

# The concept of basic clocks and simultaneity in TSR (v1, 2019-04-10)

Per Hokstad

[phokstad@gmail.com](mailto:phokstad@gmail.com)

**Abstract.** The concept of ‘basic clocks’ was introduced in Hokstad (2018b). These are moving clocks to be applied in the theory of special relativity (TSR). They experience an initial synchronization at a common location, and provide the basis for defining time and simultaneity. Here we elaborate and motivate the concept somewhat further, also utilizing symmetry. We further allow having a successive chain of initial synchronizations. This is required in the ‘travelling twin’ example, in order to handle the turning of the twin. Finally, the approach is used to discuss the interpretation of the case of the  $\mu$ -mesons.

*Key words:*, Simultaneity, Lorentz transformation, symmetry, time vector, travelling twin,  $\mu$ -mesons.

## 1 Introduction

Today one establishes simultaneity within a single inertial reference frame (RF) by synchronized clocks, *e.g.* see standard textbooks like Giulini (2005) and Mermin (2005). We say that events with the same clock reading, ‘time’,  $t$ , on a specific RF, are *simultaneous in the perspective* of this frame. However, from a holistic viewpoint this is not a very satisfactory definition, as the various RFs will disagree with respect to simultaneity; also see further discussions in Debs and Redhead (1996).

The present work elaborates and combines ideas presented in Hokstad (2018a) and Hokstad (2018b). We still restrict to consider one space coordinate,  $x$ , and assume the existence an infinite set of (imagined) RFs at various velocities. All clocks on each RF are synchronized in the standard way. In particular, each RF has located a clock at its origin, and at an initial time, 0, the origins of these RFs are at the same location. At this instant, we synchronize all these so-called *basic clocks* (BCs), (allocating the value 0). So, this synchronization is carried out when the clocks are at *the same location at the same time*, and we may refer to this as a ‘point of initiation’.

We now assume that each of these BCs *remain* synchronized. Any two of them move away from each other at a constant speed, but there is a symmetric situation; so there is no way to claim that one of the two clocks will go faster than the other.

So when the BCs at the origins of two specific RFs show the same time, this corresponds (in some sense) to simultaneous events ‘at a distance’. Actually, we could consider this to be a consequence of the standard assumption of symmetry between the RFs. This leads to a rather strong form of simultaneity, as all observers will agree on this. The argument does not restrict to consider just two RFs.

Ch. 3 includes a discussion of the Lorentz Transformation (LT), and how the BC can be used to formulate a symmetric version of this; *cf.* Hokstad (2018a). We see this as a motivation for the definition of the suggested time vector, but it may not be essential for the following text. In an appendix we further present two alternative versions of the LT, both somewhat related to the present approach.

In Ch. 4 we provide our definition of the time vector of an event, *cf.* Hokstad (2018b); now specified as a complex variable. The real part equals the clock reading of the BC present at the event, and its absolute value equals the clock reading. Ch. 5 elaborates on the measure of simultaneity. Previous results are extended, as we also look at a sequence of ‘points of initiation’. This is needed *e.g.* when we consider the travelling twin paradox; as the turning of the twin requires the introduction of such a new ‘point of initiation’.

We suggest that the given approach might affect the way we interpret time and simultaneity in TSR, and include a discussion of a couple of standard examples (Ch. 6); the travelling twin paradox and the case of the  $\mu$ -mesons.

## 2 Basic clocks (BCs) and notation

Consider an arbitrary RF,  $K$ . There are synchronized clocks at virtually any position. When there is a clock reading,  $t$  at a position,  $x$ , we will specify this as an event,  $(t, x)$  relative to  $K$ . Further, we introduce

$$w = x/t$$

This equals the velocity of an object that has moved from the origin of  $K$  and has arrived to the position  $x$  at ‘time’,  $t$ . In some respects it is useful to define an event by  $(t, w)$  rather than  $(t, x)$ .

If a RF moves relative to  $K$  with velocity  $w$  we will denote it  $K_w$ . Further,  $(t_w, x_w)$  will specify an event on  $K_w$ . Now assume that we have an infinite set of auxiliary  $K_w$ ’s, ( $-c < w < c$ , where  $c$  is the speed of light). Initially there is an event, where the origins of all  $K_w$ ’s are located at the same position, and we refer to this event as the ‘point of initiation’. We refer to the clocks located at the origins of the RFs as basic clocks (BCs), and these are all set to 0 by this ‘point of initiation’. Further, all clocks on each RF are synchronized with its own BC in a standard way.

We note that at any later event there will now be present a BC. In particular, the BC at the origin of the  $K_w$ , ( $w = x/t$ ), will be present at the event  $(t, x)$  on  $K$ , see Fig. 1. We refer to this BC reading as  $t_w^{BC}$ . Thus,  $t_w^{BC}$  equals the clock reading,  $t_w$  on  $K_w$  at the position,  $x_w = 0$ . So, at each event there will be exactly one such BC reading. If we do not specify the RF where the BC is located, we simply write  $t^{BC}$ . Now it is a well-known consequence of the Lorentz Transformation (LT) that

$$t_w^{BC} = \sqrt{t^2 - (x/c)^2}, \quad (w = x/t) \quad (1a)$$

Alternatively

$$t_w^{BC} = t \sqrt{1 - (w/c)^2}, \quad (w = x/t) \quad (1b)$$

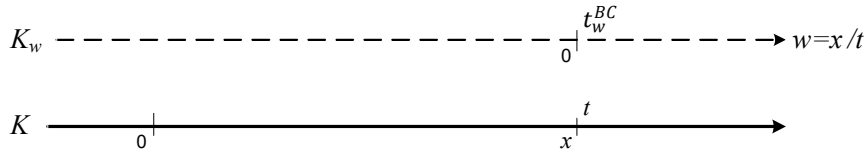


Figure 1 An event  $(x, t)$  on  $K$ , and the corresponding basic clock reading,  $t_w^{BC}$  of this event. Thus,  $K_w$  carries the BC for this event, as it moves relative to  $K$  at a velocity  $w = x/t$ .

## 3 The Lorentz Transformation (LT) and symmetry

As an introduction, we consider some aspects of the LT, being related to BCs and to the symmetry of RFs. So now we consider *two* specific RFs,  $K$  and  $K_v$ .

