To calculate the anomalous magnetic moment – which is actually an anomalous g-ratio<sup>9</sup> - we need the electric current I =  $q_e \cdot \omega$ . The current does *not* depend on *v* or *r*: qe is just the (naked) charge, and  $\omega$  is the same angular frequency  $\omega = E/\hbar = v/r$ . As mentioned, we assume *v* and *r* may vary but their *ratio* remains the same. The magnetic moment is equal to the current times the area of the loop and is, therefore, equal to:

$$\mu = \mathbf{I} \cdot \pi r^{2} = q_{e} \frac{mc^{2}}{h} \cdot \pi r^{2} = q_{e} c \frac{\pi r^{2}}{2\pi a} = q_{e} c \frac{r^{2}}{2a}$$

We substituted m*c*/h for  $\lambda_c = 2\pi \cdot a$  in the formula above. For a = r, the formula simplifies to the one you know:

$$\mu = q_e c \frac{\pi a^2}{2\pi a} = \frac{q_e c}{2} \frac{\hbar}{mc} = \frac{q_e}{2m} \hbar$$

However, we don't simplify here. Let us have a look at the formula for the g-ratio:

$$g_r = \frac{\mu_r}{L_r} = \frac{I \cdot \pi r^2}{m_v \cdot r \cdot v} = \frac{I \cdot \pi \cdot r}{m_v \cdot v}$$

What can we do with this? Nothing much. However, note that we introduced a subscript  $(g_r)$  to distinguish the *actual* value for g from its theoretical value, which we get from equating r to a and v to c:

$$g = \frac{\mu}{L} = \frac{I \cdot \pi a^2}{m_v \cdot a \cdot c} = \frac{q_e \cdot c \cdot \frac{a^2}{2a}}{m \cdot a \cdot c/2} = \frac{q_e}{m}$$

You will say this doesn't look like the *g*-factor for the pure spin moment, and you are right. The convention is to write the g-factor as a multiple of  $q_e/2m$ , so it is a pure number:

$$\boldsymbol{\mu} = -g\left(\frac{q_e}{2m}\right) \mathbf{L} \Leftrightarrow \frac{q_e}{2m} \hbar = g\frac{q_e}{2m}\frac{\hbar}{2} \Leftrightarrow g = 2$$

We think this convention obscures the matter<sup>10</sup>, so we'll just stick with our ratio – which is a real gyromagnetic ratio instead of some number – and let's see what happens. The anomaly is usually defined as the *difference* between real gyromagnetic ratio and the theoretical value  $(g_r - g)$  but we'll also write it as a ratio:

<sup>&</sup>lt;sup>8</sup> We could write it with a subscript (m = m<sub>e</sub>) but, for the sake of keeping the notation as simple as possible, we refrained from that.

<sup>&</sup>lt;sup>9</sup> The gyromagnetic ratio is the ratio of the magnetic moment and the angular momentum. As mentioned, the anomalous magnetic moment is actually a misnomer. First, it is *not* a magnetic moment: it is the g-ratio. Second, as we try to show here, it may actually not be anomalous at all!

<sup>&</sup>lt;sup>10</sup> See: Jean Louis Van Belle, The Not-So-Anomalous Magnetic Moment, 21 December 2018 (http://vixra.org/abs/1812.0233).

$$\frac{g_r}{g} = \frac{\frac{q_e c \frac{r^2}{2a}}{m \cdot r \cdot v/2}}{\frac{q_e c \frac{a^2}{2a}}{m \cdot a \cdot c/2}} = \frac{r^2}{a^2} \frac{a}{r} = \frac{r}{a}$$

This is a wonderful result: the anomaly is just the ratio between the *actual* or *effective* Zitterbewegung radius and its theoretical value. We can write it very simply:

$$g_r = (r/a) \cdot g$$

We know Schwinger's first-order value for the anomaly is  $\alpha/2\pi \approx 0.00116141$ . We also know and we know – *from experiments that measure this g-ratio* – that this first-order correction explains 99.85% of the anomaly. The second-, third-, or *n*<sup>th</sup>-order corrections that one gets only need to explain 0.15%.

The  $\alpha$  in the formula is the fine-structure constant ( $\alpha \approx 1/137$ ), and it also relates the Compton radius to the Thomson radius. The Thomson radius is the classical electron radius:  $r_e = \alpha \cdot a \approx a/137 \approx 2.818 \times 10^{-15}$  m. We get this radius from *elastic* scattering experiments. They are referred to as elastic because the photon seems to bounce off some hard *core*: there is no interference. In contrast, Compton scattering is usually explained by some electron-photon interference. It involves high-energy photons (the light is X-or gamma-rays) whose energy will be briefly absorbed before the electron comes back to its equilibrium situation by emitting another photon. The wavelength of the emitted photon will be longer. The photon has, therefore, less energy, and the difference in the energy of the incoming and the outgoing photon gives the electron some linear momentum.

