A New Study on Unification of Electromagnetic Field and Gravitational Field

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Abstract: We specified the difficulties of Maxwell's electromagnetic theory and Einstein's gravitational theory in detail, established a new unified theory of field including consistent electromagnetic theory and gravitational theory on the basis of new starting postulates, and applied these theoretical methods to quantum electrodynamics. We accepted a new geometrical space in which metric tensor and all main physical functions becomes implicit function, called "KR space" conforming to our starting postulates, and normalization of implicit functions in order to connect all physical functions defined in this space with real world, experimental measures. As a result, we naturally unraveled problem of radiation reaction, a historical difficult problem of Maxwell-Lorentz electromagnetic theory, built new quantum electrodynamics without renormalization procedure and also predicted some new theoretical consequences which could not find in traditional theories.

Keyword：Unified theory of field, Gravitational field, Electromagnetic field, KR space, Breaking of gauge symmetry, Quantum electrodynamics, Classical field

Contents

Introduction

The classical theory of fields consists of two parts, i.e. Maxwell's theory of electromagnetic field and Einstein's theory of gravitational field (theory of general relativity). Moreover, modern theory of quantum fields stands on the basis of classical theory of fields. This paper is aimed at finding out the internal relation of electromagnetic field and gravitational field, establishing new classical theory of fields with unification of two fields and, based upon it, rebuilding several main things of quantum theory of fields.

For evolution of this theory, the followings are regarded as main principles.

(1) The conservation law of energy-momentum

As a natural fact, this law is, even at any case, an indestructible main law of physics and then other principles (for example, gauge invariance principle, equivalence principle in Einstein's general theory of relativity, etc.), in spite of their importance, possess real meaning only in case of satisfying the law of conservation of the energy and momentum. Here, another important thing for argument of conservation law of the energy-momentum is that physical quantities that characterize energy and momentum should possess real physical meaning in any theory. That is to say, real physical meaning of energy and momentum discussed in a theory is a necessary and enough condition for establishment of conservation formula of the energy and momentum.

(2) The finiteness of physical quantity

Physical quantity that characterizes a finite material system should be always, at any case, finite. Supposing that physical quantity of finite material system is infinite, it is put beyond consideration of physics based upon quantitative analysis. Moreover, this is a premise for establishment of conservation law of energy-momentum. In fact, conservation is meant by conservation of finite quantity and if a finite material system has infinite energy, conservation of energy loses its meaning.

(3) The consistency of theory

The principles, axioms consisting of basis of theory, conclusions and laws following from them must not be contradictory each other. This, in a word, is rule of logics to be necessarily observed and introduced in all theories themselves.

(4) The principle of correspondence in physics

According to the principle of correspondence, a new theory should involve some former-old theory as an approximate form or special form. Very natural is that theory established in opposition to principle of correspondence is disqualified as a theory of physics. However, seeing through the present classical theories of fields, we can find several problems that the above-mentioned principles are not enough embodied. Actually, as well known, there are several contradictory problems such as the divergence of energy of electrostatic field and radiation damping in Maxwell's theory, the divergence of scattering matrix in quantum electrodynamics, and absence of physical meaning of energy-momentum tensor of gravitational field in Einstein's general theory of relativity (GR), etc. Until now, for solutions to these divergences and the unification of electromagnetic field and gravitational field, string theory and superstring theory have been widely studied. These theories, concluding that occurrence of infinite quantities is rooted in taking a particle to be a point, were built and studied, based on the starting point that a particle has such finite size as a string but not a point. However, in spite of expending so much time and efforts on these studies, the result is not so satisfactory. At present, many scientists think that the number of string theory known until now are so many and there has not yet been a confirmable theory to be perfect through strict experimental verification, moreover the theories involve some drawbacks. About this, Mandel Sachs described as follows; "Unfortunately, after many years of theoretical studies, the string theory has not yet yielded a mathematically consistent (finite) quantum field theory of matter, which was its original purpose, nor has successfully predicted any observable facts." [1]

A basic reason which the string theory gives these unavoidable problems consists in that the string theory only presented a mathematical form of "string", attempting to fit the theories to mathematical form, without finding a new physical principle that underlies in removing divergence of physical quantities and clarifying internal relation of electromagnetic-gravitation field. In history of physics, appearance of new physical content brought about the birth of new mathematical form compatible to it and in this case, the harmonious combination of new content and form gave birth to miraculous physical conclusions. For example, the principle of invariance of velocity of light, one of starting ideas in Einstein's special theory of relativity led to the introduction of Minkowski space and with appearance of equivalence principle in GR was introduced four dimensional Riemann space, unprecedented in history of physics.

By contrast, for purpose of building of a new physical theory, introduction of new mathematical form about behavior of matter require to discover and receive the new physical content (or principle) which underlies and backs up it. But, though string theory presented new existence form (mathematical form) of matter called "*string*", it has not new physical content (physical principle) which is compatible to it and underlie it. Consequently, while string theory attached importance to mathematical form of string and mathematical formalism with absence of physical principle, it resulted in unavoidable inconsistency and difficulties. That the number of string theories is so many is also caused by not discovering a unified physical principle that underlies model of string. This, in a word, shows that string theory is a not-closed theory without unification between content and form.

Our theory was built on the basis of the starting idea that all divergence does not occur by taking a particle to be a point but is incurred from the difficulties and limitations of Maxwell's theory and General Relativity (GR). That is why our theory is quite different from string theory or superstring theory at starting point.

We, evolving theory based on the above-mentioned principles, obtained following conclusions.

First, On the basis of the starting idea that the total energy of free particle and fields created by it is equal to mc^2 *(presented in sect. 1), we found out a unified Lagrangian of electromagnetic field and gravitational field and gave theoretical predictions about experimental effects relevant to the dependency of electric field on gravitational field and vice versa.*

Second, We gave an inartificial solution, with no theoretical inconsistency, to such problems as divergences in electrostatic field, radiation damping and high order terms of scattering matrix of quantum electrodynamics (historical knotty points in physics), and did the consistent theoretical analysis of already known experimental facts, including annihilation and production of particles.

Third, We also rebuilt several main things of quantum electrodynamics and made some theoretical predictions of nonlinear quantum effects.

