Simultaneity and Light Propagation in the context of the Galilean Principle of Relativity

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Abstract

The intent of this work is to present a discussion of the Galilean Principle of Relativity and of its implications for what concerns the characteristics of light propagation and the nature of simultaneity. It is shown that by using a clock synchronization procedure that makes use of isotropically propagating signals of generic nature, the simultaneity of distinct events can be established in a unique way by different observers, also when such observers are in relative motion between themselves. Such absolute nature of simultaneity is preserved in the passage from a stationary to a moving reference frame also when a set of isochronous generalized coordinates is introduced. These transformations of coordinates can be considered as a generalization of the Lorentz transformations to the case of synchronization signals having characteristic speed different from the speed of light in vacuum.The specific invariance properties of these coordinate transformations with respect to the characteristic speed of propagation of the synchronization signals and of the corresponding constitutive laws of the underlying physical phenomenon are also presented, leading to a different interpretation of their physical meaning with respect to the commonly accepted interpretation of the Lorentz transformation as a space-time distortion. On the basis of these results, the emission hypothesis of W. Ritz, that assumes that light is always emitted with the same relative speed with respect to its source and that is therefore fully consistent with the Galilean Principle of Relativity, is then applied to justify the outcomes of the Michelson-Morley and Fizeau interferometric experiments by introducing, for the latter case, an additional hypothesis regarding the possible influence of turbulence on the refractive index of the fluid. Finally, a test case to verify the validity of either the Galilean or the Relativistic velocity composition rule is presented. The test is based on the aberration of the light coming from celestial objects and on the analysis of the results obtained by applying the two different formulas for the resultant velocity vector to process the data of the observed positions, as measured by a moving observer, in order to determine the actual un-aberrated location of the source.

I. The Galilean Principle of Relativity

The Galilean Principle of Relativity has been originally formulated by Galileo Galilei in the following way[1]: *"Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though doubtless when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor under you will be going in a direction opposite to your jump. In throwing something to* *your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. And if smoke is made by burning some incense, it will be seen going up in the form of a little cloud, remaining still and moving no more toward one side than the other. The cause of all these correspondences of effects is the fact that the ship's motion is common to all the things contained in it, and to the air also. That is why I said you should be below decks; for if this took place above in the open air, which would not follow the course of the ship, more or less noticeable differences would be seen in some of the effects noted."*

The principle states the invariance of physical phenomena with respect to uniform rectilinear motion, with constant velocity, of the system within which the phenomena themselves occur.

As it can be noted from the above formulation of the relativity principle, the invariance of the phenomena requires that the entire system being analyzed, including the medium in which the phenomena occur and propagate and the system's boundary conditions (the walls of the ship's cabin, the bowl where the fishes swim), all translate with the same uniform velocity. The filling medium, the atmosphere of the ship's cabin or the water contained into the bowl, present into the enclosed volume of the system being analyzed is transported by the motion of the frontier of the system in such a way that the medium remains stationary with respect to the other parts of the system also when the ship is proceeding along its course with constant velocity.

In the above formulation of the principle of relativity Galilei remarked that any phenomenon which is characterized by having an isotropic propagation speed for a given state of motion of the ship will maintain this property also when the entire system (the ship) is moving with constant uniform velocity with respect to its original condition. This invariance property is applicable both to wave-like phenomena that require a propagation medium to occur, like the circular propagation of the waves on the water surface, and to particle-like or corpuscular phenomena, involving the motion of physical objects, like the balls thrown by hand mentioned in Galilei's example, or the motion of particles originating from a given source, provided that these corpuscular physical entities are emitted by the source with the same constant speed in all directions.

Though the original formulation by Galilei made explicit reference only to some specific physical phenomena, involving in particular mechanics and fluidodynamics, the principle is considered valid also for all other physical phenomena, including electromagnetism and optics. This means that the results of any physical experiment shall not vary when the same test is repeated in a given laboratory and in another laboratory which is moving with uniform constant velocity with respect to the first one.

If we now consider an observer that is moving with constant velocity inside the cabin of the ship, we can note that for this observer the above phenomena are no longer characterized by a uniform speed in all directions: for a moving observer the velocity of propagation of the wavefronts on the water surface of the bowl will appear different along different directions, and the same happens also for the velocity of the hand launched balls which will have a different value along different directions. For such a moving observer those phenomena are not characterized by an isotropic propagation speed, whilst this property is valid for an observer which is stationary with the ship's frame, i.e. for an observer which is stationary both with the source of the phenomenon and with its propagation medium, when the presence of a propagation medium is necessary for the specific phenomenon being investigated.

Assuming that the ship is in a state of uniform rectilinear motion, a reference frame stationary with the ship's deck is an inertial reference frame. If the observer moving inside the cabin is also translating with uniform constant speed with respect to the ship's deck, then also this frame is an inertial reference frame. The difference between these two inertial frames lies in their property of conserving, or not conserving, the isotropy of propagation of the phenomena. This characteristics therefore splits the class of the inertial frames into two groups, and the distinction is applicable also to phenomena that do not require a propagation medium, it is thus applicable also in vacuum. As a consequence of this distinction, the laws describing the evolution of the physical phenomena will not be the same for *all* inertial reference frames, and for the associated observers. The laws of the same physical phenomenon will take a different form in an inertial frame for which the isotropy of propagation is conserved and into another inertial frame that is in relative motion with respect to the first one and for which the isotropy of propagation is not conserved.

An observer moving inside the ship's cabin will also notice variations of the frequency of periodic phenomena occurring into the system. The time separation between the peaks and valleys of the water waves appears different for an observer at rest with the propagation medium, the water inside the bowl, and for an observer moving on its surface. Similarly, the frequency of encounter of the water drops falling from the bottle will increase if the observer is moving upwards and decrease if the observer is moving downwards, reaching a null value when the downward speed of the observer is equal to the falling speed of the water drops. The same variation of the observed frequency affects also other phenomena not mentioned by Galilei: the tone of a sound or the colors of the spectral lines emitted by an excited substance appear different for a moving observer with respect to a stationary one.

Finally, it can be noted that whilst the Galilean Principle of Relativity has been formulated for systems that are in a state of uniform rectilinear motion, the specific example used by Galilei in its original description, i.e. the ship and the physical entities contained in its cabin, is not actually representative of such a case, since the ship, whether at rest in the harbour or cruising on the sea, is transported by the Earth's motion along a non rectilinear path. Due to the curvature of the Earth and to its angular rotation, the state of motion of the ship contains a circular component and is characterized by a non-null angular velocity. Even if the amount of the deviation from uniform rectilinear motion is quite small and can be neglected, in first approximation, for many applications, the presence of the Earth's rotation can have an influence of the physical phenomena being observed and it can indeed be detected by suitable physical experiences, for example by observing the variation of the plane of oscillation of a Foucault pendulum. This same observation is applicable to any experiment performed into a Laboratory on the Earth, since the entire experimental setup is rigidly transported by the non-rectilinear motion of our planet. In general, this accelerated state of motion could have an influence on the results of the experiment and on the measurements being conducted. The actual extent and entity of the influence will depend on the phenomenon being investigated and on the specific experimental setup, being possibly not negligible for some very accurate measurements or experiments.

II. Simultaneity and Time Intervals

In order to describe the governing laws of physical phenomena by means of mathematical expressions it is necessary to define a set of coordinates to associate each event being analyzed to a location in space and to a time of occurrence. This requires a method to determine when two or more events occurring at different locations in space are simultaneous, and a definition of a reference clock to measure the time intervals between non-simultaneous events.

Let us take a given closed system S where a reference frame K has been identified. For convenience K can be a set of three orthogonal Cartesian axes. Let us consider two simultaneous events occurring at two distinct space locations A and B. Suppose that at the time of occurrence of each event some kind of synchronization signals are emitted from the two points of space where the events occur and that such synchronization signals travel with uniform and constant speed in all directions, i.e. that they propagate isotropically into the reference frame K. Let v_c be the finite *characteristic speed* of propagation of such signals, which is assumed to be equal in all directions into the reference frame K. The two synchronization signals emitted by A and B at the time of occurrence of the two events will reach simultaneously the midpoint M of segment AB, since both signals will take the same time to cover the distance that separates M from A and M from B, being such two distances equal by construction. If we now consider another couple of signals, emitted from A and B at the same time of first two, but which are characterized by a different value of the propagation speed with respect to the first ones, also these two signals will reach the midpoint M of AB simultaneously. The coincidence of the time of arrival time of the A and B signals is therefore verified for any value of the characteristic speed of propagation v_c of the synchronization signals, the only difference being that the faster signals will arrive at the midpoint M earlier than the slower ones.

It is this thus possible to formulate the criterion for the simultaneity of events in the following general way:

Two events are simultaneous if and only if the two synchronization signals, emitted from the points A and B at the time of occurrence of the corresponding events, reach simultaneously the mid-point M of segment AB [1] for any finite value of the characteristic speed v_c of the selected signals.

When this condition is verified we can say that, into the specified reference frame K being considered, the time t of the two events is the same, i.e. we can state $t_A = t_B$.

The simultaneity of events is therefore a characteristics that is invariant with respect to the speed of propagation v_c of the synchronization signals, thus resulting independent from the specific kind of signal being selected.