### 3.1 The Lorentz Transformation (LT)

The LT states that when the event  $(t_v, x_v)$  on  $K_v$  is equivalent to the event  $(t, x)$  on  $K$ , then

$$t_v = \frac{t - (v/c^2)x}{\sqrt{1 - (v/c)^2}} \quad (2)$$

$$x_v = \frac{x - vt}{\sqrt{1 - (v/c)^2}} \quad (3)$$

Fig. 2 gives an illustration of the relation, (2) for events where the clock readings equal  $t$  all over  $K$ . The corresponding clock readings on  $K_v$  equal  $t_v = t_v(x)$ . Thus, we may replace  $t_v$  in (2) by  $t_v(x)$ , to stress its dependence of the position,  $x$ .

The BC readings on these two RFs are found at  $x = 0$  and  $x = vt$ , respectively, (marked as ‘o’ in the figure). Further, at position,  $x^*$ , we have  $t_v = t$ , (cf. Hokstad 2018a). Finally, we also indicate the clock readings,  $t_v(x)$  of the RF,  $K_v$  by a red, stippled line. This, of course, is symmetric to  $t_v(x)$  around the vertical axis.

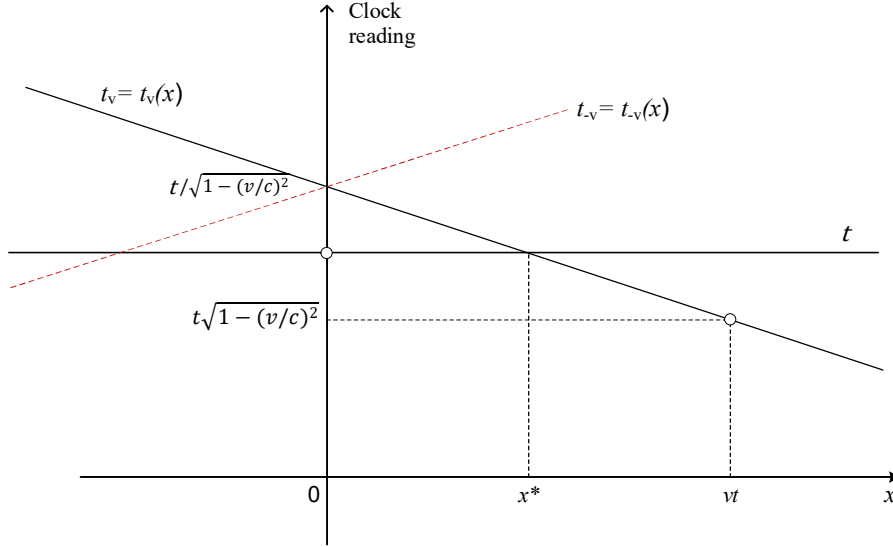


Figure 2 Illustration of the LT. Perspective of the RF,  $K$ : Clock readings equal  $t$  all over  $K$ . The clock readings on  $K_v$  equals  $t_v$ , as a function of the position,  $x$  on  $K$ . The Basic Clocks at the origins of  $K$  and  $K_v$  are marked as small circles, 'o'.

### 3.2 The symmetry of the two RFs

Fig. 2 gives an asymmetric illustration of the LT. The clock readings equal  $t$  all over  $K$ , which means that we have chosen the perspective of  $K$ . In Fig. 3 we give a symmetric presentation of the same relation. We may achieve this by introducing an auxiliary RF with its origin always located at the midpoint between the origins of  $K$  and  $K_v$ , and then choose the perspective of this auxiliary RF; *cf.* (Hokstad 2018a). Fig. 3 presents the clock readings of these three RFs as a function of  $x_{Aux}$ . By increasing 'time', the clock readings of all these RFs moves upwards along the vertical axis.

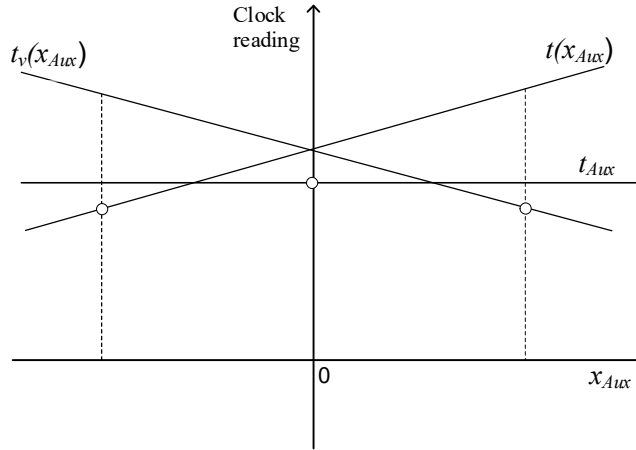


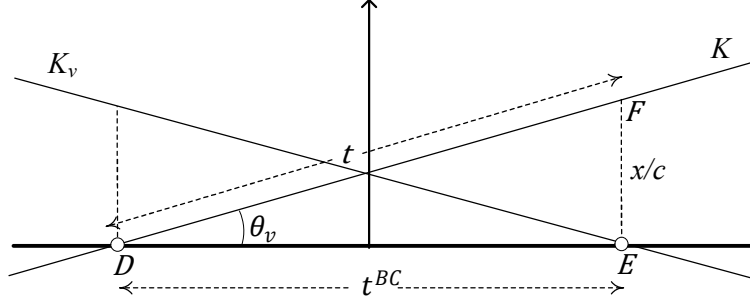
Figure 3 Alternative illustration of the LT. Perspective of an auxiliary, RF,  $K_{Aux}$ : with its origin always located at the midpoint between the origins of  $K$  and  $K_v$ . The origins of the three RFs (with a BC) are marked as 'o'.

Fig. 4, gives an alternative symmetric illustration of the clock readings on  $K$  and  $K_v$ , focusing on the observations at the origins of these RFs, (with the BCs again marked as 'o'). At the 'point of initiation' these BCs are at the same location, and by increasing 'time', they will in the figure move apart along a fixed horizontal line. Further, the figure has two lines, representing  $K$  and  $K_v$ . They have fixed slopes, given by the angle  $\theta_v$ , where

$$\sin \theta_v = v/c \quad (4)$$

Now consider the triangle  $DEF$ . First, we choose the distance,  $DE = t^{BC}$ . It follows that  $DF = t$ , and finally  $EF = \sqrt{t^2 - (t^{BC})^2} = x/c$ . Thus, all distances in the figure have a simple interpretation, measured in time units; ( $x/c$  equals the time required for a light flash to go the distance,  $x$ ).