This picture is fully consistent with the *Zitterbewegung* model of an electron: the hard core is just the pointlike charge itself. It is, effectively, pointlike ( $10^{-15}$  m is the *femto*meter scale) but, as we can see, pointlike does not mean dimensionless. So what is going on here, and can we explain Schwinger's  $\alpha/2\pi$  factor for the anomaly?

## A classical explanation for the anomaly?

Figure 5 is not to scale but illustrates the geometry of the situation. We think of the naked charge as a charged sphere with radius  $\alpha \cdot a$  moving in a circular orbit with radius r = a.

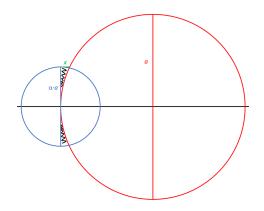


Figure 5: Geometry of *zbw* charge and electron (1)

The points in the two triangular areas will move at a velocity v which is slightly higher than  $c = a \cdot \omega$ . Hence, the effective *center* of charge is slightly changed. If we want the charged sphere – on average and as a whole – to move around the center at the speed of light (*c*), then we have to reduce the effective *zbw* radius somewhat. This correction is approximated by the distance x/2 in Figure 6.

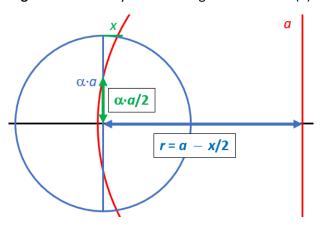


Figure 6: Geometry of *zbw* charge and electron (2)

All that remains to be done is to prove that the correction is equal (or not) to  $\alpha/2\pi$ . That should not be so difficult using the formula for the length of an arc (L =  $\theta \cdot r$ ) but, as yet, we have not been able to figure this out. Let's explore the geometry somewhat further. To facilitate calculations, we scaled everything by dividing the radii of the two circles ( $\alpha \cdot a$  and a) by a, so the large circle is the unit circle, and the radius of the sphere of charge is  $\beta = \alpha$ .<sup>11</sup> Also, our length *x* now becomes y = x/a.

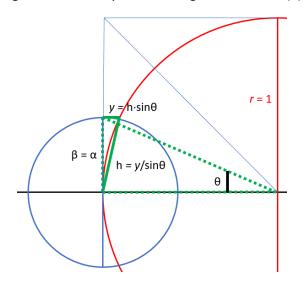


Figure 7: Geometry of zbw charge and electron (3)

<sup>&</sup>lt;sup>11</sup> We insert a new symbol ( $\beta$ ) so that you would think of it as a variable rather than as a constant (the fine-structure constant). It is, strictly speaking, not necessary, but it helped my thinking so I hope it helps yours too.

It now becomes obvious that we have two *similar* triangles here.<sup>12</sup> The first triangle is the triangle in the large circle, which represents the orbit of our *zbw* charge. The height of the first triangle – whose base is now equal to r = 1 (its base was *a* before re-scaling) – is equal to the base of the second triangle, which is the triangle in the small circle, which represents the circumference of the *zbw* charge, which was equal to  $2\pi \cdot \alpha \cdot a$  before re-scaling. Hence, the height of the large triangle ( $\beta = \alpha$ ) is the base of the small triangle.

The length of the hypothenuse (which we will denote by h as per the usual convention<sup>13</sup>) is equal to the ratio of y and sin $\theta$ . Conversely, the length we want to calculate (y, which we can then scale back to find x and x/2), will be equal to  $y = h \cdot \sin \theta$ . Now, because the large circle is the unit circle, we know that sin $\theta$  will be equal to  $\beta = \alpha$ . Hence, we can write:

#### $y = h \cdot \alpha = h \cdot \alpha$

But what is the length of that hypothenuse? Not sure, but it is easy to see that it depends on  $\theta$ , or on  $\beta$ . But *how* exactly? If  $\beta = 1$ , the  $\theta$  angle will be equal to  $\pi/4$ , so that's one eighth of the circumference of the circle:  $\beta = 1 \Leftrightarrow \theta = 2\pi/8 = \pi/4$ . We can now take smaller values of  $\beta$  to approximate the actual  $\beta = \alpha$ , and it is easy to see we have a proportional relation here, as shown in Table 1.