1. Difficulties of Classical Theory of Fields

Since the presentation of Einstein's GR in 1916, until now, many scientists has put a lot of effort into studying on the unification of classical theories of fields. But these studies have hardly focused on revealing internal physical relation existing objectively between two fields and then mostly tried to discover some unified mathematical means and formalities of space-time geometry which can put two theories in a vessel. The main starting idea of these studies is that two theories of fields are, in classical viewpoint, complete and perfect without any room for touching on and accordingly mathematical means and techniques to combine or unify two theories are the key to solution of all problems. Of course, we know well that all of these attempts led to in failure and conclusions that stood against reality.

Maxwell's theory and Einstein's GR are quite distinguished in its physical contents and mathematical form of description (in view of main principle). It is explicitly impossible to find any physical relations between two fields described by these theories. Actually, in classical theories of fields, electric charge, source of electromagnetic field, has no relation with mass, source of gravitational field. On the other hand, energy of electromagnetic field and energy of gravitational field are conserved separately, and then there can be no argument about unified conservation involving mutual conversion between them.

But now let us make the following assumption. Supposing that electromagnetic field has some physical relation with not only electric charge, source of electro-magnetic field, but also mass, source of gravitation, there appears real conversion of gravitational field into electromagnetic field and vice-versa, and these can be verified experimentally and then universally valid physical arguments for them can be established, what will result in? This presents new unprecedented tasks before classical physics. In a word, it lead to the conclusion that the present classical theories of fields failed to establish some physical relation between two fields should be reconsidered from a critical viewpoint. That one reveals the objective relation between two fields and realizes the unification of two theories is, that is to say, to complete and develop the present classical theories of fields into a new stage, which should be accompanied by clarification and solution of the internal inconsistencies and difficulties of the present classical theories of fields.

Sect. 1 New reasoning-thinking (starting idea)

Everything starts from an extremely simple thing. In this section is described very simple thinking giving birth to new theory. In the view of classical physics, a charged particle creates electromagnetic field and gravitational field around it. But considering the state of this particle and field in the light of the present classical theory of field, some serious inconsistencies are found.

Let us consider annihilation of particle-antiparticle well known in physics. The non-relativistic approximate formula of energy conservation can be written as follows:

$$m_0c^2 + \frac{1}{2}m_0v_1^2 + m_0c^2 + \frac{1}{2}m_0v_2^2 = 2\hbar\omega \qquad (1-1)$$

where m_0c^2 is the energy of free particle, a main result of special theory of relativity (SR). Actually, since the presentation of SR, until now, in physics, m_0c^2 has been considered only as energy confined to particle. But, in formula (1-1), it should be surely considered that m_0c^2 includes energy of particle, as well as energy of field created by the particle. Why should be viewed like that? It, in a word, is based upon the idea according to which the total energy of particle and field created by it should be always conserved. In case of considering m_0c^2 to be energy confined to particle only, the conservation formula of total energy of particle and field can be represented as follows:

$$\left(m_0c^2 + \frac{1}{2}m_0v_1^2 + \varepsilon_1\right) + \left(m_0c^2 + \frac{1}{2}m_0v_2^2 + \varepsilon_2\right) = 2\hbar\omega + \varepsilon \qquad (1-2)$$

where ε_1 is the energy of electromagnetic field and gravitational field created by a particle and ε_2, the energy of two fields created by an antiparticle. From formula (1-1) and (1-2), we have

$$\varepsilon_1 + \varepsilon_2 = \varepsilon \qquad (1-3)$$

where ε is the energy of another matter newly appeared except photon after annihilation of a system of particle-antiparticle. But, until now has not yet been found any experimental data which, except photon, another matter occurred. Therefore, if there is something except photon after annihilation of particle-antiparticle, it is no alternative but to conclude that only energy of fields remained as it is, as invariant not measured. On the other hand, as long as particle-antiparticle is annihilated, the mass, m_0, and charge, e, also vanish and accordingly static gravitational field and electric field created by their source - rest mass and charge, respectively, should also disappear. Therefore, the result is

$$\begin{cases} \varepsilon = 0 \\ \varepsilon_1 + \varepsilon_2 > 0 \end{cases} \qquad (1-4)$$

$$\varepsilon_1 + \varepsilon_2 \neq \varepsilon$$

Consequently, considering m_0c^2 only as energy confined to particle, we, with occurrence of photon, lead to the conclusion that energy of electromagnetic field and gravitational field should vanish, which obviously stands against conservation law of the energy.

On this context, in this paper, that the total energy of a free particle and fields created by it is just equal to m_0c^2 is regarded as a starting point for evolving our theory. From the new starting point is followed the logical conclusion about physical relation between mass and electromagnetic field which in the present classical theories of fields has been considered to have no relation so far, and about mutual conversion of electromagnetic field and gravitational field. In fact, the occurrence of photon as the result of annihilation of particle and antiparticle means occurrence of electromagnetic wave. With occurrence of electromagnetic wave disappears gravitational field of particle-antiparticle. In view of conservation law of energy, this shows that gravitational energy of particle-antiparticle is converted into a part of electromagnetic wave. On the contrary, in case of pair creation, it proves that a part of the energy of electromagnetic wave is converted into the energy of gravitational field.

We can obtain following conclusions, based upon all above-mentioned argument.

1. When one considers a system of particle and field, the measured mass m_0 is equivalent to the total energy of particle and all fields (electro-magnetic field, gravitational field and nuclear field created by it) but not energy of particle only.

2. The electromagnetic field and gravitational field are mutually converted; accordingly, there exist a unified conservation law of the total energy of particle and electromagnetic-gravitational field, involving mutual conversion.

This conclusion, of course, was never drawn in terms of abstract assumptions. This conclusion is based on experimental data about annihilation of particle-antiparticle well known and recognized in particle physics, and rooted in conservation law of energy - foundation of physics. But, unfortunately, from the present classical theories of fields cannot be obtained these conclusions, and arguments about these problems leads to contradiction. In this regard, we consider difficulties of classical theories of fields separately according to respective theories and then make a final analysis as a whole.