According to the above criterion, if an event A is simultaneous with a second event B and also with a third event C, than also the two events B and C are simultaneous. The synchronization signals selected to assess the mutual simultaneity between the three events can be different for each couple of events, the outcome of the process will be the same.

The physical nature of the specific signals being used for the synchronization is not relevant for the method, they could be particle-like or wave-like phenomena, nor the value of their characteristic propagation speed v_c, which is only assumed to be finite and equal in all directions. The only assumption required for the validity of the method is that the selected synchronization signals propagate isotropically with respect to the reference frame K.

For example, in vacuum one could imagine to employ small particles, emitted in every direction with the same relative speed with respect to the source by a spring-loaded launching device, or one could consider to perturb an ideal string tensioned between its endpoints A and B and use the propagation of the resultant waveform as synchronization signal. In both cases we can imagine of being able to tune the value of the signal speed ideally to whatever finite value v_c, by properly adjusting the characteristics of the governing parameters of the selected physical phenomenon (string tension, spring and mass values). In presence of a ho-

[1] In general, the location of the control point M need only to be selected in such a way that it is equidistant from A and B, i.e. it can be located at the center of a sphere having points A and B on its surface. The midpoint of the segment AB represents the minimum distance choice.

mogeneous and isotropic medium, other kind of signals could also be employed like, for example, acoustic waves traveling in the air at the speed of sound.

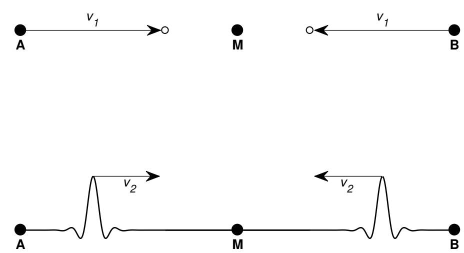

Figure 1: *Simultaneity assessment by means of particle-like (top) or wave-like (bottom) isotropic synchronization signals traveling with characteristic speed $v_c = v_1$ and $v_c = v_2$, respectively*

In order to guarantee the isotropy of propagation of the synchronization signals, according to what stated by the Galilean Principle of Relativity, it is necessary that the frame of reference K identified to represent the coordinates of the two events A and B is stationary both with the source of the signals and with the propagation medium (for those phenomena that require a medium to propagate). In the above examples this means that the spring-loaded launcher of the particle-like objects, in one case, and the entire ideal string, in the other case, have to be stationary with respect to the frame K.

The process can be applied to any pair of geometrical points in the space and to the corresponding couple of events. In such a way, it can be used to synchronize pairs of clocks placed at distinct space locations. Without losing generality we can assume that the origin of the reference frame K is coincident with one of the two points selected as the source of the synchronization signals. By using this method, therefore, it is possible to synchronize a *"master"* clock located in the origin of the reference frame K with a clock placed at any point of the entire space S, i.e. at any point of the entire space domain. This synchronization of the clocks guarantees also that the two clocks run at the same pace, spanning the same time intervals at the two different space location, i.e. it allows to state that $\Delta t_B = \Delta t_A$.

Repeating the same process for all points of the entire space domain it is possible to synchronize all the clocks located at the different geometrical locations of K with the time reference of the master clock located in the origin. All clocks will therefore beat in unison, spanning the same time intervals of the master clock. In this manner it is thus possible to associate, in a unique way and consistently with the Galilean Principle of Relativity, the space and time coordinates, expressed into the reference frame K, to any event occurring into the system S being observed, and the process is not dependent neither on the type of physical signal used to perform the synchronization nor on its characteristics speed v_c, the only requirement for the validity of the synchronization method being that such signals are isotropically propagating with respect to the K frame.

Let us now consider a second reference frame, K', that is in a state of uniform rectilinear motion with respect to the previous one, with a velocity having magnitude V, as measured in the reference frame K, and direction parallel to segment AB, oriented from A to B. Let A' and B' be the position of the two geometrical points expressed in the reference frame K' that coincide, respectively, with the position of A and B at the time of occurrence of the corresponding events. Let A'B' be the segment joining these two points and let M' be the midpoint of this segment which is at rest into the frame K'. In order to assess the simultaneity of the two events A and B, an observer stationary with the K' frame cannot use the same two synchronization signals that have been adopted by the observer of the K frame. In fact, due to the finite value of the characteristic signal speed v_c, the two K-based signals will meet together at M after some time from their emission from A and B. During this time period the midpoint

M′ will have traveled a certain amount of distance from M. Since the two signals cannot meet both in M and in M′, it follows that the K′ observer would incorrectly judge the two events as being non-simultaneous.

In order to avoid this issue and to correctly evaluate the simultaneity of the two events also into the reference frame K′, it is necessary to make use of signals that propagate isotropically into this moving frame. This requires, according to the Galilean Principle of Relativity, that the sources of the signals and the propagation medium (for example, the ball launchers or the tensioned string) are both stationary with respect to the reference frame of the observer. A moving observer K′ can therefore assess the simultaneity of events A and B by using other two synchronization signals, distinct from the ones used by the observer of the K frame, emitted from the space locations A′ and B′, that are coincident with A and B at the time of occurrence of the corresponding events, provided that such signals travel isotropically with respect to his reference frame K′. The nature of these two *'primed'* signals and the corresponding characteristic speed v'_c, could be the same of the one used for synchronization in frame K, or it could be different, provided that it is isotropic in K′. For example, one could image to use the traveling balls in frame K and the waveform propagating on the string in frame K′, or viceversa. In this way the two events A and B will be declared simultaneous also into the K′ reference frame, since the two *"primed"* signals, propagating with the same speed v'_c from A′ to M′ and from B′ to M′, will reach simultaneously the midpoint M′ of segment A′B′ [2]. In this way, whenever two events are declared simultaneous in one reference frame K, they result simultaneous also in the moving frame K′, and this conclusion regarding the coincidence in time of the two events is independent from the specific nature and the related characteristic speed v_c, or v'_c, of the synchronization signals being used in the two reference frames.

When the specific physical signal chosen for the synchronization procedure needs some form of medium to propagate in the surrounding space, the requirement of isotropic propagation can be guaranteed only for a reference frame that is stationary with the specific propagation medium being considered. For example, in case of acoustic signals traveling in the atmosphere, only the clocks of those reference frames which are stationary with the air can be synchronized using such acoustic signals. The clocks referred to any other reference frame in relative motion with respect to the previous ones, and therefore in motion with respect to the air, cannot be synchronized by means of acoustic signals, since for such a moving frame the speed of sound would no longer be same in all directions, i.e. it would not be isotropic.

The synchronization procedure described above, and the related considerations, are valid also when light signals are used to establish the simultaneity of events, provided that the light sources being considered and the transparent light propagation medium, if present, are both stationary with respect to the reference frame of the observer and with the clocks that are being synchronized. The emission hypothesis formulated by W. Ritz[3], that assumes that light is emitted in all directions with the same relative speed, equal to c in vacuum, with respect to its source, being fully consistent with the Galilean Principle of Relativity and therefore also compliant with the above requirements of the synchronization procedure, justifies the usage of light signals to synchronize the clocks.

The assessment of simultaneity of the events with respect to the moving reference frame can also be implemented in the following, more direct way. At each instant of time a generic geometrical point P′ belonging to the moving frame K′ happens to be coincident with one geometrical point P of the reference frame K. When the two geometrical points are coincident, $P' \equiv P$, the time indicated in that moment

[2] As previously noted, since the segment A′B′ is at rest into frame K′, which is moving with velocity V with respect to frame K, the location in space of its midpoint M′ at the time of detection of the primed signals will not be coincident with M

by the clock located at P can be readily extended also to P′ since, being the two points coincident, there is no delay associated with the transfer of the information regarding the time readout between two different space locations, separated by a non null physical distance. It is therefore possible to associate to P′ the same time indicated by the clock associated to P. Since the clocks of the entire space K are all synchronized between them, they all indicate the same time. This same time stamp can therefore be assigned also to all geometrical points of K′ because each point of the K′ space domain will be coincident with one and only one location of the K space and will therefore take from it the corresponding time indication. In other terms, it is possible to assign to all geometrical points of the moving frame K′, the same time indicated by the "stationary" clocks synchronized into reference frame K. This conclusion is valid for any geometrical location of the reference frame K′, thus allowing to establish, also for the observers of this "moving" reference frame, the same time basis of the "stationary" one, i.e. it is possible to set $t' = t$, from which it also follows $\Delta t' = \Delta t$.

III. Isochronous transformations of coordinates

In this paragraph it will be shown that the absolute nature of simultaneity can also be consistently assessed by a moving observer through the use of a class of coordinate transformations similar to the Lorentz transformations.

Let us consider two events occurring at two distinct locations A and B of the space and be K a reference frame stationary with respect to the points A and B, having its origin located at the midpoint of segment AB and the x axis parallel to AB and directed towards B. Let v_c be the characteristic propagation speed of the isotropic signals that have been selected to synchronize the clocks into this reference frame. According to the previously described synchronization method, the two events are simultaneous if the synchronization signals emitted from A and B at the time of occurrence of the corresponding events reach simultaneously the midpoint of segment AB, which in this case is the origin of the reference frame K.