Here  $E$  and  $F$  represent the same event described by  $K$  and  $K_v$ , respectively. The point  $F$  represents the event  $(t, x)$  on  $K$ , and point  $E$  the identical event  $(t_v, 0)$  on  $K_v$ . Thus,  $t_v$  equals the  $t^{BC}$  at this position  $E/F$ . Similarly, point  $D$  corresponds to the event  $(t^{BC}, 0)$  on  $K$ .



**Figure 4** A fully symmetric illustration of the LT between RFs,  $K$  and  $K_v$ , focusing on the location of the two BCs.

So, by this illustration we split up the clock reading,  $t$ , in two orthogonal components,  $x/c$  and  $t^{BC} = \sqrt{t^2 - (x/c)^2}$ . Further, contrary to Fig. 2, we see that Fig. 4 is symmetric with respect to the two BCs (the two RFs).

#### 4. The time vector and Basic Clocks (BCs)

Here we introduce and present basic features of the time vector, suggested in Hokstad (2018b). Also see Appendix A, for a further motivation.

We return to the situation described in Ch. 2, focusing on a specific RF,  $K$ , and consider an event  $(t, x)$  with  $w = x/t$ . Thus, the relevant BC of the event is located on the RF,  $K_w$ ; cf. Fig. 1. By including a full set of RFs,  $\{K_w\}_{-c < w < c}$ , there will be a BC present at any event.

For an event  $(t, x)$  on  $K$  we now introduce a two-dimensional time vector,  $\vec{t}$ . We present it as a complex variable, and let the real part be the clock reading of the BC of the event (cf. (1)):

$$\text{Re } \vec{t} = t^{BC} = t_w^{BC} = \begin{cases} \sqrt{t^2 - (x/c)^2} = t\sqrt{1 - (w/c)^2}, & t > |x|/c \\ 0, & t \leq |x|/c \end{cases} \quad (5)$$

The imaginary part equals

$$\text{Im } \vec{t} = x/c$$

Thus, we define this time vector as

$$\vec{t} = t^{BC} + i \cdot x/c \quad (6)$$

Now,  $w = x/t$  equals the velocity (relative to  $K$ ) of the RF where the relevant BC is located. If we want to specify this  $w$ , we write  $t_w^{BC}$  rather than  $t^{BC}$ . However, we note that  $t^{BC}$  is a generic feature of the event; being independent of which RF we choose for specifying it.

The imaginary part,  $x/c$ , however, depends on the chosen RF,  $K$ . It equals the time required for a flash of light to go from the origin of  $K$  to the position of the event ( $x$ ). This represents a distance in time, from the location on this  $K$ , where the BC calibration took place.

The absolute value of the time vector becomes:

$$|\vec{t}| = \begin{cases} |x|/c, & t < |x|/c \\ t, & t \geq |x|/c \end{cases} \quad (7)$$

Thus, it is only when  $t > |x|/c$  that  $|\vec{t}|$  equals the clock reading,  $t$  of the event. The reason is that we only observe  $t^{BC} > 0$  for  $t > |x|/c$ ; that is when  $|v| < c$ . Actually, at a fixed position,  $x$ , there will be no BC present until  $t > |x|/c$ . Further, the first BC that arrives to a position will (in the limiting case,  $v=c$ ) read  $t^{BC} = 0$ . This is obviously a weakness of the time vector, with respect to providing a full description of the event  $(t, x)$ . Alternative definitions of the time vector for  $t < |x|/c$  is possible; alternatively we could restrict to define the time vector for  $t > |x|/c$ , only.

There are various alternative formulations of the time vector. First, we can write it in polar form. Defining  $\varphi \in (-\pi/2, \pi/2)$  by

$$\sin \varphi = w/c (= x/ct) \quad (8)$$

we have

$$\vec{t} = te^{i\varphi} = t(\cos \varphi + i \sin \varphi) \quad (9)$$

When  $\theta = 0$ , we have  $w = x = 0$ . Then the event in question occurs at the origin of  $K$ , and the relevant BC is the one located on  $K$  itself. In this case, only, the time vector becomes a real number.

We observe that there is a strong link between the above approach and Minkowski's approach to space-time; cf. space-time distance given as  $\sqrt{c^2t^2 - x^2 - y^2 - z^2}$  in his four-dimensional space, (Minkowski, 1909). As given in Petkov (2012), Minkowski refers to our BC reading, eq. (1), as 'proper time', and our  $t$  as 'coordinate time'.

Further, expressed in terms of the event,  $(t, x)$ , we get

$$\vec{t} = \sqrt{t^2 - (x/c)^2} + i \cdot (x/c)$$

Finally, in terms of  $(t^{BC}, w)$ ; or equivalently, by  $(t^{BC}, \varphi)$ , we have

$$\vec{t} = (1 + i \cdot \tan \varphi)t^{BC}$$

We refer to Appendix A.2 on the formulation of the LT in terms of the variables  $(t^{BC}, w)$ ; for still another alternative see Appendix of Hokstad (2018b).

## 5 Simultaneity

We now consider simultaneity of events and how this relates to the time vector, (6).

### 5.1 The simultaneity concept

We talk about simultaneity '*in the perspective of K*' for events having the same clock reading,  $t$  on this RF. This represents simultaneity in a rather narrow sense; as clocks on different RFs will not agree on the 'time' of a specific event.

However, we can use the above concept of Basic Clocks (BCs) to define simultaneity in a stronger sense; (we refer to Simultaneity *Type II*). For an event  $(t, x)$  it is present a BC with the clock reading,  $t^{BC}$  (provided  $t > |x|/c$ ). As all the BCs move relative to each other at constant speed, there is no way to claim that one goes faster than another does. We note that this BC reading is a feature of the event; being independent of the chosen RF.

Thus, our criterion for events being simultaneous (*Type II*), is simply that  $Re \vec{t} = t^{BC}$  of the events are identical. Below we exemplify this by considering a couple of special cases, illustrated in Figs. 5, 6. First, in Section 5.2 we have one RF and a multitude of BCs at various positions. Next, in Section 5.3 there is one BC and a multitude of RFs.