Sect. 2 The difficulties of Maxwell's theory of electromagnetic field

The classical theory of electromagnetic field or Maxwell's electrodynamics, as the unique theory of electromagnetic phenomena, until now, has been regarded as a perfect and completed theory. But this theory also involves some unavoidable inconsistencies.

(1) In Maxwell's theory, divergence of energy of electrostatic field seems to be an unavoidable difficulty, and accordingly the conservation law of total energy of particle-its field is always meaningless.

The classical theory of field evolves with taking a particle to be a pointis, from demand of Special theory of Relativity in which particle cannot have finite size. But, in case of regarding a particle as a point, the energy of electrostatic field always diverges. The law of energy conservation is based on the idea that energy of a finite material system is always finite. This is because of the fact that conservation law is meaningful only for finite quantity and can be studied quantitatively. That is why, for finite material system with infinite energy, the conservation law of energy leads to absence of meaning. This shows clearly that, in Maxwell's theory, the finiteness of energy - conservation law of energy is not valid and so, not well qualified as a scientific theory.

Understanding Maxwell's theory in the viewpoint of logics, one also can find inconsistency. For building of a consistent closed theory, starting definitions, all conclusions and laws following from them should not be inconsistent each other. But that there is inconsistency between basic definition regarding a particle as a point and conservation law of energy in Maxwell's theory shows that this theory is a not-closed theory with inconsistency

In the past, divergent problem of energy was solved within Maxwell's theory as follows. One artificially removed the term relevant to the divergence in energy of field, regarding it to be absence of physical meaning. Of course, this obviously is in opposition to rule of logics. On the other hand, in case of the divergence of energy of electrostatic field when action radius of electric field approaches zero, confining the applicable region of classical electrodynamics to electron radius, e^2/m_0c^2, outside this region of application it was concluded that not classical theory of field but quantum theory of field is meaningful. But, this "measure of solution" is also wrong. Actually, region of application of theory is defined in accordance with what exactly the theory can describe experiments, namely, by applicable limitation in which can give answer to experiment but not by some specific limitation that the theory falls to logical inconsistency. When theory is not closed and has logical inconsistency, we fall to the poor situation that cannot distinguish whether disagreement between some consequences of the theory and experiments is based on internal inconsistency of the theory itself or actual limitation of application of the theory related to that the theory can give no perfect answer to experiments. Consequently, Maxwell's theory stands against the finiteness of energy and conservation law, and accordingly is a not-closed theory that involves contradiction.

This difficulty of Maxwell's theory is represented as the more serious form on the stage of quantum electrodynamics. In this regard, Stephan Weinberg wrote:

"Earlier experience with classical electron theory provided a warning that a point electron will have infinite electromagnetic self-mass. Disappointingly this problem appeared with even greater severity in the early days of quantum field theory, and although greatly ameliorated by subsequent improvements in the theory, it remains with us to the present day.

The problem of infinites in quantum field theory was apparently first noted in the 1929-30 papers of Heisenberg and Pauli. Soon after, the presence of infinites was confirmed in calculations of electromagnetic self-energy of a bound electron by Oppenheimer, and of a free electron by Ivar Waller. …But it had become accepted wisdom in the 1930s, and a point of view especially urged by Oppenheimer, that quantum electrodynamics could not be taken seriously at energies of more than about 100MeV, and that the solution to its problems could be found only in really adventurous new ideas." [2]

It is the obvious fact that even in the quantum electrodynamics in which region of application of theory cannot be limited, the energy of electrostatic field is divergent. The artificial "measure of solution" by what defines limitation which in classical electrodynamics the theory can be applied to can no longer apply to quantum electrodynamics. It is no alternative but to conclude that this difficulty is rooted in the inconsistency of Maxwell's theory.

(2) In Maxwell's theory, because of the divergence of field energy, one cannot give answer to the experimental fact that the total energy of particle plus its field is equal to the finite quantity, m_0c^2.

It was Albert Einstein who gave scientific answer to the relation of mass and field for the first time. He, evolving the theory of gravitational field, proved that in gravitational field of central symmetry the total energy of particle-gravitational field is equivalent to the inertial mass of a system. This shows that equivalence of mass and energy of a particle in SR is more generalized into equivalence of total energy of particle-field and inertial mass of a system. However, this equivalence is confined to within the theory of gravitational field and, until now, the interrelation between total energy of particle-its electromagnetic field and mass has not yet been studied. On the other hand, the experiment formula (1-1) shows explicitly that the total energy of particle and electromagnetic field-gravitational field is equal to the finite quantity, m_0c^2. This implies again the serious contrariety of Maxwell's theory.

(3) In Maxwell's theory, the consideration of radiation damping by radiation of an electric charge leads to a serious difficulty.

In case of considering electromagnetic wave radiated by a uniformly accelerated electric charge, it was experimentally verified that radiated field reacted on the electric charge. But in Maxwell's theory, the description of radiation damping always leads to a serious difficulty. This problem has been regarded as "the greatest crisis in Maxwell's theory" [3].

We now proceed the discussion of radiation damping in Maxwell's theory. The expansion of power series of four dimensional field potential in $\mathbf{V}/c$ can be written as follows:

$$\varphi = \frac{e}{R} + \frac{e}{2c} \cdot \frac{\partial^2 R}{\partial^2 t} = \varphi^{(1)} + \varphi^{(2)} \qquad (2-1)$$

$$\mathbf{A} = \frac{e}{c} \cdot \frac{\mathbf{V}}{R} - \frac{2}{3c^2} e\dot{\mathbf{V}} = \mathbf{A}^{(1)} + \mathbf{A}^{(2)} \qquad (2-2)$$

where $\varphi^{(3)}$ is zero by the gauge transformation [4]. From this, damping force by radiation is represented as

$$\mathbf{F}^{in} = e\mathbf{E}^{in} = -\frac{1}{c}\dot{\mathbf{A}}^{(2)} = \frac{2}{3c^2}\dddot{\mathbf{d}} \qquad (2-3)$$

$$m\dot{\mathbf{V}} = e\mathbf{E}^{ex} + \frac{e}{c}[\mathbf{V} \cdot \mathbf{H}^{ex}] + \frac{2}{3c^2}\dddot{\mathbf{d}} \qquad (2-4)$$

where $\mathbf{E}^{ex}$ and $\mathbf{H}^{ex}$ are strengths of external fields and $\mathbf{F}^{in} = e\mathbf{E}^{in}$ is the force by field of point charge itself. Consequently, $\mathbf{A}^{(2)}$ was considered only as the additional field that contributes to damping force by radiation. But this argument is followed from the following premise.