We can now introduce, into the reference frame K, the *characteristic interval*, s_c, that "separates" the two events A and B and that is defined by the following relation containing the value of the signal propagation speed v_c as a parameter:

$$s_c = \sqrt{v_c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2} \qquad (1)$$

where $\Delta t = (t_B - t_A)$, and $\Delta x = (x_B - x_A)$, $\Delta y = (y_B - y_A)$, $\Delta z = (z_B - z_A)$.

Let us now consider a second reference frame K′, having its axes parallel to those of K, and let K′ be translating with constant speed V along the positive direction of the x axes with respect to frame K. We will call K the stationary frame and K′ the moving frame. It is possible to introduce, into the moving frame K′, a new set of four generalized space-time coordinates, that will be indicated with $(\varepsilon', \eta', \zeta', \tau')$, and that are functions of the (x, y, z, t) coordinates of the stationary reference frame K:

$$(\varepsilon', \eta', \zeta', \tau') = f(x, y, z, t) \qquad (2)$$

In the stationary frame K we can calculate, according to definition (1), the characteristic interval between the events of emission of the synchronization signal from either geometrical points A or B and of their detection at the midpoint of segment AB. For convenience we can set the origin of time to be coincident with the time of occurrence of the two simultaneous events, therefore $t_A = t_B = 0$. Indicating then with t_D the time of detection of the synchronization signals at the midpoint of segment AB, it results $t_D > 0$.

We want to find the specific form of the functions f defining the primed coordinates that makes invariant the characteristic interval between these two events in the passage from K to K′, and viceversa.

Since the y and z axes of the two reference frames are parallel by construction, the corresponding coordinates of the two frames can be set equal to each other: $\eta' = y$ and $\zeta' = z$, and since the two events A and B being considered are located on the x axis of the K frame, it is $y = z = 0$, so it results also $\eta' = \zeta' = 0$. Being the geometrical locations of the two events A and B symmetric with respect to the origin, it is $x_A = -x_B$. The differences between each coordinate that appear in the definition of the characteristic interval (1) thus result: $\Delta x^2 = (x_A - x_M)^2 = (x_B - x_M)^2$, $\Delta y^2 = \Delta z^2 = 0$ and $\Delta t^2 = (t_D - t_A)^2 = (t_D - t_B)^2 = t_D^2$.

In this way the problem reduces to that of finding the relations between the (ε', τ') and (x, t) coordinates. We are therefore looking for the specific form of the transformation of coordinates that gives:

$$(v_c \Delta t)^2 - (\Delta x)^2 = (v_c \Delta \tau')^2 - (\Delta \varepsilon')^2 \qquad (3)$$

The transformation that satisfies this invariance property of the characteristic interval is:

$$\varepsilon' = \frac{x - Vt}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \qquad \eta' = y; \qquad \zeta' = z;$$

$$\tau' = \frac{t - \frac{V}{v_c^2} x}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \qquad (4)$$

and the inverse transformation, from the generalized space-time coordinates of K′ to K, is:

$$x = \frac{\varepsilon' + V\tau'}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \qquad y = \eta'; \qquad z = \zeta';$$

$$t = \frac{\tau' + \frac{V}{v_c^2} \varepsilon'}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \qquad (5)$$

When light signals propagating in vacuum are chosen as synchronization signals, the characteristic speed is equal to the speed of light in vacuum, $v_c = c$, and the above transformation of coordinates coincide with the Lorentz transformations.

It can be noted that the transformations (4), and the corresponding inverse (5), are not defined for $V = v_c$, whereas for $V > v_c$ the two generalized coordinates ε' and τ' become complex, having a non null imaginary part. Even in this case, these complex coordinates still preserve the invariance of the characteristic interval s_c, as it can be verified by direct substitution of (4) into equation (3). The invariance of the characteristic interval is therefore verified for all values of $V \neq v_c$ and it holds true for any finite value of the characteristic speed v_c of the selected isotropic signal used to synchronize the clocks into the stationary frame K. When the speed V of the K′ frame is very small compared to the characteristic speed v_c of the selected synchronization signals, the speed ratio V/v_c tends to zero and the transformation of coordinates of eq. (4) tends, in the limit $V/v_c \to 0$, to the Galilean one:

$$\varepsilon' = x - Vt; \quad \eta' = y; \quad \zeta' = z; \quad \tau' = t; \quad (6)$$

Let us now consider two simultaneous events A and B into frame K occurring at two generic points of the space and let (x_A, y_A, z_A) and (x_B, y_B, z_B) be the coordinates of the geometrical locations of the two events and $t_A = t_B$ the corresponding time of occurrence. According to the definition of simultaneity given before, the synchronization signals emitted by A and B will reach simultaneously, at time $t_M > t_A$, the midpoint M of segment AB, with M having coordinates:

$$(x_M, y_M, z_M) = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}, \frac{z_A + z_B}{2}\right)$$

The characteristic intervals, into frame K, between the two simultaneous events A and B being considered and the event of detection of the arrival of their synchronization signals at the midpoint M are given by:

$$s_A^2 = (v_c \Delta t_A)^2 - L_A^2$$

$$s_B^2 = (v_c \Delta t_B)^2 - L_B^2 \qquad (7)$$

where:

$$\Delta t_A = (t_M - t_A); \quad \Delta t_B = (t_M - t_B)$$

$$L_A^2 = (x_A - x_M)^2 + (y_A - y_M)^2 + (z_A - z_M)^2$$
$$L_B^2 = (x_B - x_M)^2 + (y_B - y_M)^2 + (z_B - z_M)^2$$

Since in the stationary frame K the two points A and B are located symmetrically with respect to the midpoint of the segment, it is $L_B = L_A = L/2$, where L is the length of segment AB, and since the time of emission of the signals is the same, $t_A = t_B$, it follows that $\Delta t_A = \Delta t_B$ and therefore it results:

$$s_A = s_B \tag{8}$$

This result shows that, in the stationary frame K, two distinct events A and B are simultaneous when they are separated by the same characteristic interval from the event of detection of the arrival of their synchronization signals at the midpoint of segment AB. Expression (8) can thus be considered as the mathematical formulation of the clock synchronization method described in the previous section, based on isotropic signals propagating with characteristic speed v_c into frame K.

The above considerations can be repeated for any other finite value of the parameter v_c, leading always to the same result expressed by relation (8), thus showing that simultaneity is invariant with respect to the propagation speed v_c of the selected synchronization signals.

Since the characteristic interval s_c is invariant under the generalized coordinate transformation defined above, it follows that two events that are simultaneous in the stationary frame K are simultaneous also in the moving frame K', according to the same criterion of equality of the emission-detection characteristic intervals. This invariance of simultaneity holds true for any value of the relative speed V between the two moving frames and for any kind of physical signals selected to synchronize the clocks, so it holds true for any finite value of their characteristic speed v_c and is thus consistent with the definition of simultaneity given in the previous section. It appears therefore that simultaneity is an absolute characteristic of the events, that can be defined and assessed univocally by different observers that are in a state of uniform relative motion one with respect to the other, by applying the same general criterion of equality of the emission-detection characteristic intervals.

Let us now calculate, in frame K', the generalized coordinate τ' of the two simultaneous events A and B. According to (4), we have:

$$\tau'_A = \frac{t_A - \frac{V}{v_c^2} x_A}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{9}$$

and

$$\tau'_B = \frac{t_B - \frac{V}{v_c^2} x_B}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{10}$$

Taking into account that $t_B = t_A$ it is possible to rewrite τ'_B as follows:

$$\tau'_B = \frac{t_A - \frac{V}{v_c^2} x_A - \frac{V}{v_c^2}(x_B - x_A)}{\sqrt{1 - \frac{V^2}{v_c^2}}} =$$

$$= \tau'_A - \frac{V}{v_c^2} \frac{(x_B - x_A)}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{11}$$

Therefore it appears that the coordinate τ' of two simultaneous events A and B, evaluated in the moving frame K', has not the same value for the two events, being, in general:

$$\tau'_B \neq \tau'_A \tag{12}$$

In other words, the generalized coordinate τ' cannot be used to assess the simultaneity of events in the moving frame K', since two simultaneous events A and B turn out as being characterized by a different value of the corresponding generalized coordinate τ'. The only particular case for which $\tau'_B = \tau'_A$ occurs when $x_B = x_A$, i.e. when the two simultaneous events A and B are located in a plane orthogonal to the direction of the velocity vector V of frame K', and therefore in a plane orthogonal to the x axis of the K frame, being it parallel to V by construction.

We can now consider the governing laws that describe the isotropic propagation of the signals selected to synchronize the clocks. In particular let us consider the case of the tensioned ideal string. It is known that for small amplitudes, the transverse displacement u of the points of the string is determined by the solution of the d'Alembert equation:

$$v_s^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0 \; with \; u = u(x,t) \tag{13}$$

where $v_s = \sqrt{N/\lambda}$ gives the speed of propagation of the perturbations along the string as a function of the applied axial tension N and linear mass density λ of the string.