β	θ
β = 1	$\theta = 2\pi/8 = \pi/4$
β = 1/2	$\theta = (2\pi/8) \cdot (1/2) = (2\pi/8)/2 = \pi/8$
$\beta = 1/4$	$\theta = (\pi/4) \cdot (1/4) = \pi/16$
β = α	θ = (π/4)·α

**Table 1:** The proportionality between  $\beta$  and  $\theta$ 

The  $y = h \cdot \sin \theta = h \cdot \alpha$  and  $h = y/\sin \theta = y/\alpha$  relations are nice but give us a tautology: y = y. We need to try something else. Let us try the small-angle approximation. Indeed, our ratio  $\alpha$  is very small and, hence, we can write:  $\sin \theta \approx \theta = (\pi/4) \cdot \alpha$  (the latter identity is given in the table above). In addition, we can also use the small-angle approximation to write h as  $h \approx \alpha$ , so we get:

$$y = h \cdot \sin \theta \approx \alpha \cdot \theta = \alpha \cdot (\pi/4) \cdot \alpha = (\pi/4) \cdot \alpha^2$$

Let's scale everything back to the actual size by multiplying all lengths by a. We get:

$$x = a \cdot y = (\pi/4) \cdot \alpha^2 \cdot a \Leftrightarrow x/2 = a \cdot y/2 = (\pi/8) \cdot \alpha^2 \cdot a$$

Now, we are interested in the anomaly, which we can now write as:

$$\frac{g-g_r}{g} = \frac{a-r}{a} = \frac{x/2}{a} = \frac{(\pi/8) \cdot \alpha^2 \cdot a}{a} = \frac{\pi \cdot \alpha^2}{8}$$

<sup>&</sup>lt;sup>12</sup> Two triangles are similar – i.e. they have the same *shape* – if every angle of one triangle has the same measure as the corresponding angle in the other triangle. The corresponding sides of similar triangles have lengths that are in the same proportion, and this property is also sufficient to establish similarity.

<sup>&</sup>lt;sup>13</sup> There is not much scope for confusing h with Planck's constant here, so we should not invent some other symbol here.

This result is good, but it is not good enough. It is good because it is a result that is expressed in terms of  $\alpha$  and  $\pi$  – and, importantly, *nothing else* – but the formula differs from Schwinger's  $\alpha/2\pi \approx 0.00116141$  factor. It's not even close numerically:  $\pi \cdot \alpha^2/8 \approx 0.000021$ . The result is off by a factor that's equal to  $4/\pi^2 \alpha \approx 55.5$ , so that's a factor of the order of  $1/\alpha \approx 137$ . That's not a disaster for a first attempt at calculation but it's, obviously, not good enough.

Is there any way out? We may think it's got something to do with the fact that we imagine the *zbw* charge to be some *sphere* of charge. Indeed, unlike our electron, we do *not* picture it as some disk. Our little triangle is a little *cone* of charge, so perhaps we should calculate the *ratio* between the volume of our cone of charge and the total volume of charge. We can do that. The formula for the volume of a cone is equal to  $V = \pi \cdot b^2 \cdot h/3$ . The b and b here are the base (b) and the heigth (h) of the triangle that defines the volume of rotation here. Hence, b and h are equal to x and  $\alpha \cdot a$  here.

$$\frac{V_r}{V} = \frac{\frac{\pi \cdot x^2 \cdot \alpha \cdot a}{3}}{\frac{4\pi \cdot (\alpha \cdot a)^3}{3}} = \frac{\pi \cdot (\pi \cdot \alpha^2 \cdot a/4)^2 \cdot \alpha \cdot a}{4\pi \cdot (\alpha \cdot a)^3} = \frac{\pi^2 \cdot \alpha^2}{4^3}$$

As expected, this line of reasoning just confirms we get an anomaly that is of the same order:

$$\frac{g-g_r}{g} \propto \alpha^2$$

In contrast, Schwinger's factor is of the order  $\alpha$ : no square.

#### Conclusions

Critics of the *Zitterbewegung* model often ask what *predictions* come out of the model. The answer to this question is quite simple: in order to gain credibility, the model would need to explain the anomalous magnetic moment as measured in, for example, the Harvard single-electron cyclotron experiments. If it could do this, then it should be recognized as a valid and alternative interpretation of quantum mechanics.

This paper explored the geometry of the *zbw* model in very much detail and calculated the order of magnitude of the anomalous magnetic moment assuming the naked *zbw* charge has *zero* rest mass. We found an anomalous magnetic moment that is off by a factor of the order of  $1/\alpha$  (as compared to the experimentally established value in, for example, the Harvard experiments). This is encouraging because the calculations show the result will be some function of  $\alpha$  and  $2\pi$ , and of those two factors only. More research is needed to analyze other factors, which include the classical coupling between spin-only and orbital moment which one would expect to see in a one-electron cyclotron.