1) In the Lagrangian of interaction of particle and field, four dimensional vector $(\mathbf{A}, \varphi)$ of field should be viewed as the sum of external field and field of particle itself (created by particle itself), namely,

$$\boldsymbol{A} = \boldsymbol{A}^{ex} + \boldsymbol{A}^{in} \qquad (2-5)$$

$$\varphi = \varphi^{ex} + \varphi^{in} \qquad (2-6)$$

where $\mathbf{A}^{ex}$ and φ^{ex} are external fields and $\mathbf{A}^{in}$ and φ^{in} are fields of point charge itself.

2) For finding the force that the field produced by point charge acts on itself, in formula (2-1) and (2-2), radius of action by field should go to zero, namely,

$$\mathbf{A}^{in} = \lim_{R\to 0} \mathbf{A} \ , \quad \varphi^{in} = \lim_{R\to 0} \varphi \qquad (2-7)$$

But one can easily understand that, under the above mentioned premise, the theory immediately results in inconsistency. Actually, converging radius of action R to zero, (2-1) and (2-2) yield divergence of $\varphi^{(1)}$ and $A^{(1)}$. Consequently, within Maxwell's theory, introduction of field created by electric charge itself necessarily gives divergent terms. This inconsistent conclusion, as mentioned above, is rooted in the fact that the energy of electrostatic field leads to divergence.

In order to overcome this difficulty, in consideration of damping force by radiation, divergent terms, $\varphi^{(1)}$ and $\mathbf{A}^{(1)}$, were artificially subtracted, concluding that they are insignificant terms, and only $\mathbf{A}^{(2)}$ independent of radius of action was regarded as the significant term relevant to radiation damping. Of course, this "measure of solution" is obviously in opposition to logical rule for construction of theory. Many experimental data show that reaction of radiation by electric charge, radiation damping, appears in reality and affects motion of electric charge. But in Maxwell's theory, the fact which introduction of reaction effect by radiation leads to contradiction shows that this theory is not closed one involving inconsistencies.

(4) Because of the principle of gauge symmetry that underlies Maxwell's theory, the energy of material system loses physical meaning and accordingly energy conservation of material system arrives at absence of its meaning.

The principle of gauge symmetry that underlies classical electrodynamics, as well as quantum theory of field involves a serious problem to be reconsidered. In SR, the relation among mass of a free particle, its energy and momentum is as follows:

$$m_0c^2 = \frac{E^2}{c^2} - P^2 \qquad (2-8)$$

As referred to in many papers and textbooks, in case of a complex system which consists of elements (or subsystems), formula (2-8) holds [5]. In this case, the energy of the system is

$$E = \sum_i \varepsilon_i + U \qquad (2-9)$$

where U is the interactional energy of constituent particles and $\varepsilon_i = m_ic^2 + T_i$, the sum of rest energy and kinetic energy, and then momentum is

$$\mathbf{P} = \sum_i \mathbf{P}_i \qquad (2-10)$$

Now, if one chooses a coordinate system allowing momentum $\mathbf{P} = 0$ in which inertia center is placed at origin of coordinate system, the result is

$$m_0 = \sum_i m_i + \frac{1}{c^2}\sum_i T_i + \frac{U}{c^2} \qquad (2-11)$$

If $\sum_i T_i \ll |U|$, namely constituent particles maintain relative stability and kinetic energy of particles is supposed to be very small, formula (2-11) arrives at

$$m_0 = \sum_i m_i + \frac{U}{c^2} \qquad (2-12)$$

From this, difference or deficit of mass is as follows:

$$\Delta m = m_0 - \sum_i m_i \qquad (2-13)$$

$$U = \Delta mc^2$$

This conclusion, of course, was verified by many experiments relevant to fission. That is to say, formulas (2-12) and (2-13) are correct results proved by experiments. But in case of applying the principle of gauge invariance (gauge symmetry) to formula (2-12) and (2-13), at once, we arrive at inconsistency. Actually in formula (2-12), as long as the interaction term, U, includes potential term of electric interaction, and from the principle of gauge symmetry, any constant can be either added to or subtracted from potential φ. In this case, the mass and energy of a system cannot be uniquely determined and further by choosing properly a constant included in φ, the mass and energy of a system can be transformed to zero or even negative value. Consequently, from the principle of gauge symmetry, the energy of material system leads to loss of physical meaning. On the other hand, only when $U < 0$, system becomes stable, but as long as U leads to zero or positive value according to constant chosen, discussion about the criterion of stability and instability is impossible.

In Newton's classical mechanics, the energy of a rest object is not defined uniquely and is positive value or negative value. In contrast, in SR the energy of a free particle is always determined uniquely as positive value and equivalent to rest mass. If one follows the principle of gauge symmetry, the formula (2-12), a main conclusion of SR that has already verified by experiment should be rejected and energy of material system leads to absence of physical meaning. Consequently, one arrives at failure in arguing conservation law of energy. If one receives, as a truth, the equivalence of mass and energy verified experimentally and the fact that the energy of material system can be neither zero nor negative value, the principle of gauge symmetry should be reconsidered.

(5) Quantum electrodynamics regarding Maxwell's theory as the unique basis raises problem of divergence of scattering matrix within region of large momentum or small area of space.

The occurrence of divergent terms in approximation of higher order of scattering matrix presents unavoidable knotty points before electrodynamics. In fact, even though first order approximation in scattering theory is well conformed with experimental results, if the approximations of high order diverges, even correctness of first approximation is put into doubt and accordingly this theory leads to loss of qualification as a scientific theory of physics.