As discussed in the first paragraph related to the phenomenological description of the Galilean Principle of Relativity, any experimental determination of the string properties and of its response will give identical results when the same characterization tests are repeated into two different laboratories that are uniformly translating one with respect to the other. Therefore, the string behaviour will be represented by the same governing laws in both cases, i.e. the same equation (13) will be determined both by the observer of the stationary laboratory and by the observer of the moving one, and the propagation of the perturbations along the string will remain isotropic and will have the same characteristic speed v_s in both reference frames.

The situation is different if we consider, into a given laboratory, a moving observer with its associated moving reference frame. Let K be a reference frame stationary with the laboratory, that is therefore stationary also with respect to the string, and let K' be another reference frame translating with velocity V parallel to the string axis. For this frame, which is in relative motion with respect to the string, the perturbations on the string will be no more propagating isotropically, their speed being greater than v_s along one direction and lower than v_s in the opposite direction. Correspondingly, also the governing laws of the string will change when expressed into the moving frame K'. In this case therefore the governing law of the string, expressed by equation (13), should not be invariant in the transformation from the stationary frame K to the moving frame K'.

Let us now see how the wave equation (13) transforms in the moving frame K' when the generalized coordinates with characteristic speed v_c, as defined by (4), are used. The d'Alembert equation (13) can be reformulated as:

$$\left(v_s\frac{\partial}{\partial x}+\frac{\partial}{\partial t}\right)\left(v_s\frac{\partial}{\partial x}-\frac{\partial}{\partial t}\right)u=0 \qquad (14)$$

and the two generalized coordinates (ε', τ') can be written in a more compact form as:

$$\varepsilon' = \gamma_c(x - Vt), \quad \tau' = \gamma_c(t - Vx/v_c^2) \qquad (15)$$

where $\gamma_c = 1/\sqrt{1 - V^2/v_c^2}$. This change of variables can be applied to the d'Alembert equation by taking into account that:

$$\frac{\partial}{\partial x} = \frac{\partial \varepsilon'}{\partial x}\frac{\partial}{\partial \varepsilon'} + \frac{\partial \tau'}{\partial x}\frac{\partial}{\partial \tau'} = \gamma_c\left(\frac{\partial}{\partial \varepsilon'} - \frac{V}{v_c^2}\frac{\partial}{\partial \tau'}\right)$$

$$\frac{\partial}{\partial t} = \frac{\partial \varepsilon'}{\partial t}\frac{\partial}{\partial \varepsilon'} + \frac{\partial \tau'}{\partial t}\frac{\partial}{\partial \tau'} = \gamma_c\left(\frac{\partial}{\partial \tau'} - V\frac{\partial}{\partial \varepsilon'}\right)$$

In this way, the wave equation (14) takes the form:

$$\gamma_c^2\left(A\,\frac{\partial^2 u}{\partial \tau'^2} + B\,\frac{\partial u}{\partial \varepsilon'}\frac{\partial u}{\partial \tau'} - C\,\frac{\partial^2 u}{\partial \varepsilon'^2}\right) = 0 \qquad (16)$$

where the three terms A, B and C are given by:

$$A = \left(1 - \frac{V^2 v_s^2}{v_c^4}\right)$$

$$B = 2V\left(\frac{v_s^2}{v_c^2} - 1\right)$$

$$C = (v_s^2 - V^2)$$

From these expressions it can be noted that when $v_s = v_c$ it results $B = 0$ and $A = 1/\gamma_c^2$; $C = v_c^2 - V^2$. Substituting these terms into (16), gives:

$$v_c^2\frac{\partial^2 u}{\partial \varepsilon'^2} - \frac{\partial^2 u}{\partial \tau'^2} = 0 \; with \; u = u(\varepsilon', \tau') \qquad (17)$$

Therefore, when the parameter v_c contained into the generalized coordinate transformation (4) is equal to the speed of propagation v_s of the phenomenon being described, the corresponding equation governing the evolution of the perturbations along the string is invariant in the passage from the stationary frame K to the moving frame K'. This invariance property, however, is no longer verified when the characteristic speed v_c used in the coordinate transformation is different from v_s. In this case, in fact, the term B is not null, and the equation resulting from the change of coordinates has no longer the same form of the original wave equation.

This peculiar invariance property of the generalized coordinates is valid not only for the monodimensional case of the string equation that has been considered here, but also for the tridimensional case of the wave equation that has the same form of (13). Also in this more general case, the invariance of the governing equations is satisfied only when the parameter v_c that appears in the definition of the coordinate transformation has the same value of the characteristic speed of the isotropically propagating phenomenon being represented. If some of the physical properties characterizing the phenomenon changes, thereby changing the corresponding physical speed of propagation, then the wave equation will take a different form in the passage from the stationary to the moving observer and to the corresponding frame of reference and its solutions will be different along different directions.

The situation is similar when we consider light signals as the means to synchronize the clocks. In this case the governing laws associated to the underlying physical phenomenon are the set of Maxwell equations. In vacuum these equations represent the isotropic propagation into space of the influence generated by the electrical charges and currents, which are the sources of the electromagnetic fields. The speed of propagation of the solutions of the Maxwell equations is equal to the speed of light, which in vacuum is given by $c = \sqrt{\varepsilon \nu}$. When this value is used as characteristic speed parameter, $v_c = c$, the generalized coordinate transformations (4) reduce to the Lorentz transformations, and the set of Maxwell equations result invariant in the passage from a stationary frame K to a moving frame K' under such transformation of coordinates. However, if we consider the same electromagnetic phenomenon in the presence of a homogeneous and isotropic transparent medium different from vacuum, it is known that in this case the speed of light takes a lower value with respect to the vacuum case, it being $v_c = c_o = \sqrt{\varepsilon_o \nu_o} = c/n < c$, where $n \geq 1$ is the index of refraction of the medium. In presence of such a medium the propagation of light is still represented by the Maxwell set of equations, now containing the values of the dielectric ε_o and magnetic permeability ν_o of the specific medium being considered. For this situation, similarly to the case of the wave equation for the string, the Maxwell set of equations in a medium different from vacuum will be no longer invariant with respect to the Lorentz coordinate transform, given the difference between the value of the characteristic speed, the speed of light c, that appears in the Lorentz coordinate transformation with respect to the value of the propagation speed that appears in the set of Maxwell equations, which for a medium different from vacuum is $c_o = c/n \neq c$.

Let us now evaluate the relationship between the generalized velocity $\mathbf{w}'$, expressed into frame K' on the basis of the coordinate transformation defined by (4), and the expression of the velocity into frame K. This can be done by evaluating, from eqs. (5), the differentials:

$$dx = \frac{d\varepsilon' + V d\tau'}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \quad dy = d\eta'; \quad dz = d\zeta';$$

$$dt = \frac{d\tau' + \frac{V}{v_c^2} d\varepsilon'}{\sqrt{1 - \frac{V^2}{v_c^2}}}; \tag{18}$$

Through the definition of the velocity in the stationary frame:

$$\mathbf{v} = (\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt})$$

and, by analogy, of the generalized velocity in the moving frame:

$$\mathbf{w}' = (\frac{d\varepsilon'}{d\tau'}, \frac{d\eta'}{d\tau'}, \frac{d\zeta'}{d\tau'})$$

it follows that

$$v_x = \frac{w'_x + V}{1 + \frac{V w'_x}{v_c^2}};$$

$$v_y = \frac{w'_y \sqrt{1 - (V^2/v_c^2)}}{1 + \frac{V w'_x}{v_c^2}};$$

$$v_z = \frac{w'_z \sqrt{1 - (V^2/v_c^2)}}{1 + \frac{V w'_x}{v_c^2}}; \tag{19}$$

The expression of the generalized velocity $\mathbf{w}'$ into frame K' is found by inverting the above relations, giving:

$$w'_x = \frac{v_x - V}{1 - \frac{V v_x}{v_c^2}};$$

$$w'_y = \frac{v_y\sqrt{1-(V^2/v_c^2)}}{1 - \frac{V v_x}{v_c^2}};$$

$$w'_z = \frac{v_z\sqrt{1-(V^2/v_c^2)}}{1 - \frac{V v_x}{v_c^2}}; \tag{20}$$

From these expressions it turns out that when the magnitude of the velocity in frame K is equal to the characteristic speed, i.e. when $|\mathbf{v}| = v_c$, it follows that also the magnitude of the generalized velocity in the moving frame K' has the same value: $|\mathbf{w}'| = v_c$. In fact, considering the case $v^2 = v_x^2 + v_y^2 + v_z^2 = v_c^2$ and evaluating the magnitude of $\mathbf{w}'$ from equations (20), it results:

$$|\mathbf{w}'|^2 = (w'_x)^2 + (w'_y)^2 + (w'_z)^2 =$$

$$= \frac{(v_x^2 + v_y^2 + v_z^2) - 2v_x V + V^2 - \frac{V^2}{v_c^2}(v_y^2 + v_z^2)}{(1 - v_x V/v_c^2)^2} =$$

$$= \frac{[v_c^2 - 2v_x V + \frac{V^2 v_x^2}{v_c^2}]}{(1 - v_x V/v_c^2)^2} =$$

$$= v_c^2 \frac{[1 - 2\frac{v_x V}{v_c^2} + \frac{v_x^2 V^2}{v_c^4}]}{(1 - v_x V/v_c^2)^2} = v_c^2$$

The generalized coordinates defined by equation (4) can therefore be considered as an isochronous characteristic speed transformation. Considering the case of particle-like signals used to synchronize the clocks (for example the spring-loaded launcher device considered in the previous section), and applying the characteristic coordinate transformation (4) to calculate the generalized speed of the particles into the moving frame K', it turns out that also these particle-like signals, that propagate isotropically with speed v_c in the stationary frame K, will be propagating isotropically, with the same value v_c of the generalized speed w', also in the moving frame K'. This invariance of the characteristic speed is valid for any finite value of the speed of the specific synchronization signal being considered and when the value of characteristic speed is equal to the speed of light in vacuum, $v_c = c$, the above result corresponds to the invariance of the speed of light under the Lorentz coordinate transformations.