In general, we may combine these two cases. There can be various BCs at different locations, and each of the BCs is common to all RFs. However, all events  $(t_v, x_v)$  on any RF,  $K_v$ , satisfying

$$\sqrt{t_v^2 - (x_v^2/c^2)} = t^{BC} = \text{Const.}; (\text{any } v)$$

are simultaneous (*Type II*). This gives a consistent definition, applying 'at a distance' and across RFs.

## 5.2 Simultaneity on a single RF

As in Ch. 2 we consider a single RF,  $K$ , having a series of auxiliary RFs,  $K_w$ , ( $-c < w < c$ ). Fig. 5 illustrates the situation that the BCs being present at the various locations ( $x$ ) of  $K$  all show the same value,  $t^{BC}$ ; see stippled blue vertical line; that is, the events are simultaneous *Type II*. We consider one specific of these events,  $(t, x)$ , and as above, let  $w = x/t$ . Then  $\varphi$  is given by (8). So, the angle  $\varphi$  specifies the relative velocity of the RF,  $K_w$  carrying the BC of this event. In particular, for  $\varphi = 0$ , ( $x = 0$ ), the relevant BC is that at the origin of  $K$ .

A time vector will have the angle  $\varphi$  with the (horizontal)  $t^{BC}$ -axis; see Fig. 5. Since the corresponding BC is located on  $K_w$ , we write  $t_w^{BC}$  for the BC reading of this time vector, and we have

$$t_w^{BC} = \sqrt{t^2 - (x/c)^2}$$

Now, events,  $(t, x)$  on  $K$  with BC reading,  $t_w^{BC}$ , which equals a specified value,  $t^{BC}$  are simultaneous (*Type II*).

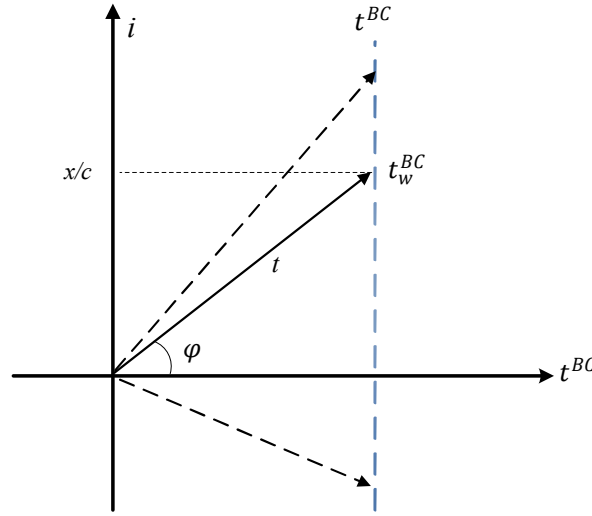


Figure 5 Time vectors of a single RF,  $K$ , with different BCs at various locations, but with the same BC reading,  $t^{BC}$ ; thus, being simultaneous *Type II*. One of the time vector is fully drawn, two others are stippled.

## 5.3 A single BC and simultaneity

Next, in Fig. 6 we illustrate one single event, viewed in the perspective of various RFs; *i.e.* the situation described by the LT; *cf.* Ch. 3. So now there is just a single BC involved, and for simplicity, we assume that it is located on  $K$ , (corresponding to  $v=0$ ). The angle  $\theta_v$  defined in (4), corresponds to the event  $(t_v, x_v)$  on  $K_v$ . We can say that Fig. 6 illustrates the LT between  $K$  and  $K_v$  for all  $v$ . By choosing a velocity,  $v$ , we also choose a  $\theta_v$ , and then vi directly find the time vector for the event, as specified on  $K_v$ .

On any RF,  $K_v$  the event  $(t_v, x_v)$  with the chosen BC reading,  $t^{BC}$  satisfies

$$\sqrt{t_v^2 - (x_v^2/c^2)} = t^{BC}$$

Thus, having the same value. Here we have the same event, just being described by various RFs; therefore, we have all reasons to claim simultaneity.

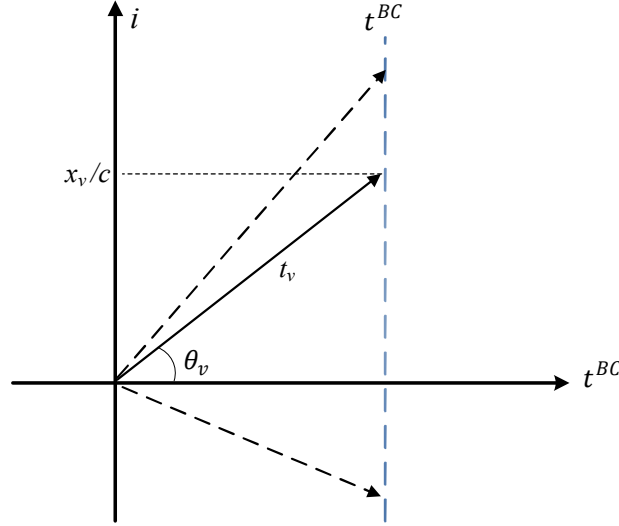


Figure 6 Time vectors of various RFs, related to a single BC reading; (cf. the LT). For convenience, we assume that this single BC is located on  $K$ .

#### 5.4 Restrictions regarding use of the simultaneity measure

Of course, defining simultaneity by applying  $t^{BC}$ , only, does not give the full picture. For instance, the first BC arriving (at speed,  $c$ ) to a location,  $x$ , will read  $t^{BC} = 0$ , as if a flash of light requires no time to go the distance  $x$ ; while the local clock at this position will read  $t = x/c$ . However, while  $t$  provides a good measure of simultaneity with respect to a given RF, we argue that the  $t^{BC}$  for  $t > |x|/c$  will provide a good symmetric measure relative to ('in the perspective of') the 'point of initiation'.

So, when we define this simultaneity relative to a specific 'point of initiation' ('p.o.i.'): it does not represent an 'absolute simultaneity'. Now it would be of interest to relax on this assumption, for instance by allowing discontinuities in the chain of events, by specifying several 'p.o.i.'. We then go outside the framework of the TSR, but we can speculate on such an extension of the approach.