As well known, in classical electrodynamics also is raised problem of divergence within classical radius of electron $r_0 = e^2/m_0c^2$ but within small area of Compton wavelength degree, recognizing that quantum theory only is significant, by the way of confining applicable region of classical electrodynamics to Compton wavelength, this inconsistency was overcome. However, as far as quantum electrodynamics considers interaction of particles within any area of space, divergence occurred in some area cannot be solved by the same way as in classical electrodynamics. In this regard, in present quantum electrodynamics, this difficulty was "solved" as follow: At first, one defined boundary momentum L and next, separated infinite quantity from the main expression and then by including these in electric charge and mass, renormalized electric charge and mass to yield finite quantity only. But this cannot certainly be the right measure of solution. In fact, as recognized by many scientists, this measure of solution is very artificial and harm the logical system of the theory.

P.A.M. Dirac was strongly against the procedure of neglecting infinity by renormalization:

"This small correction is interpreted as giving the Lamb shift in the case of the energy levels of hydrogen or an extra magnetic moment of the electron, the anomalous magnetic moment, for an electron in a magnetic field. These calculations do give results in agreement with observation.

Hence most physicists are very satisfied with the situation. They say: 'Quantum electrodynamics is a good theory, and we do not have to worry about it any more.' I must say that I am very dissatisfied with the situation, because this so-called 'good theory' does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it turns out to be small-not neglecting it just because it is infinitely great and you do not want it!

…

There must be some drastic change introduced into them so that no infinites occur in the theory at all and so that we can carry out the solution of the equations sensibly, according to ordinary rules and without being bothered by difficulties. ⋯ I feel that the change required will be just about as drastic as the passage from the Bohr orbit theory to the quantum mechanics." [6]

Mandel Sachs said as follows:

"While this (method of renormalization) is taken to be a success of the quantum theory, it is still not satisfactory because the renormalization procedures are not mathematically consistent. That is, while some predictions are correct, by changing the method of subtracting the infinities from the divergent series solutions, one may predict any other numbers for the same physical effects! This violates the scientific requirement that there is a unique prediction for any given experimental fact.

Thus it has been my contention, as well as some others in the field (such as one of the original founders of quantum field theory, Paul Dirac) that quantum electrodynamics is not in a satisfactory state as a bona fide theory." [1]

In this paper, we clarified that occurrence of this inconsistency from quantum electrodynamics was rooted in problem of divergence in classical electrodynamics and built a basis of consistent quantum theory without any inconsequence.

Sect. 3 The difficulties of GR

Difficulties of GR, for the first time in history of physics, was presented by Schrodinger in 1918, and since then, was stated by many physicists like Fock, and discussed collectively in "the relativistic theory of gravitation" (English Edition , 1989) co-written by Logunov and Valssov etc., physicists of former Soviet Union.

With reference to all arguments in preceding research, summarizing difficulties of GR is as follows.

(1) The energy-momentum tensor defined in Einstein's GR has no physical meaning.

As demonstrated for the first time by Schrodinger, by choosing properly coordinate system, energy-momentum tensor of gravitational field vanishes outside a ball. From this follows the inconsistent conclusion that the energy-momentum of gravitational field cannot be localized and accordingly energy-momentum density of field existing in any point of space-time cannot be defined and then only total energy-momentum integrated through total space can be well-defined. As stated by many scientists, in this case, propagation of gravitational energy from one place to another is impossible and description of gravitational wave leads to principled inconsistency [8]. Actually, in GR, energy-momentum tensor τ^{lm} of field is defined as pseudo tensor and in this case, by selecting an appropriate system of coordinates, one can nullify all the components of τ^{lm} at any point of space [7]. On the other hand, in GR, energy-momentum conservation formula of integral form also possess a limitation. In this theory, energy conservation of matter and field can be written as follows.

$$\partial_n(T_i^n + \tau_i^n) = 0 \qquad (3-1)$$

where T_i^n is energy-momentum density tensor of matter and τ_i^n energy-momentum density tensor of field. If matter is concentrated only in a volume *V*, Eq (3-1) implies that

$$\frac{d}{dx_0}\int_V (T_i^0 + \tau_i^0)\, dV = -\oint \tau_i^\alpha dS_\alpha \qquad (3-2)$$

At present, there exist a whole series of exact solutions to the vacuum Hilbert-Einstein equations for which the stresses, τ_0^α, are everywhere null [9]. Thus, for exact wave solution to Hilbert-Einstein equations that nullifies the components of the energy-momentum pseudo-tensor, Equation (3-2) yields

$$\frac{d}{dx^0}\left\{\int_V (T_i^0 + \tau_i^0)\, dV\right\} = 0 \qquad (3-3)$$

that is, the energy of matter and gravitational field inside V is conserved. This means that there is no flow of energy outward from V and, therefore, there can be no action on test bodies placed outside V. And vice versa, in case of absence of gravitational field, i.e. flat space-time, that is, when the metric tensor g_{ni} of the Riemann space-time is equal to the metric tensor, γ_{ni}, of pseudo-Euclidean space-time, components of energy-momentum pseudo-tensors may not vanish although there is no gravitational field and all components of the curvature tensor are zero. For example, in the spherical system of coordinates of the pseudo-Euclidean spacetime is given following formula

$$R_{klm}^i = 0, \quad g_{00} = 1, \quad g_{rr} = -1, \quad g_{\theta\theta} = -r^2, \quad g_{\varphi\varphi} = -r^2 \qquad (3-4)$$

In this case, component τ_0^0 of Einstein's pseudo-tensor for energy density of the field yields

$$\tau_0^0 = -\frac{1}{8\pi} \qquad (3-5)$$

It is clear that the total energy of gravitational field in this system of coordinates would diverge because of $\tau_0^0 < 0$. In this case, Landau-Lifshitz pseudo-tensor demonstrates a different energy distribution in space [8].

$$(-g)\tau^{00} = -\frac{r^2}{8\pi}(1 + 4sin^2\theta) < 0 \qquad (3-6)$$

Consequently, In GR, energy-momentum density of field, the main physical quantity characterizing the field is not determined by real field itself but by choice of coordinate system. That is to say, by a suitable choice of coordinate system, the field can vanish in spite of existence of real field or appear even in case of absence of real field. This shows obviously that Einstein's GR failed to have a main character which must possess as scientific theory.

(2) The principle of equivalence, starting idea of GR is not qualified enough as scientific principle of physics.