It can be noted that this kind of coordinate transformations do not preserve the invariance of the relative velocity between two physical objects in the passage from a given reference frame, K, another one, K', that is in a state of uniform rectilinear motion with respect to the first one. For this reason, the Lorentz transformations, that are a particular case of eqs. (4) for which the characteristic speed of the synchronization signals is equal to the speed of light in vacuum, i.e. for which $v_c = c$, appear incompatible with the Galilean Principle of Relativity, since any physical phenomenon that depends from the relative velocities of the involved entities will turn out as being different for two different observers, in contrast with the invariance postulated by Galilei in his formulation of the principle.

Let us now consider the case of a signal, particle-like or wave-like, traveling along the x axis of the stationary frame K with constant speed v, that is: $v_x = v$ and $v_y = v_z = 0$. Let K' be a moving reference which is also traveling along the direction of the x axis of K with the same uniform speed v of the signal, that is: $V = v$. According to the Galilean rule of speed composition, the velocity of the signal with respect to such moving frame K' is null, since it is given by $v' = v - V = v - v = 0$ for any value of the common speed v of both the signal and the reference frame. We want now to evaluate the generalized velocity of the signal into the moving frame K' according to the formulas (20) previously established. By putting $v_x = V = v$ and $v_y = v_z = 0$ it results:

$$w'_y = w'_z = 0$$

and

$$w'_x = \frac{v_x - V}{1 - \frac{V v_x}{v_c^2}} = \frac{v - v}{1 - \frac{v^2}{v_c^2}} = v_c \frac{\beta - \beta}{1 - \beta^2}$$

where $\beta = v/v_c$. Therefore it results $w'_x = 0$ $\forall\, v \neq v_c$ whilst in the case $v = v_c$, for which $\beta = 1$, the previous expression gives an undetermined form of the type $0/0$ that can however be evaluated by applying the l'Hopital's rule. Putting $f(\beta) = \beta - \beta$ and $g(\beta) = 1 - \beta^2$ it gives:

$$\lim_{\beta \to 1} w'_x = \lim_{\beta \to 1} v_c \frac{f}{g} = \lim_{\beta \to 1} v_c \frac{f'}{g'} = v_c \frac{0}{-2} = 0$$

Thus, also for the case $v = v_c$ the generalized speed of the signal traveling with speed v, as evaluated by an observer comoving with it at the same speed, $V = v$, is zero. When applied to the case of a light signal, or a photon, traveling in vacuum with velocity $v = c$, this result shows that the generalized speed of a light signal evaluated by a luminal observer, i.e. the speed of light evaluated by a reference frame moving at the same speed of light in vacuum, is null. This result appears therefore in contrast with the postulate of invariance of the speed of light which is at the base of the Special Theory of Relativity[2], since it shows that there is at least one observer, the luminal observer, for which the speed of light, calculated according to the rules determined by the theory itself, is zero instead of being equal to c as required by the postulate.

In summary, the generalized coordinate transformation defined by (4) is characterized by the following peculiar properties in the passage from a reference frame K to another frame K′ that is translating with constant velocity V:

1. it makes invariant the characteristic interval s_c defined by equation (1);

2. it leaves invariant the constitutive laws representing isotropic propagation of a phenomenon having characteristic speed v_c in the stationary frame K;

3. it maintains, for the generalized speed of propagation in frame K′, the same value of the characteristic speed v_c that such phenomenon has in frame K.

The above properties are verified for any finite value of the characteristic speed v_c and correspond, for the case $v_c = c$, to the same properties of the Lorentz transformations that are valid for the propagation of light in vacuum and for the corresponding governing laws as described by the Maxwell equations of electromagnetism. Being valid for any value of the selected characteristic propagation speed v_c, these invariance properties of the generalized coordinate transformation (4) can be considered as a peculiar mathematical characteristics of this type of coordinate transformations, rather than a manifestation of space-time distortion or rather than a specific property associated to a single type of physical phenomenon, i.e. as a specific property of light.

It has been shown above by relation (12) that, in the general case, two simultaneous events do not have the same value of the generalized coordinate τ' which therefore cannot be used as a time identification of the events. This coordinate, instead, can be interpreted in a different way as follows. Let us consider, in frame K, a generic event P occurring at a given point (x, y, z) of the space, and at a given time t and let us consider a second reference frame K′ moving with uniform velocity V along the x axis, and having its origin O' coincident with the origin O of frame K at time $t = 0$. The amount of time needed by the synchronization signal emitted from P in order to reach the $x = 0$ plane of frame K is $\Delta t = x/v_c$. In this same amount of time, the origin of frame K′ will have traveled a distance, along the x axis of frame K, equal to $\Delta x = V\,\Delta t = Vx/v_c$, and a synchronization signal, traveling with characteristic speed v_c in frame K, would take a time interval equal to $\Delta t_c = \Delta x/v_c = Vx/v_c^2$ in order to cover such distance. It is therefore possible to write the expression of the generalized coordinate τ' in the following way:

$$\tau' = \gamma_c\,(t - \Delta t_c) \tag{21}$$

where $\Delta t_c = Vx/v_c^2$ and $\gamma_c = \sqrt{1 - V^2/v_c^2}$. Equation (21) shows that τ' represents a *retarded* (or *advanced*, depending on the sign of the characteristic time delay Δt_c) and *scaled* time coordinate which is a function of the position x of the event along the direction of motion of the moving frame K', of its velocity V, and of the characteristic speed v_c of the specific synchronization signal that has been considered. Looking now to the definition of the generalized coordinate ε' associated to the moving frame K', it turns out that it can be interpreted as a *scaled* version of the position $x' = (x - Vt)$ of the event P along the x axis of frame K', that uses the same value of the non-dimensional scaling factor γ_c that enters into the definition of the generalized coordinate τ', that is:

$$\varepsilon' = \gamma_c \, x' \tag{22}$$

For low values of the speed ratio V/v_c the characteristic delay Δt_c tends to zero and the characteristic scaling factor γ_c tends to one, thus the generalized coordinate transformation reduces to the Galilean one in the limit of low values of the speed V of the moving frame K' with respect to the characteristic speed v_c.

IV. Physical experiences on the speed of propagation of light

In this section two interferometric experiments on light propagation will be examined, comparing the experimental measurement results with the expected outcomes deriving from the Ritz emission theory that is, as already mentioned, fully in agreement with the Galilean Principle of Relativity and with the associated rule of vector sum for the velocities with respect to moving frames. The validity of the Galilean velocity composition can be verified by means of these optical tests since they allow measuring the difference of the speed of light through the analysis of the interference patterns that are generated in the presence of phase differences between independent light beams travelling along different optical paths.

The first, well-known, experience being considered is the Michelson-Morley interferometer that typically has two orthogonal arms. This experiment has been designed to identify the potential dependency of the speed of light from the velocity of the observer. At the time of its first realization it was mainly devoted to investigate the possible effects of the motion of the Earth along its orbit on the so-called luminiferous aether, or simply aether, that was thought as being the propagation medium of light. In this experiment all the optical components of the setup - the beam splitter, the mirrors, the target plane where the fringes can be observed - and the propagation medium of the light, when present, are rigidly transported by the motion of the Earth along its trajectory. Because of its geometrical layout, shown schematically in Figure 2, the area of the optical path is null, therefore the angular motion of the Earth does not produce any shift of the fringes due to the Sagnac effect which is proportional to the product ΩA, where Ω is the component of the angular speed orthogonal to the plane of the optical path and A the corresponding area.

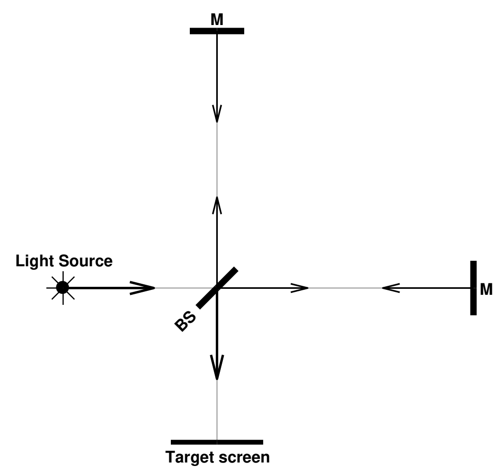

Figure 2: *Schematic layout of the Michelson-Morley interferometer. M indicate the mirrors and BS the beam-splitter*

The results of the experiment, performed under a variety of conditions and in different geographical locations and times of the year, have always revealed no effect on the interference pattern deriving from either the speed or the orientation of the interferometer. The same null results have been obtained both in vacuum and in presence of a transparent medium having an index of refraction greater than one, for which the speed of light is less than c. These null outcomes of the test are fully consistent with the Galilean Principle of Relativity and can be immediately explained just on its basis. In fact, if we imagine to perform a baseline test in a given reference frame obtaining a given baseline fringe pattern, on the basis of the Galilean Principle of Relativity the same exact fringe pattern should be found also when the test is repeated in a laboratory that is moving with constant velocity with respect to the baseline one. Any discrepancy between the test results obtained in the two cases would constitute either a violation of the Galilean Principle of Relativity or an evidence of the existence of a propagation medium for light, the aether, that is not following the same state of motion of the optical components. The experimental evidence gained so far leads to exclude both cases.