We give the following example; starting out with an event  $(t', x')$  on  $K$  with a BC reading,  $t^{BC}$ . The BC is located at the origin of a  $K_v$ , where  $v = x'/t'$ . However, at this event, conditions are changing: The velocity of  $K_v$  changes from  $v$  to  $v^*$ ; or in other words, we 'replace' the RF,  $K_v$  with a new RF  $K_{v^*}$ . In doing so, we also perform a new calibration; *i.e.* specify a new 'p.o.i.'. Then we should go through the following steps:

1. First, we make a record of the current clock readings of the RFs at the given position. For  $K_v$  this equals the BC reading,  $t_v = t^{BC}$ , and for  $K$  it equals  $t' = \sqrt{(t^{BC})^2 + (x'/c)^2}$ . These values will be added to future clock readings, which we obtain after the new 'p.o.i.'. In particular, the BC reading,  $t_v = t^{BC}$  on  $K_v$  will be added to the future clock reading on  $K_{v^*}$ .
2. We introduce new basic clocks, denoted  $BC^*$ , at this new 'p.o.i.', and assign the values  $t^{BC^*} = 0$ .
3. All clocks on  $K$  and  $K_{v^*}$ , respectively are calibrated in the traditional way; based on the new  $t^{BC^*}$ .
4. We recalibrate the  $x$ -values on  $K$ , as the new origin is located at  $x'$ . For instance, the 'old' origin, ( $x = 0$ ), on  $K$  is now assigned the value  $-x'$ , with the time vector,  $\vec{t} = t^{BC^*} - i \cdot x'/c$ .

We note that such an updating/recalibration is required, *e.g.* for a proper handling of the travelling twin example; see next chapter.

## 6 Discussion. Examples

To illustrate our approach to time and simultaneity in STR we look at the two standard examples, the *travelling twin* and the  *$\mu$ -mesons*.

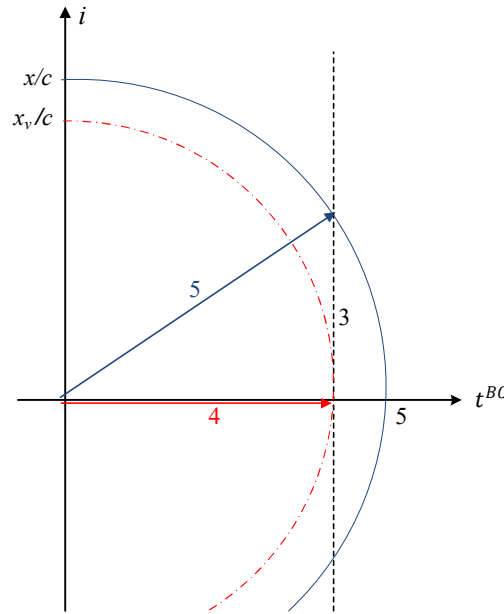
## 6.1 The travelling twin

The so-called travelling twin paradox is frequently discussed, *e.g.* see Schuler and Robert (2014), and we also discussed it in Hokstad (2018a, b).

The earthbound twin has a RF,  $K$  with the origin on the earth, and the travelling twin has a RF,  $K_v$  with the origin at his rocket. The start of the travelling twin's journey is our 'point of initiation', and so both clocks read 0, and both twins are located at the origin of their RFs; thus, both clocks are BCs.

Now we use the numerical example of Mermin (2005). The distance from the earth to the star equals  $x = 3$  light years, *i.e.*  $x/c = 3$  years. Further, the velocity of the rocket is  $v = 0.6c$ , giving  $\sqrt{1 - (v/c)^2} = 0.8$ . It follows that by the arrival to the star the clock at the star, which belong to the earthbound twin, will read  $t = x/v = 3/0.6 = 5$  years, (assuming that the he has a synchronized clock, located on the star). At the same instant the clock of the travelling twin reads  $t_v = t^{BC} = \sqrt{t^2 - (x/c)^2} = t\sqrt{1 - (v/c)^2} = 5 \cdot 0.8 = 4$  years.

Fig. 7 illustrates the relevant time vectors by the arrival; the blue vector for the earthbound twin, and the red for the travelling twin, and. The semicircles indicate events (time vectors) that are simultaneous in *the perspective of*  $K$  and  $K_v$ , respectively



**Figure 7. Time vectors by the arrival at the star: Blue for the earthbound twin. Red for the travelling twin. The relevant vectors are  $\vec{t} = 4 + 3i$  (with absolute value 5), and  $\vec{t} = 4$ , respectively.**

Further, when the travelling twin's clock shows 4 years by his arrival, this is simultaneous (*Type II*) with the event that the clock on the earth also shows 4 years. In the literature one often restricts to point out the simultaneity 'in the perspective of'  $K$  and  $K_v$ , respectively. However, considering the symmetry of the situation, 4 years is the only feasible result regarding this simultaneity at a distance, (Hokstad, 2018a, b).

The main problem of the paradox is rather how to handle the abrupt discontinuity in the chain of events caused by the turning of the travelling twin. This event contradicts the assumptions of TSR, and requires the introduction of a third RF, with a third BC, which is then brought back to the earth. So if we shall treat this within the framework of the TSR, it is not the twin himself that comes back to the earth, but rather a clock that had the identical reading to his own by the turning at the star. We gave an approach in Appendix A.3 of Hokstad (2018a), based on symmetry, *cf.* Fig. 3. However, we now apply a more direct, less detailed argument, following the discussion of Section 5.4.

So, at the instant when the travelling twin arrives to the star, we introduce a third RF,  $K_v$ , which moves at a velocity  $-v$  relative to  $K$ . We first make a record of the clock readings of the event; that is,  $t' = |\vec{t}| = 5$  years on  $K$ , and  $t^{BC} = 4$  years on  $K_v$ . The value 5 is ‘kept for the record’ for  $K$ , while the value, 4 on  $K_v$  is ‘transferred’ to  $K_v$ .

Further,  $K$  and  $K_v$  are assigned new BCs, located on the star, (at the new origins). These are denoted  $BC^*$ , and are allocated the value  $t^{BC^*} = 0$ . Further, all clocks on the two RFs,  $K$  and  $K_v$ , respectively, are calibrated according to these new  $BC^*$ . Finally, positions,  $x$  on  $K$  are replaced by  $x-3$ , to account for the new origin on  $K$ . Then we are back to exactly the same situation as when the travel from the earth started.