According to principle of equivalence, by choice of system of coordinate, inhomogeneous gravitational field in space-time cannot totally vanish, but in any infinitely small region of space, coordinate system can always be chosen in such a way that the gravitational field in the region vanishes, and accordingly in this region gravitational field can be replaced completely by field of inertia. Just this idea reflected main character of Riemann space in which curved surface, by any choice of coordinate system, cannot transform into flat surface but, for infinitesimal region, into Euclidean infinitesimal space, which just was the main reason that Einstein chose Riemannian space as a form of space-time for evolution of the theory of gravitational field.

But as argued by Logunov, Vlassov and many physicists including Schrodinger and Fock, the above mentioned difficulty (loss of physical meaning of gravitational field) is rooted in equivalence principle. In fact, in GR metric tensor g_{mn} is both metric of space-time and function of field. Therefore, equivalence principle that metric, g_{mn}, in any point of space-time can be transformed to Euclidean metric (constant metric) leads us to the inconsistent conclusion that energy-momentum density tensor of field localized in a point of space-time can become zero and gravitational field occurs even in Euclidean spherical system of coordinate (empty space without gravitational field), as long as energy-momentum tensor of field consists of metric tensor g_{mn}.

Now let us make the following imaginary thought experiment. Supposing that there is homogeneous and static gravitational field, in this field the particle accelerates to radiate gravitational wave, gravitons. On the other hand, in view of equivalence principle, static and homogeneous field, by an appropriate transformation of coordinate system, can be the state of "null-gravitation". Of course, in this empty space or the state of null-gravitation is followed an inconsistent conclusion that with uniformly and rectilinear motion of a particle vanishes gravitational wave radiated by particle, i.e. graviton. This shows that equivalence principle reflects the inconsistent idea that can either create or remove such objective matter as static field or gravitons. Besides, in GR gravitational mass is not invariant under transformation of three-dimensional spatial coordinate system, and so the descriptions about three effects of gravitation (Red shift of Light, Reflection of Light and Shift of Mercury's Perihelion) have not uniqueness in view of theoretical analysis, and moreover by choice of coordinate system, the radiation strength of gravitational wave can be either zero or negative value. These obviously are inconsistent.

It is our contention, as well as Logunov, that these difficulties are rooted in equivalence principle [8]. The conservation law of energy-momentum is a main idea of physics, whereas the equivalence principle is very essential for building of GR. Sacrificing conservation law of energy, this principle cannot certainly find any foundation for its existence as the principle of physics.

(3) In GR, the conservation law of the total energy-momentum of matter and field is not based on a main principle of physics relevant to homogeneity and isotropy of space and time, and, moreover, has not physical meaning.

From Newton's time to now, the relation between conservation law of energy-momentum and homogeneity and isotropy of space-time has been recognized as a main principle in all theories of physics including classical electrodynamics and quantum electrodynamics. But in case of applying this principle to GR, one reaches the inconsistent conclusion. The Lagrangian density of matter and field and action integral formula can be written as follows.

$$L_M = L_M(g_{mn}, \phi_A), \qquad L_g = \sqrt{-g}R, \qquad S = \int \left(L_M + L_g\right) d\Omega \qquad (3-7)$$

where L_M is Lagrangian density of matter and L_g Lagrangian density of field, g_{mn} metric tensor, and ϕ_A field of other matter. In this case, infinitesimal transformation of space-time $x'^i = x^i + \delta x^i$ results in infinitesimal transformation of metric $g'^i = g^i + \delta g^i$, and from $\delta S = 0$ is followed

$$T^{ni}_{(M)} + T^{ni}_{(g)} = 0 \qquad (3-8)$$

where

$$T^{ni}_{(M)} = -2\, \delta L_M / \delta g_{ni}$$

is the symmetric energy-momentum tensor of matter,

$$T^{ni}_{(g)} = -2\, \delta L_g / \delta g_{ni} = -\frac{C^4}{8\pi G} \cdot \sqrt{-g}\left[R^{ni} - \frac{1}{2} g^{ni} R\right]$$

is the energy-momentum tensor of field. Equation (3-8) also implies that all components of the energy-momentum tensor density of the symmetric gravitational field, $T^{ni}_{(g)}$, vanishes everywhere outside matter, Thus, these results imply that the gravitational field in GR does not possess properties inherent in electromagnetic field [8]. Consequently, drawing conservation law of energy-momentum (field plus matter) from general principle of homogeneity and isotropy of space-time naturally, we reach the inconsistent conclusion. If so, why results in the inconsistent conclusion. In GR g_{mn} is both metric of space-time and variables of field, and accordingly obtaining the equation of field from variation of field δg^{mn} coincide mathematically with drawing conservation formula of energy-momentum from variation of metric δg^{mn} following by variation of space-time $\delta x'$. The conservation formula drawn by this method is invalid as showed in formula (3-8).

In order to avoid this inconsequence, in GR the concept of energy-momentum was defined by the illogic and artificial method as follows. The field equation of Hilbert-Einstein can be written as:

$$-\frac{C^4}{8\pi G}\cdot g\left[R^{ik}-\frac{1}{2}g^{ik}R\right]=-gT^{ik} \qquad (3-9)$$

where $\det g_{ik}=g$, R^{ik} Ricci tensor and T^{ik} the energy-momentum tensor of matter. Then, the left-hand side can be represented as the sum of two non-covariant quantities

$$-\frac{C^4}{8\pi G}g\left[R^{ik}-\frac{1}{2}g^{ik}R\right]=\frac{\partial h^{ikl}}{\partial x^l}+g\tau^{ik} \qquad (3-10)$$

where $\tau^{ik}=\tau^{ki}$ is the energy-momentum tensor of gravitational field and $h^{ikl}=-\ h^{ilk}$ spin pseudo-tensor. This transforms Hibert-Einstein equations (3-10) into equivalent form

$$-g\left(T^{ik}+\tau^{ik}\right)=\frac{\partial h^{ikl}}{\partial x^l} \qquad (3-11)$$

From the obvious fact that

$$\frac{\partial^2 h^{ikl}}{\partial x^k \partial x^l}=0 \qquad (3-12)$$

Hilbert-Einstein equation yields the following "differential conservation law"

$$\frac{\partial}{\partial \tau^k}\left[-g\left(T^{ik}+\tau^{ik}\right)\right]=0 \qquad (3-13)$$

which formally is similar to the conservation law for energy-momentum in electrodynamics [4]. Of course, this argument is never made on the basis of homogeneity and isotropy of space-time. Moreover, energy-momentum tensor defined in conservation formula (3-13) as pseudo-tensor can vanish by a suitable choice of coordinates system or diverges in Euclidean spherical coordinates, and so is invalid as a physical quantity that characterizes physical field. On the other hand, as showed in formula (3-3), conservation of energy-momentum of integral form also leads to difficulty and in case of discussion of "energy-momentum" of system we also arrives at the inconsistent conclusion which it or inertial mass depends on choice of spatial coordinate [8].