The null results of the experiment can be explained immediately by assuming, as in the emission hypothesis of W. Ritz, that light is always emitted and propagated with the same relative speed, equal to c/n, with respect to the optical components of the test setup, along both arms of the interferometer. This conclusion is valid both in vacuum and in presence of a transparent medium, the only difference between the two cases being the actual value of the relative speed of light. For an observer that is at rest with respect to the test apparatus, the light propagation will remain isotropic, having the same speed along the two orthogonal arms of the interferometer also when the test setup is moving with a non-null constant velocity V. On the contrary, an observer that is in a state of uniform relative motion with respect to the test apparatus would notice a non-isotropic propagation of light, with different values of the speed along different directions, in agreement with the Galilean vector sum of the velocity vectors. However, since also the optical components of the test setup would have different velocities with respect to this observer, determined by the same vector sum rule, the calculation of the time taken by the light to go through the optical path along the two arms of the interferometer will give the same results obtained by the observer at rest, and therefore no alteration of the fringe patterns has to be expected from such calculation, in agreement with the experience.

In the Fizeau experience[4], the propagation of light into a stream of water flowing within pipes has been investigated by analyzing the fringe patterns generated at the recombination of two light beams that are counter-propagating in the fluid stream. The analysis of the test results obtained by Fizeau with its original apparatus, shown schematically in Figure 3, led to the following expression for the relative velocity W_F of the light with respect to the stationary system of the laboratory:

$$W_F = w + v(1 - \frac{1}{n^2}); \qquad (23)$$

where v is the velocity of the fluid flowing into the pipes having circular cross-section, and $w = c/n$ is the speed of light into the fluid being utilized for the test, characterized by an index of refraction equal to n when such fluid is stationary.

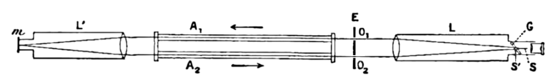

Figure 3: *Original layout of Fizeau's experiment*

We want now to investigate if the experimental result of Fizeau (23) can be obtained by applying the Ritz emission hypothesis and the associated Galilean vector sum of the velocities of light into the propagation medium and the velocity of the fluid. This requires the determination of the actual value of the speed v of the fluid flow to be used in the vector sum formula, since in this kind of experiment the propagation medium used, typically water, is not characterized by a common and uniform state of motion of all its particles inside the volume occupied. The motion of the fluid in fact cannot be represented as a pure rigid body translation with constant speed, therefore there is not just a single value of the velocity for the entire fluid, but rather a velocity field with a distribution that varies from point to point inside the volume of the pipes.

In addition, the specific geometrical layout of the Fizeau test setup introduces several factors that can affect the characteristics of the interference fringes formed at the recombination of the two counter propagating beams, in particular:

1. the area of the optical path is not null, therefore interference fringes can arise also in absence of fluid motion (and actually also without any fluid) due to the Sagnac effect associated to the Earth's rotation;
2. the radial shape of the velocity profile of the fluid motion at the various sections of the pipes;
3. the axial flow speed of the fluid which varies along the pipe length and therefore along the optical path;
4. the non-axial components of the fluid velocity associated to a turbulent regime of the flow.

The Sagnac effect can be considered as a constant bias, since both the angular velocity of the laboratory where the experiment is performed and the area of the optical path, determined by the geometrical layout of the test setup are not varied during the execution of the measures, therefore the product ΩA remains constant.

The radial shape of the axial velocity profile of the fluid flow can cause a distortion of the shape of the incident wavefront of the light beam. For the beam propagating in the same direction of the fluid stream an incident planar wavefront could be deformed in a way similar to that of a plane-concave lens, since the equivalent optical length of the light rays closer to the centerline of the pipes will be shortened by the dragging effect due to the fluid flow more than that of the rays travelling farther from the pipe centerline. Conversely, the wavefront deformation associated to the light beam traveling against the fluid stream should be similar to the one generated by a plane-convex lens. On the recombining plane, the interference pattern generated by the two counter-propagating beams would be affected by the actual shape of the two distorted wavefront and in particular this effect could generate a variation of the fringe spacing if the shape of the radial velocity profile changes as a result of changes of the overall flow rate. This effect should be more pronounced comparing the distortion associated with a laminar flow regime, characterized by a parabolic velocity profile, with respect to that of a turbulent flow regime, where the velocity profile in the central portion of the pipes is more flat. The two different shapes of the velocity profiles for laminar and turbulent regimes are shown qualitatively in Figure 4

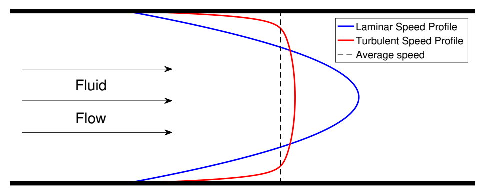

Figure 4: *Laminar vs turbulent velocity profiles inside circular pipes*

The axial velocity of the flow is also not constant along the length of the pipes, and therefore along the optical path traveled by the light beams into the moving fluid. The total amount of dragging effect to be added, or subtrated, to the speed of propagation of the light would be dependent from the integral, across the entire length of the optical path, of all the local values of the axial fluid velocity on every section of the pipes. This calculation is not straightforward, being the actual velocity field quite complex, especially in the transition regions close to the end of the tubes. Using longer tubes could help in reducing the sensitivity of the results to the local effects concentrated at the ends of the tubes, but even along the central straight portion of the pipes the flow regime would reach a stable, fully developed configuration with a constant radial velocity profile distribution, only after some distance along the pipe. Overall, this variation of the speed profile along the length of the pipes will have the effect of reducing the average axial speed seen by the light beam at the center of the pipes, with respect to the value determined at a specific section, typically located towards the exit end of the pipes, where the actual velocity profile measurement is performed and the value of the velocity at the center of the flow is determined.

In the case of turbulent flow regime, which is the actual flow regime used for the measures in the original Fizeau experiment, the velocity field of the fluid flow is characterized by having also radial components of the fluid velocities in addition to the axial ones. These radial components are associated to the presence of fluid vortices, typical of the turbulent flow regime, having different scales, and which can be randomic and non-stationary. In this regime, the motion of the fluid particles with respect to the stationary frame of the laboratory does not corresponds to a pure axial motion along the axis of the pipes, but contains also circular components, due to the vorticity of the flow, with the associated accelerations. Under these conditions the invariance of the physical phenomena asserted by the Galilean Principle of Relativity is no longer applicable, therefore it may be possible that the physical properties of the entities involved in the test are somehow affected by the accelerated state of motion of the particles that constitute the system being observed, thereby changing to some extent the values of their physical characteristics with respect to the corresponding values determined under stationary conditions. In particular, in the case of the Fizeau's experiment, the specific state of motion associated to turbulence could have an impact on the light propagation inside the transparent medium flowing into the pipes. On average it could introduce an additional *'dragging'* term, generated by the circular motion of the fluid into the turbulent vortices, that creates an additional delay of the axial propagation of the light beam. In other terms, the turbulent motion of the fluid, with the associated vortices, can have the effect of reducing the average equivalent propagation speed of the light beam inside the fluid which therefore would have a greater index of refraction in turbulent conditions with respect to the stationary case.

In order to separate this term from the ones associated to the variation of the axial components of the fluid velocity, it can be taken into account by including into the equation an *'equivalent'* index of refraction n^*, which would be dependent on the level of turbulence of the fluid, and would take values greater than the one corresponding to the stationary fluid, i.e. $n^* \geq n$. Being associated to the presence of a turbulent flow regime, such equivalent index of refraction can be expressed as a function of the Reynolds number Re that is used to characterize the level of turbulence of the flow: $n^* = n^*(Re)$. For low values of the Reynolds number, within the laminar range, n^* would be equal to the index of refraction of the stationary fluid, whereas for Reynolds number values greater than the threshold corresponding to the onset of turbulent flow, an increase of n^* with the Reynolds number could be expected. Taking into account of these effects, the expression of the relative speed of light W with respect to the stationary observer, calculated on the basis of the classical Galilean rule of vector sum, can be written in the following form:

$$W = w^* + \bar{v} = \frac{c}{n^*} + \frac{1}{L}\int_0^L v(x)\,dx; \qquad (24)$$

where the first term accounts for the effect of variation of the refraction index, and the integral of the second term is extended to the entire length L of the optical path of each light beam. It appears therefore, in particular on the basis of items 3) and 4) above, that the actual value of the speed of light measured with respect to a stationary observer should be lower than the value predicted by the Galilean formula of speed composition, when that formula is evaluated using the peak value of the fluid speed inside the pipes and the nominal value of the refraction index of the stationary fluid, and this result is consistent with the outcome of the experiment. The actual amount of deviation would depend on the specific characteristics of the experimental setup being considered, in particular for what concerns its hydraulic characteristics and parameters.