We illustrate this in Fig. 8, (see Fig. 4 regarding the interpretation). After the departure, (a), the  $K_v$  moves to the right and  $K$  to the left, (or rather they move apart), ending up in position, (b);. After the recalibration, (c),  $K_v$  moves to the left and  $K$  to the right, ending up with the final position (d); illustrating the return of the travelling twin.

In (c) and (d) of Fig. 8 we add (+5) and (+4) to  $K$  and  $K_v$ , respectively, referring to the clock readings, at the arrival, (b); so, these values are added to the clock readings on  $K$  and  $K_v$  by the return, (d).

We note that the time vector at the earth after the change of p.o.i. (c) equals  $\vec{t} = t^{BC^*} - i \cdot 3$ , as this position now has the distance  $x = 3$  light years from the new origin. So, by the arrival of the  $BC^*$  on  $K_v$  to the earth, this time vector equals  $\vec{t} = 4 - 3i$ , with absolute value  $t=5$ . By adding the clock readings, recorded by the arrival to the star, we arrive at the result  $5+5 = 10$  years. Further, the clock on  $K_v$  by the return reads  $t^{BC^*} = 4$  years, giving  $4+4 = 8$  years for the travel both ways. So, our approach provides the standard answer on the paradox.

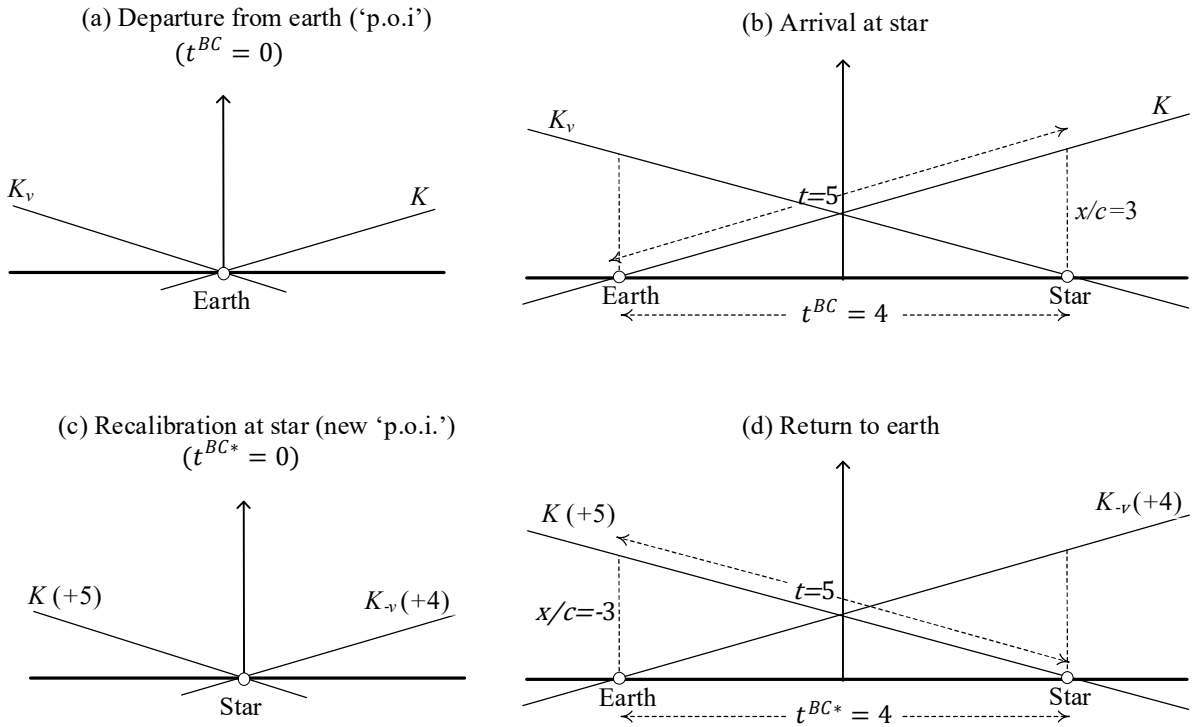


Figure 8. Illustrating the time vectors of the two twins in four steps. In (c) and (d), we also indicate the recorded clock readings at the arrival to the star in parentheses (5 and 4 years, respectively); to be added to clock readings of the recalibrated clocks,  $BC^*$ .

## 6.2 The $\mu$ -mesons

The example of the  $\mu$ -mesons is frequently referred, *e.g.* see Mermin (2005), where we have extracted the following text. The  $\mu$ -mesons are produced by cosmic rays in the upper atmosphere. When 'at rest'

they have a lifetime of about 2 microseconds, so if their internal clocks ran at a rate independent of their speed, even if they travelled at the speed of light about half of them would be gone after they had travelled 2.000 feet. Yet about half of the  $\mu$ -mesons produced in the upper atmosphere (about 100.000 feet up) manage to make it all the way down to the ground. This is because they travel at speed so close to the speed of light that the slowing down factor equals 1/50, and they can survive for 50 times as long as they can when being stationary.

The phenomenon is easily described by the LT. We have the RF of the earth,  $K$ , and that of the  $\mu$ -mesons, denoted  $K_v$ . The creation of the  $\mu$ -mesons in the upper atmosphere, is denoted event A; then  $x = x_v = 0$  and  $t = t_v = 0$ . So, this is the ‘point of initiation’, specifying two BCs; one on  $K$ , remaining in the upper atmosphere, one on  $K_v$ , going to the ground with the  $\mu$ -mesons.

Event B is the arrival at the ground; then  $x_v = 0$  and  $t_v = 2 \cdot 10^{-6}$  sec. So this  $t_v$  equals the BC reading of event B, and  $t^{BC} = t_v = 2 \cdot 10^{-6}$  sec.