(4) In case of considering matter as point, in GR the energy of material system also would diverge.

When one, neglecting macro-character of objects that comprise material system, regarding them as points and calculating energy of the system, in addition to interactional energy dependent on spatial distribution of objects, there appear such divergent terms relevant to self-energy as in Maxwell's theory [4].

(5) Radiation damping (radiation reaction effect) by gravitational wave created by an accelerated particle arrives at serious choplogic.

Until now, in spite of so many studies concerning gravitation, radiation damping by gravitational wave has hardly been studied. It is because intensity of gravitational wave is too small to measure and then what accounts for radiation damping effect much smaller than it has no significance. On the other hand, owing to the characteristic of nonlinear equation of gravitational field, it is impossible to obtain the correct solution of the equation and accordingly the strict theoretical consideration of radiation damping effect by radiation wave cannot be given.

But, now let us make an imaginary thought experiment about what radiation damping will result in. In GR, gravitational field is always attraction field and accordingly interactional force between object and field has negative value. Therefore, force of radiation damping, namely interactional force with self-field produced by object also has negative value. Consequently, from this is drawn the inconsistent conclusion that an object, with radiation of gravitational wave, does not lose energy by damping force but by the force in direction of motion comes to obtain energy. For actual approximate calculation, one can get formula for radiation field of gravitational wave by a power series under approximate condition of weak field. In this case, the first term of expansion is proportional to $-1/r$ and as for the interaction with self-field, term of interaction with attraction field has negative infinity. Of course, although this argument is not based on a rigorous calculation and more or less imaginative, the result of the thought experiment presents the main difficulty of GR.

All of these shows that Einstein's GR is also not a closed theory. Until now we considered the main difficulties that Maxwell's theory and Einstein's GR involve. Summarizing all argument above mentioned leads to the following conclusions.

1. In present classical theory of fields, total energy-momentum conservation law does not has physical meaning

In case of Maxwell's electrodynamics, the energy of field created by an electric charge always diverges and owing to principle of gauge symmetry, the energy of material system arrives at absence of physical meaning. In Einstein's GR, energy-momentum tensor does not possess physical meaning, and the total energy-momentum conservation formula also leads to inconsequence.

2. It is impossible that within Maxwell's theory and Einstein's GR give solution to experimental data which total energy of particle-field is equal to $m_0 c^2$.

If one sticks to that the present theories are consistent and closed ones, we should give up main principles of physics for total energy conservation of matter-field and the finiteness of physical quantities, which of course has not objective validity and is never possible.

2. Starting Postulates and Lagrangian for Unified Theory of Field (Electromagnetic Field and Gravitational Field)

The unification of the different two theories, i.e. theory of electromagnetic field and theory of gravitational field that have been systemized separately is indeed the establishment of a new consistent classical theory of field. On basis of the analysis of difficulties and inconsistencies (of classical fields), we present the following principles and methodology for solutions to these.

First of all, the principles are as follows:

In classical and quantum electrodynamics, even in case of considering a particle as a point, theories should be constructed so that do not appear any divergence of physical quantities,.

In case of gravitational field theory, limitation of application of equivalence principle should be, to some extent, given and then the theory be constructed so that conservation formula of energy-momentum possesses physical meaning.

Theory should be built so that the total energy of particle-fields (electromagnetic field and gravitational field) becomes m_0c^2.

Next, the methodology is as follows:

The construction of a new theory should base on the assumption which the Lagrangian of Maxwell's electrodynamics is the first approximation of a new Lagrangian.

As for gravitation, one should find some application limitation within which equivalence principle is valid, and then construct a new gravitation theory that involves Einstein's GR established as the approximate form within it (application limitation of equivalence principle).

In the new Lagrangian, variable of field should include field functions of electromagnetic-gravitation in a unified form, and then Lagrangian should be represented such that can obtain the unified conservation law of electromagnetic-gravitational field.

Sect. 4 Starting postulates

The starting postulates of the unified classical theory of electromagnetic-gravitational field or new classical theory of field are as follows.

1. In all inertial reference frames, all laws of physics are equivalent.

The inertial reference system regards inertial law as the foundation of self-existence. From this law, in theory of field is drawn conclusion that free particle cannot accelerate automatically to produce wave of radiation, and then damping force by radiation is based upon existence of external field only. This is just understanding of inertia which the theory of matter-field is based and is more generalized expression of mechanical inertial law. On the other hand, as long as the unified theory of field is evolved in inertial system, Lagrangian for matter-field is invariant by Lorentz transformation only. Therefore, in case where, without external field, only self-field of particle is given, we lead to the conclusion that its Lagrangian should be Lagrangian for a free particle in the special theory of relativity (SR).

Though this conclusion seems clear at first glance, it has very important significance. In SR, mc^2 is the energy confined to particle only but mc^2, in our theory, is the energy which includes not only the energy of a particle but also the energy of electrostatic field. And then it is because in case of introducing term of interaction between particle and its self-field to new Lagrangian for free particle, this term, unlike Maxwell's theory, does not diverge but become zero and the Lagrangian for a free particle in Minkowski space should be obtained.

2. In all inertial reference frames, the velocity of light is invariant.

3. The total energy of matter of free state (or free particle) and all kinds of the fields created by it is equal to measured mass m (rest mass m_0, motion mass $m = m_0/\sqrt{1-\beta^2}$) of system consisted of matter-fields multiplied by c^2.