Recent repetitions of the Fizeau experiment have highlighted that the effects due to turbulence could be the major contributor to the fringe shift observed as a result of the variation of the fluid flow rate and average velocity. In particular Lahaye *et al.* [5] explicitly mention that for low value of the fluid speed, $\bar{v} < 1\,m/s$, it has not been possible to acquire any valid test point because of the difficulties in getting stable pictures on the digital sensor used to detect the fringes and their variation. Also in a similar work from Maers *et al.*[6] the experimental data of fringe shift versus flow velocity-difference have been measured only for water velocities in the range $0.5 < \bar{v} < 3.6\,m/s$ for which the flow is fully in the turbulent regime, having Reynolds number in the range $12.700 < Re < 91.400$.

The expression of the Reynolds number for the flow into circular pipes is:

$$Re = \frac{\rho D}{\mu}\, v = \frac{\nu}{D}\,\bar{v} \tag{25}$$

where ρ is the density of the fluid, μ and ν its dynamic and kinematic viscosity, D is the pipe diameter and $\bar{v}$ the macroscopic velocity of the fluid flow. According to the previously described assumption we could write the dependency of the equivalent refraction index from the Reynolds number as follows:

$$n^* = \begin{cases} n & if \quad Re < Re_L \\ n + \alpha(Re - Re_T) & if \quad Re > Re_T \end{cases} \tag{26}$$

where α is a constant to be determined, and Re_L and Re_T represent, respectively, the Reynolds numbers corresponding to the end of the laminar flow regime and to the onset of the turbulent one. Considering velocities of the fluid flow in the turbulent range, $\bar{v} > v_T$, it is therefore possible to put the expression of the equivalent index of refraction of the turbulent fluid in the form:

$$n^* = n(1+\delta)$$

where $\delta = \delta(Re) = \frac{\alpha(Re - Re_T)}{n} \ll 1$.

The evaluation of the resultant speed of light into the moving turbulent flow using expression (24) would require calculating the integral of all the local axial velocities of the fluid along the optical path, but this in turn would require the precise knowledge of the flow field in each point into the pipes, which is not available. Due to this ignorance of the detailed flow field, it will be assumed, as stated also in[6], that the velocities of the fluid are constant in the straight sections of the tubes through which the light beams travels, and have a radial profile typical of a turbulent regime. In this way the expression of the resultant speed of light becomes:

$$W = \frac{c}{n(1+\delta)} + \bar{v}; \tag{27}$$

Taking into account that $\delta \ll 1$, it is possible to expand the first term of (27) into powers of δ. Making then use of the definition of δ and truncating the expansion to first order it results:

$$W = \frac{c}{n}(1 - \delta + \delta^2 ...) + \bar{v} \simeq \frac{c}{n}(1 - \alpha\frac{\Delta Re}{n}) + \bar{v} =$$

$$= \frac{c}{n} - \alpha\frac{c}{n^2}\Delta Re + \bar{v}$$

where $\Delta Re = (Re - Re_T) = \frac{\nu}{D}(\bar{v} - v_T)$. Substituting this into the previous equation gives:

$$W = \frac{c}{n} + \bar{v} - \alpha\frac{c}{n^2}\frac{\nu}{D}(\bar{v} - v_T) \tag{28}$$

Putting now

$$\alpha = \frac{D}{\nu c} \tag{29}$$

the expression of the light speed with respect to the stationary observer finally becomes:

$$W = \frac{c}{n} + \bar{v}(1 - \frac{1}{n^2}) + \frac{1}{n^2}v_T \tag{30}$$

that corresponds to the expression obtained by Fizeau taking into account that the term v_T/n^2 can be neglected, being much smaller, by several orders of magnitude, than the other constant term c/n.

The above derivation shows that it is possible to provide an interpretation based on the Galilean vector sum of velocities of the experimental results obtained by Fizeau, without the need to invoke any space-time distortion.

The proposed approach is based on the hypothesis that the index of refraction of the fluid is altered by the turbulent flow regime. This hypothesis could be verified by further experimental investigations of the optical properties of fluids under turbulent flow regimes, or by a theoretical analysis that would however require to have a very detailed model representing the complex velocity field of the fluid under such flow regime.

V. A test case for the velocity composition rule

In this section a test case is proposed to investigate the validity of the Galilean velocity vector addition rule versus the relativistic one that derives from the Lorentz transformations. The test is based on the analysis of the phenomenon of stellar aberration, i.e. on the observed variation of the position of the celestial objects as a function of the motion of the observer and of its velocity, motion that coincide with that of the Earth along its orbit in the case of a terrestrial telescope. Being the two formulas for the composition of the velocity of the light with the velocity of the observer different, the expected variation of the position of the star evaluated by means of the relativistic rule is different from that obtained with the classical one, and the amount of the difference depends on the value of the ratio of the speed of the observer with respect to the speed of light. Since the orbital velocity of the Earth is about 10^4 times smaller than c, such differences are very small and their analysis therefore requires very accurate measurements of the observed position of the celestial objects in order to resolve the differences between the two cases.

Let us consider the light coming from a very far celestial source, such that the corresponding wavefront can be considered planar over the entire area of the Earth's orbit. For an observer at rest into the center of mass of the Solar system the position of this source is fully characterized by two angles which can be expressed as the in-plane azimuth angle and out-of-plane elevation angle with respect to the plane of the Earth's orbit (ecliptical plane).

Let $\mathbf{V}$ be the velocity vector describing the motion of an observer that is moving into the ecliptical plane. Let $\mathbf{c}$ be the vector defining the velocity of propagation of the light with respect to the stationary frame, and let us consider a moving reference frame having its x axis aligned with the direction of the velocity vector $\mathbf{V}$ of the observer and the y axis lying into the plane formed by the direction of the incoming light and $\mathbf{V}$. The resultant vector $\mathbf{c}'$ that defines the apparent position of the light source for the moving observer, will also lie into the xy plane according both to the Galilean vector-sum rule and to the relativistic velocity-composition rule. However, the observed variation of the angle of incidence, i.e. the amount of *aberration*, is different in the two cases. It can be calculated by applying the two velocity composition rules and focusing the analysis on the x and y components of the vectors.

Let us define, in the reference frame of the Sun, the direction of the light source by the angle θ that the incoming light vector makes with the direction of the velocity of the observer. Let v by the speed of the observer, which is assumed to be directed along the positive direction of the x axis of the observer's reference frame, and $\beta = v/c$ be the ratio of the observer speed with respect to the speed of light. Let us indicate with θ' the aberrated direction of the source as seen by the moving observer. The relationship between the angles θ and θ', derived respectively from the classical vector sum and from the relativistic velocity composition rule, is given by the following two exact trigonometric expressions:

$$sin(\theta - \theta'_G) = \beta \frac{sin(\theta)}{\sqrt{1+\beta^2+2\beta cos(\theta)}} \tag{31}$$

and

$$\begin{aligned} sin(\theta - \theta'_R) = {} & \beta \frac{sin(\theta)}{1+\beta cos(\theta)} \\ & + \beta^2 \frac{sin(2\theta)}{2\big(1+\beta cos(\theta)\big)\big(1+\sqrt{1-\beta^2}\big)} \end{aligned} \tag{32}$$

For very small values of the observer speed, compared to the speed of light, the difference between the two angles θ and θ' is also very small, therefore it is possible to determine the solution of the above expressions by approximating the sine function with its argument, $sin(\theta-\theta') \simeq (\theta-\theta')$, thus giving:

$$\theta'_G = \theta - \beta \frac{sin(\theta)}{\sqrt{1+\beta^2+2\beta cos(\theta)}} \tag{33}$$

and

$$\theta'_R = \theta - \beta \frac{sin(\theta)}{1+\beta cos(\theta)} - \beta^2 \frac{sin(2\theta)}{2\big(1+\beta cos(\theta)\big)(1+\sqrt{1-\beta^2})} \tag{34}$$

The two expressions (33) and (34) allow to calculate the expected apparent position θ' of the light source for the moving observer when the corresponding position θ of the celestial object into the stationary frame is known.

Conversely, in order to perform the calculation of the *un-aberrated* position of the source starting from the one observed into the moving frme, it is necessary to use the inverse relationships between θ and θ' that are given by:

$$\theta_G = \theta' + \beta sin(\theta') \tag{35}$$

and

$$\theta_R = \theta' + \frac{\beta sin(\theta')}{1-\beta cos(\theta')} + \frac{sin(2\theta')}{2} \frac{\sqrt{1-\beta^2}-1}{(1-\beta cos(\theta'))}$$

When $\beta \ll 1$ this last expression can be rewritten as a power series of β truncated to the term of second order, giving:

$$\theta_R \simeq \theta' + \beta sin(\theta') + \frac{1}{4}\beta^2 sin(2\theta') \tag{36}$$

The comparison of equations (35) and (36) shows that the reconstructed position of the light source calculated using the relativistic formula differs from the the one obtained from the Galilean vector sum by a term which is quadratic into β. For a given value of v, the amplitude of this term depends on the angle between the incident light and the direction of the velocity vector of the observer, being maximum when $|sin(2\theta')| = 1$, therefore when $\theta' = \pi/4 + k\pi$, and being null when the observer velocity forms a right angle with respect to the direction of the incident light.