The electron in the Penning trap that is used in these experiments is, effectively, not a spin-only electron. It follows an orbital motion – that is one of the three or four layers in its motion, at least – and, hence, if some theoretical value for the g-factor has to be used here, then one should also consider the g-factor that is associated with the orbital motion of an electron, which is that of the Bohr orbitals (g = 1). In any case, one would expect to see a classical coupling between (1) the precession, (2) the orbital angular momentum and (3) the spin angular momentum, and the situation is further complicated

because of the electric fields in the Penning trap, which add another layer of motion. The complexity of the situation is illustrated below.<sup>14</sup>

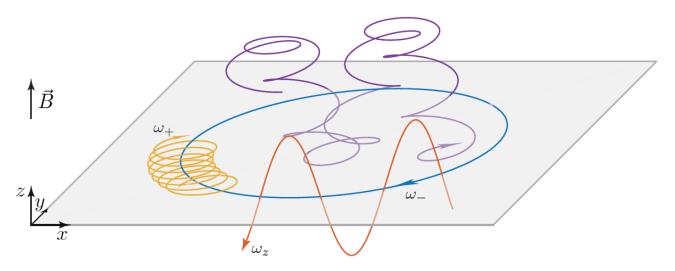


Figure 8: The three principal motions and frequencies in a Penning trap

Hence, we are hopefully that some more research will be able to narrow the gap between the *zbw* explanation and QFT calculations. One obvious way out is to question our assumption: the rest mass of the naked (*zbw*) charge may be *close* to zero, but not quite. This is easy and, therefore, attractive way out – but the associated calculations will be more complicated and – critics would say – surely somewhat more random because it would like we're making the theory fit experiment.

To these critics, I'd say the following: the model on which these calculations are based would seem to have more appeal than the hocus-pocus on which current QFT calculations are based.

Jean Louis Van Belle, 4 June 2019

<sup>&</sup>lt;sup>14</sup> We found the following course material particularly enlightening: *Cylotron frequency in a Penning trap*, Blaum Group, 28 September 2015, <u>https://www.physi.uni-heidelberg.de/Einrichtungen/FP/anleitungen/F47.pdf</u>. The motions are complicated because the Penning trap traps the electron using both electric as well as magnetic fields.

## References

This paper discusses general principles in physics only. Hence, references were mostly limited to references to general physics textbooks. For ease of reference – and because most readers will be familiar with it – we often opted to refer to:

1. Feynman's *Lectures* on Physics (http://www.feynmanlectures.caltech.edu). References for this source are per volume, per chapter and per section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

One should also mention the rather delightful set of Alix Mautner Lectures, although we are not so impressed with their transcription by Ralph Leighton:

2. Richard Feynman, The Strange Theory of Light and Matter, Princeton University Press, 1985

Specific references – in particular those to the mainstream literature in regard to Schrödinger's *Zitterbewegung* – were mentioned in the footnotes. We should single out the various publications of David Hestenes and Francesco Celani:

- 3. David Hestenes, Found. Physics., Vol. 20, No. 10, (1990) 1213–1232, *The Zitterbewegung Interpretation of Quantum Mechanics*, http://geocalc.clas.asu.edu/pdf/ZBW\_I\_QM.pdf.
- 4. David Hestenes, 19 February 2008, *Zitterbewegung in Quantum Mechanics a research program*, https://arxiv.org/pdf/0802.2728.pdf.
- 5. Francesco Celani et al., *The Electron and Occam's Razor*, November 2017, https://www.researchgate.net/publication/320274514\_The\_Electron\_and\_Occam's\_Razor.

We would like to mention the work of Stefano Frabboni, Reggio Emilia, Gian Carlo Gazzadi, and Giulio Pozzi, as reported on the phys.org site (<u>https://phys.org/news/2011-01-which-way-detector-mystery-double-slit.html</u>). In addition, it is always useful to read an original:

6. Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf

We should, perhaps, also mention the following critical appraisal of the quantum-mechanical framework:

7. *How to understand quantum mechanics* (2017) from John P. Ralston, Professor of Physics and Astronomy at the University of Texas.

It is one of a very rare number of exceptional books that address the honest questions of amateur physicists and philosophers upfront. We love the self-criticism: "Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research." (p. 1-10)

Finally, I would like to thank Prof. Dr. Alex Burinskii for taking me seriously. He is in a different realm – and he has made it clear that my writings are extremely simplistic and probably serve pedagogic purposes only. However, his confirmation that I am not making any *fundamental* mistakes while trying to understand the fundamentals, have kept me going on this. We refer to his publications (Burinskii, 2008, 2016, 2017) in the body of our paper itself.