Further, the RF of the  $\mu$ -meson is moving relative to earth at a speed,  $v \approx c$ , with  $\sqrt{1 - (v/c)^2} \approx 0.02$ . Thus, in event B, we have  $t \approx t_v / 0.02 \approx 10^{-4}$  sec. The  $\mu$ -mesons have then gone the distance  $x = 10^5$  feet, (measured on  $K$ ), giving  $x/c \approx 10^{-4}$  sec. Thus, the time vectors of event B for the two RFs, respectively, equal

$$\vec{t} = t^{BC} + i \cdot (x/c) = (2 \cdot 10^{-6} + i 10^{-4}) \text{ sec}; \text{ (thus, } |\vec{t}| = t \approx 10^{-4} \text{ sec)}$$

$$\vec{t}_v = t^{BC} = 2 \cdot 10^{-6} \text{ sec}; \text{ (thus, } |\vec{t}_v| = t^{BC} \approx 2 \cdot 10^{-6} \text{ sec)}$$

In summary, the theory (the LT) is in full agreement with the observations. Now, according to the present approach the BC reading of event B, ( $2 \cdot 10^{-6}$  sec) is simultaneous with the event that the BC in the upper atmosphere shows the same value. However, my main concern is how we should talk about/interpret the above result.

It is a fact that the clock following the  $\mu$ -mesons goes slower by a factor 50, *when it is compared with two clocks, which are stationary relative to the earth*. However, I find it strange that Mermin (2005) formulates this as: "The atomic particles can go much further because their internal clocks that govern when they decay are running much more slowly in the frame in which they rush along at speed close to  $c$ . This is a real effect, and it plays a crucial role in the operation of such particles accelerators." We also note the above formulation: "...if their internal clocks ran at a rate independent of their speed ...".

There are two statements here, that I should like to comment on. First, what does it actually mean that this is a *real effect*. Secondly, how should we interpret the statement that the experiment confirms that the *internal clocks* of the atomic particles are running more slowly, (that is, depends on their speed)? These are questions, perhaps of philosophical, rather than physical character, but are nevertheless of considerable interest.

First, regarding this being a *real effect*. It is often said that ‘moving clock goes slower’. This occurs when a single clock is moving between two clocks which are at a fixed distance from each other. However, we should stress (*cf.* discussion in Hokstad 2018a), that it is the *single* clock that always goes slower, when compared to *two* clocks on another RF. This is the case, also if we see the single clock to be ‘at rest’, and the two other clocks as moving past. Thus, rather than being a real effect, we could say that it is a consequence of the chosen observational principle; *i.e.* the (relative) velocity of the RF which we choose as the basis for the observations.

Further, what should be the physical meaning of the *internal clocks* of the atomic particles? Actually, if the observer is at rest (moves together) with respect to the phenomenon, then the average ‘lifetime’ of  $\mu$ -mesons equals 2 microseconds. So, this is rather the duration which we should consider to express the ‘inner clock’ of the  $\mu$ -mesons. Thus, provided the conditions of the TSR are valid, the  $\mu$ -mesons should ‘see’ themselves as being at rest. Further, this ‘inner clock’ should hardly be affected by passing observers making observations. And the passing observers (at various speeds,  $v$ ) will make ‘all kind’ of

observations. Obviously, my view is that we should rather see  $t^{BC} = 2 \cdot 10^{-6} \text{ sec}$  to represent the ‘true’ lifetime also in the present experiment.

So, a main question is to what extent time dilation should be considered a ‘true’ physical phenomenon or an observational phenomenon. It is a fact that different observers obtain different results regarding the time duration for the same phenomenon. However, it seems unreasonable to claim that an observer will ‘truly’ affect the experiment / phenomenon as such.

## 7 Conclusions

In our approach we introduce an infinite set of reference frames (RFs), at any velocity. All of the (possibly imagined) RFs have a synchronized set of clocks at virtually any position. Further, we synchronize the clocks at the origins of the RFs at a common instant and location, and we refer to this event as the ‘point of initiation’. From symmetry, the clocks at the origins of the various RFs now remain synchronized, and we denote them as basic clocks (BCs).

It follows that one of these BCs will be present at any later event  $(t, x)$  on any RF, (provided  $t > |x|/c$ ). Now we define time as a complex variable, where

1. The real part equals the clock reading ( $t^{BC}$ ) of the BC currently at this position.
2. The imaginary part equals the time required for a light flash to go from the origin of the RF, (where another BC is located), to the current position,  $(x/c)$ .
3. The absolute value of this time variable equals the clock reading,  $t$ , of the event, (when  $t > |x|/c$ ).

It is well known that when events on the same RF have the same clock reading,  $t$ , then the events are simultaneous ‘in the perspective’ of this RF. From a holistic point of view, however, time variables with the same BC reading ( $t^{BC}$ ) imply simultaneity in a stronger sense. They provide a form of symmetry relative to the given point of initiation, and this simultaneity is not affected by the RF(s) on which we specify the events. All events with the same  $t^{BC}$  will exhibit this form of simultaneity, also for events ‘at a distance’, possibly defined on different RFs.

The present work provides further motivation, and elaborates on previous results; *e.g.* discussing two alternative formulations of the Lorentz Transformation (LT); *cf.* Appendix.

We discuss an extension of approach, by allowing a sequence of ‘points of initiation’ *in series*. An example is provided by the travelling twin paradox; where the turning of the travelling twin obviously requires the introduction of such a second ‘point of initiation’.

The approach discussed here may not provide essential new information, but should in my opinion affect the way we talk about/interpret time and simultaneity within the framework of STR. As an example, we also discuss the case of the  $\mu$ -mesons.

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## Appendix A Alternative formulations of the Lorentz Transformation (LT)

Here we consider two alternative formulations of the LT; both somewhat related to our approach. As in Ch. 3 we start out with two RFs,  $K$  and  $K_v$ .

### A.1 The LT as an orthogonal transformation

We now replace  $v$  in the LT, (2), (3) by the angle,  $\theta_v$ . As before, we define this by, (see eq. (4))

$$\sin \theta_v = v/c$$

also implying that  $\cos \theta_v = \sqrt{1 - (v/c)^2}$ . Now we can formulate the LT, (2)-(3) as:

$$t_v \cdot \cos \theta_v = t - (x/c) \cdot \sin \theta_v \quad (\text{A1})$$

$$(x_v/c) \cdot \cos \theta_v = x/c - t \cdot \sin \theta_v \quad (\text{A2})$$

As we consider one space coordinate, only, the LT involves just four state variables:  $t, x, t_v$  and  $x_v$ . If we specify any two of these, the other two will be given by the LT.