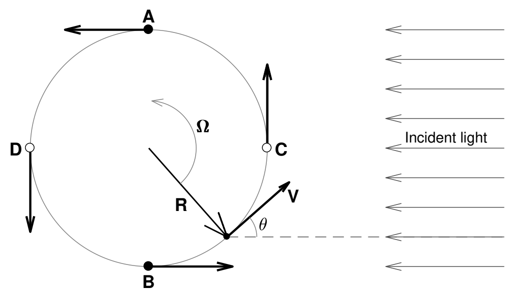

Figure 5: *Orbiting observer with complanar light source*

Let us now consider the case of an observer moving around the Sun with constant angular velocity Ω on a circular orbit having radius R, and of a distant light source located into the same plane of this orbit and stationary with respect to the Sun, as shown in Figure 5. The vector of the observer velocity always lies into the plane of the orbit, therefore in this case the aberration of the incoming light produces, for such moving observer, an apparent motion of the source which is also always lying into the same plane of the orbit. For this orbiting observer the stationary light source thus shows an apparent oscillation of its position along an horizontal line parallel to the plane of the orbit and characterized by the same time period of the orbit.

It is possible to identify four notable locations along the orbit which are significant because of their peculiar properties with respect to the aberration of the source. In the two positions labeled **A** and **B** the velocity of the observer is parallel to the incident light, therefore when the moving observer is in these points of the orbit there is no aberration of the incoming light and the observed position of the star coincides with the one observed into the stationary frame of the Sun. The position of the celestial object observed in these two points can therefore be taken as a reference position, since it requires no calculation in order to remove the aberration term.

Conversely, when the moving observer is in the two locations labeled **C** and **D**, its velocity is orthogonal to the direction of the incident light. In these two locations there is the maximum aberration of the apparent position of the star. However, the value of the aberration term is the same for both the classical and the relativistic rule. Therefore, the calculation of the un-aberrated position of the light source, by means of equations (35) or (36) leads to the same result for both the classical and the relativistic rule. In the particular case of a stationary source considered here, the position of the source calculated by the moving observer located in these two points results coincident with the position observed at locations **A**, **B**.

For any other point of the orbit, the un-aberrated position of the source calculated by means of the classical rule will be different from that obtained from the relativistic formula, and the maximum difference between the two results will occur when the moving observer is at the midpoints between **A**,**B** and **C**,**D**, i.e. at an azimuth angle along the orbit of $\psi = \pi/4 + k\pi/2$. Assuming a stationary source, since the angle between the light direction and the velocity of the observer is $\theta = \Omega t$, one of the two computed results will produce an harmonic oscillation of the horizontal position of the celestial object, having amplitude equal to $\beta^2/4$, and with period equal to one half the period of the observer's orbit. Such peculiar behaviour, characterized by a twice per revolution oscillation that constitutes its specific signature, represents an artifact of the resulting calculated source position, artifact which is due to the inconsistency of the analytical formula used with respect to the actual rule followed by the physical phenomenon.

Let us now consider the case of a terrestrial observer and let's approximate the Earth's orbit with a circle of radius $R = 150x10^6\,km$, and period T equal to one year. In this case the orbital speed is constant and its value is $v \simeq 30\,km/s$, which gives $\beta \simeq 10^{-4}$.

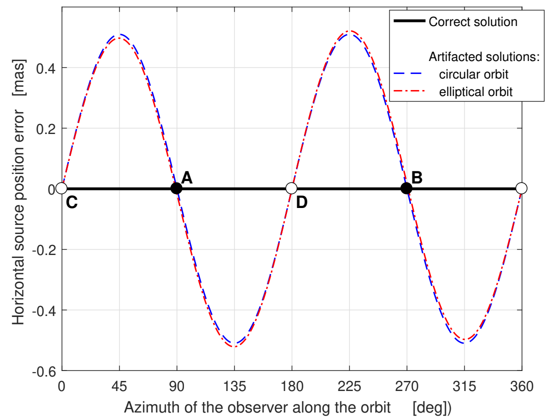

Figure 6: *Comparison of the un-aberrated position of the light source calculated by means of the two different velocity composition rules for an Earth based observer*

With these values of the orbital parameters the two resulting curves of the calculated horizontal position of the source, deriving from the application of equations (35) or (36), are shown in Figure 6. In this figure, also the resulting artifacted solution calculated taking into account the elliptical shape of the Earth's orbit is presented. Due to the low eccentricity of the actual orbit, the deviations of these results from the reference case of a circular trajectory are very small, as shown in the graph that has been calculated considering a celestial object aligned to the major axis of the ecliptic.

The values of the un-aberrated position of the source corresponding to the four notable orbital locations **A**,**B** and **C**,**D** are indicated in the figure with the same markers used in the previous figure. Both the correct and the artifacted curves pass through points **A** and **B**, since for these locations there is no aberration at all and the value of the horizontal position of the celestial object is given directly by the observed position. Both curves also give the same results for locations **C** and **D** where the velocity of the observer is orthogonal to the incoming light direction. [3]

The above described artifact, characterized by its twice per revolution frequency content, must be present in either the classical or the relativistic computed results, and has the same specific signature characteristics for any observed stationary source lying into the orbital plane, with almost the same amplitude of oscillation and with the same frequency content, independently from the specific celestial object or the specific region of the electromagnetic spectrum being observed.

When the celestial object being analyzed does not lie into the orbital plane there will be also an aberration contribution to the out-of-plane position of the source. Considerations similar to those discussed for an in-plane source apply also to this more general case: the vertical component of the calculated position of the source will contain a twice per revolution spurious term in either the classical or the relativistic results. The amplitude of the artifacted vertical component is null when the celestial object is located in the orbital plane, it then increases with the out-of-plane elevation of the source, reaching a maximum for an elevation angle of $\pi/4$, for which the term $\beta^2 sin(2\theta')$ is maximum. For elevations greater than $\pi/4$ the amplitude of the vertical spurious term will then decrease again and will become zero for circumpolar objects, for which also the in-plane component vanishes.

The presence of a twice per revolution frequency term into the computed results of the un-aberrated position of stationary celestial objects is therefore a general characteristics, a specific signature, that allows to identify the incorrect velocity composition rule between the two that have been analyzed.

VI. Conclusions

In the previous sections it has been shown that simultaneity of events can be assessed in a unique and consistent way by using a general method of clock synchronization that does not necessarily require the use of light signals. By using this method two events that are simultaneous for one observer result simultaneous also for another observer that is moving with respect to the first one. This shows that the concept of simultaneity is independent from the state of motion of the observer and from the specific clock synchronization signal that has been selected, and such absolute nature of simultaneity allows to introduce a definition of time which is common for all observers.

The above considerations have led to an alternative physical interpretation of the Lorentz transformation of coordinates and suggest that some interferometric experiments on light propagation can be explained without invoking the space-time deformation assumed by the Theory of Relativity, and by applying, instead, the Ritz emission theory[3] which assumes that light is always emitted with the same relative speed c with respect to its source.

Finally, a test case to discriminate between the Galilean and the Relativistic velocity composition rules has been proposed. The test is based on the analysis of the aberration of the light coming from stationary celestial objects as perceived by an orbiting observer, and on the different results obtained by using the two different velocity composition formulas to remove the aberration term from the observed position of the various light sources of the sky. In order to be applied to measured data, the comparison requires that the observed position of the sources is determined with high accuracy, since the differences that have to be investigated are of the order of milli-arcseconds, a level of accuracy that should be achievable by the most advanced large ground telescopes or space based astrometric instruments like, for example, the Gaia scientific satellite.

Should the outcome of the test be in favour of the classical Galilean velocity vector sum, this could constitute a supporting element to reconsider the validity of Ritz emission theory in place of the Special Theory of Relativity. Despite having radical differences in

[3] In the general case of a non stationary source, the corresponding computed value of the horizontal position of the object evaluated at **C** and **D** could differ from the one corresponding to the reference locations **A** and **B**.

their fundamental assumptions, the two theories share some important aspects that marked a sharp distinction from the approach previously adopted for the analysis of electromagnetic phenomena and for classical mechanics. Regarding the propagation of light both theories negate the existence of the aether, whilst for what concerns mechanics and the dynamics of motion of bodies, in both theories the interactions between non-coincident physical entities are not instantaneous as it was assumed in the Newtonian approach. Because of the assumption of instantaneous action at distance, the equations of motion of classical Newtonian mechanics contain, as stated by L. Landau[7], "a certain degree of imprecision". The removal of the hypothesis of instantaneous action at distance, which is inherent into the action-reaction principle when applied to physical entities having a non-null geometrical separation between them, allows both theories to provide the correct predictions of the precession of the motion of the perihelion of Mercury. It may be possible, therefore, that also other experimental observations that have been considered as being in agreement with the outcomes of the Theory of Relativity could find an alternative interpretation not based on the concept of space-time deformation.

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