The above standard version of the LT gives  $(t_v, x_v)$  expressed by  $(t, x)$ , or *vice versa*. But similarly, we could reformulate the LT to give a relation between  $(t, t_v)$  and  $(x, x_v)$ . And – as a third possibility – we can formulate the LT as a relation between  $(t, x_v)$  and  $(t_v, x)$ . Now we follow up on this third possibility. First, by combining (A1) and (A2), we also have

$$t_v = t \cos \theta_v - (x_v/c) \cdot \sin \theta_v \quad (\text{A3})$$

Now (A2) and (A3) give a new version of the LT, which in matrix form becomes

$$\begin{pmatrix} t_v \\ x_v/c \end{pmatrix} = A_v \begin{pmatrix} t \\ x/c \end{pmatrix} \quad (\text{A4})$$

Here the transformation matrix,

$$A_v = \begin{bmatrix} \cos \theta_v & -\sin \theta_v \\ \sin \theta_v & \cos \theta_v \end{bmatrix} \quad (\text{A5})$$

is orthogonal as

$$A_v^{-1} = A_v^T = \begin{bmatrix} \cos \theta_v & \sin \theta_v \\ -\sin \theta_v & \cos \theta_v \end{bmatrix} = A_{-v}$$

Next, we introduce the time vectors

$$\overrightarrow{t_1(v)} = \begin{pmatrix} t \\ x_v/c \end{pmatrix} \quad (\text{A6})$$

$$\overrightarrow{t_2(v)} = \begin{pmatrix} t_v \\ x/c \end{pmatrix} \quad (\text{A7})$$

and then write the relation (A4) as

$$\overrightarrow{t_2(v)} = A_v \overrightarrow{t_1(v)} \quad (\text{A8})$$

being an orthogonal version of the LT. It represents a rotation,  $\theta_v$ , of the  $(t, x_v/c)$  plane. So, the vectors,  $\overrightarrow{t_1(v)}$  and  $\overrightarrow{t_2(v)}$  provide identical information, (of a single event, specified by the two RFs,  $K$  and  $K_v$ ).

In chapters 2, 3, we saw that when we should describe an event relative to a RF,  $K$ , it could be helpful to utilize the clock reading of another RF, (*i.e.* the RF with the relevant BC). Now (A8), represents a generalization of this result. By choosing  $v = x/t$  in (A8), (giving to  $x_v = 0$  and  $t_v = t^{BC}$ ), this relation gives

$$\overrightarrow{t_2(x/t)} = \begin{pmatrix} t^{BC} \\ x/c \end{pmatrix} = A_v \begin{pmatrix} t \\ 0 \end{pmatrix} = \begin{pmatrix} \cos \theta_v \\ \sin \theta_v \end{pmatrix} t = \begin{pmatrix} \sqrt{t^2 - (x/c)^2} \\ x/c \end{pmatrix}.$$

So, for the special case,  $v = x/t$ , actually  $\overrightarrow{t_2(x/t)}$  specifies our time vector, (defined in Ch. 4).

## A.2 The LT expressed by BC readings. New state variables.

Again, we have an event, described as  $(t, x)$ , and  $(t_v, x_v)$ , relative to  $K$  and  $K_v$ , respectively. Further

$$w = x/t.$$

$$w_v = x_v/t_v$$

So, these are the velocities for the BC of the event, relative to  $K$  and  $K_v$ , respectively. Now we can express the BC reading of the event, either by the parameters  $(t, w)$  or  $(t_v, w_v)$ :

$$t^{BC} = t_w^{BC} = \sqrt{t^2 - (x/c)^2} = t\sqrt{1 - (w/c)^2} \quad (\text{A9})$$

$$t^{BC} = t_{w_v}^{BC} = \sqrt{t_v^2 - (x_v/c)^2} = t_v\sqrt{1 - (w_v/c)^2} \quad (\text{A10})$$

Here (A9) and (A10) refer to the same BC reading, just viewed in the perspective of  $K$  and  $K_v$ , respectively. Thus,

$$t_{w_v}^{BC} = t_w^{BC} \quad (\text{A11})$$

Now, reformulating the LT, we could choose (A11) as our first relation. To obtain a second relation, we could start out from (2), which we now can write

$$t_v = \frac{1 - vw/c^2}{\sqrt{1 - (v/c)^2}} t,$$

Replacing  $t$  and  $t_v$ , using (A9) and (A10), we directly get

$$\frac{1}{\sqrt{1 - (w_v/c)^2}} = \frac{1 - vw/c^2}{\sqrt{1 - (v/c)^2} \sqrt{1 - (w/c)^2}}$$

Recall that  $\varphi$  is defined by

$$\sin \varphi = w/c$$

and, similarly, letting  $\sin \varphi_v = w_v/c$ , we can also write this relation as

$$\cos \varphi_v = \frac{\cos \theta_v \cos \varphi}{1 - \sin \varphi_v \sin \varphi} \quad (\text{A12})$$

We could carry out exactly the same approach, starting from (3) rather than (2), then obtaining

$$\tan \varphi_v = \frac{\sin \varphi - \sin \theta_v}{\cos \theta_v \cos \varphi} \quad (\text{A13})$$

Finally, by combining (A12) and (A13) we also get

$$\sin \varphi_v = \frac{\sin \varphi - \sin \theta_v}{1 - \sin \varphi_v \sin \varphi} \quad (\text{A14a})$$

Of course, we can also write this as

$$w_v = \frac{w - v}{1 - \frac{wv}{c^2}} \quad (\text{A14b})$$

We recognize this as the formula, which applies in TSR, for the sum of the two velocities  $w$  and  $-v$ . In this respect it is an obvious result; recalling that  $w$  is the velocity of the relevant BC relative to  $K$ , and that  $w_v$  is the velocity of the same BC relative to  $K_v$ .

Thus, in total we can describe an event defined on  $K$  by the state variables  $(t^{BC}, w)$ ; which equal the BC reading of the event and the velocity of this BC relative the RF,  $K$ . Using these two state variables to specify the event, the LT indeed reduces to two fundamental results: <sup>1)</sup> the BC reading,  $t^{BC}$  is the same for both RFs; <sup>2)</sup> the velocities of this BC relative to the two RFs is related by (A14b).

We could of course choose the second variable as  $w/c$  rather than  $w$ , (which could seem reasonable, according to (A14)). This would give  $(t^{BC}, w/c)$  as the state variables.