Einstein is the first man who solved equivalence between mass and energy of particle. He, in GR, gave theoretical solution which total energy of matter and static gravitational field created by it is equivalent to inertial mass of matter. Later, this problem was proved theoretically in various forms by many physicists including R. C. Tolman, H. Weyl, and L. D. Landau [10, 11, 4]. That is why, in GR matter and field is considered as an inseperable integrity and total energy of matter-field always is equivalent to the mass of a system. But until now, such equivalence in the physics has been studied only within the theory of gravitational field.

Now let us focus on the following. The electric charge creates not only electromagnetic field but also gravitational field and as shown in experimental data of annihilation and creation of particles (see formula (1-1)), the total energy of matter and all fields created by it is equal to mc^2 (m; measured mass). Here an important thing to be explicitly emphasized is that a system consisting of particle and field should be considered as an integrity and accordingly a question about what share of mc^2 is divided into energy of matter and field, respectively, cannot be given in principle. In a word, the third postulate is the more comprehensive generalization of Einstein's theory and becomes a foundation of unified theory.

4. The physical field (energy-momentum tensor) existing in any point of space-time, under any transformation of coordinates system and even within infinitely small area of space cannot vanish.

Since the appearance of Galilean principle of relativity, there were many arguments about transformation of coordinates system. Galilean principle of relativity tacitly reflects an idea according to which although motion state of matter, under any transformation of coordinate system, can change to this way or that way but matter itself can neither vanish nor newly be created. Of course, although Galilei and later other physicists emphasized no more about this, considering transformation of coordinates system, they took it for granted.

The field is also a special form of matter. Nevertheless, supposing that gravitational field can either vanish or be created by an appropriate choice of coordinates system, this necessarily leads to a choplogic basically inconsistent with the idea of matter which is tacitly reflected in transformation theory of coordinates system, argued and inherited from Galilei time to now. In this case, the energy-momentum tensor characterizing physical field arrives at absence of meaning and accordingly the conservation law of total energy also loses its meaning. The energy-momentum conservation law is the most basic and universal idea of physics. At any case, sacrificing this law, one cannot insist reasons for existence of other secondary principles and its justness.

As considered in sect. 3, supposing that this equivalence principle is valid without any restriction, i.e. in any point of space-time and any gravitational field (weak or strong), theory is built in Riemann space in which physical field loses the meaning. This implies that Einstein's principle of equivalence has some limitation and then there exists some application limitation for it to have real meaning. If so, how should such limitation of application be considered? Considering it in the view of origin, Einstein's principle of equivalence is rooted in Newton's theory of gravitation. In this theory, gravitational force acting on an object and acceleration are

$$m\boldsymbol{a} = m\boldsymbol{g}, \qquad \boldsymbol{a} = \boldsymbol{g}$$

From this is followed the conclusion that an object subject to gravitation has identical acceleration irrespective of its mass. We can infer that when Einstein defined equivalence principle, obviously grounded upon Newton's idea of the relation between gravitational force and inertial force of object. Actually, at those days when Einstein defined equivalence principle, there was no another theory and experimental data for gravity except for Newton's theory and experimental data relevant to it. Newton's theory of gravitation is based upon experimental data obtained within nonrelativistic, static and weak field and equivalence principle can also be defined within the scope. There is no experimental and theoretical ground to conclude that this principle can be enough extended to non-static and strong field yet. Nevertheless, Einstein elevated equivalence principle grounded upon experimental results obtained within non-relativistic, static and weak field, by generalization and expansion, to an all-powerful principle of gravitation that can apply even to relativistic, non-static and strong field. As mentioned above repeatedly, unlimited expansion of this principle to any region leads to serious choplogic. Consequently, this implies that as Newton's theory of gravitation is allowed within the non-relativistic, weak and static field, the application limitation of equivalence principle should be also confined to that extent. This is essential to understand interrelation between the new unified theory and Einstein's theory of gravitation.

5. In unified theory of fields, every physical quantity is expressed as implicit functions (correctly speaking, absolute implicit functions) and the functions that have real physical meaning are determined as explicit ones uniquely corresponded to Finsler space according to rule of normalization.

This postulate is discussed in sect. 9 and 10.

Sect. 5 Lagrangian for motion of particle

In this section, we define the Lagrangian of the motion of a particle and consider the motion of a particle in electromagnetic field and gravitational field, respectively. Lagrangian integral formula for the motion of a particle can be defined based on starting postulates as follows:

$$S = -m_0 c\int ds = -m_0 c\int dt\left(g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}\right)^{1/2}$$

This formula yields

$$S = \int L ds = -m_0 c\int \left(g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}\right)^{\frac{1}{2}} = -m_0 c\int \left[\delta_{\mu\nu}\left(1+2\alpha K_\lambda u^\lambda\right)u^\mu u^\nu\right]^{\frac{1}{2}} ds \qquad (5-1)$$

where $L = -m_0 c\left(g_{\mu\nu}u^\mu u^\nu\right)^{1/2}$, $g_{\mu\nu} = \delta_{\mu\nu}\left(1+2\alpha K_\lambda u^\lambda\right)$, $\delta_{\mu\nu}$ Minkowski metric tensor, K_λ four dimensional field vector (A_λ in case of electromagnetic field and G_λ in case of gravitational field) and u^λ four dimensional velocity and α constant defined from approximate condition ($\alpha = \alpha_{(E)} = e/m_0 c^2$ in case of electromagnetic field and $\alpha = \alpha_{(g)} = m_0/m_0 c^2 = 1/c^2$ in case of gravitational field). Here metric tensor $g_{\mu\nu}$ is non-Euclidean metric dependent on four-dimensional velocity vector and coordinates of space-time.

The Lagrangian integral formula (5-1) is conformed to starting postulates mentioned in sect. 4.

Firstly, integral of action (5-1) is invariant under Lorentz information and includes the universal constant c.

As easily demonstrated, because in metric tensor $g_{\mu\nu}$, $\delta_{\mu\nu}$ is Minkowski tensor and $K_\lambda u^\lambda$ the scalar product of four dimensional field vector and four dimensional velocity vector, so the formula (5-1) is invariant under Lorentz transformation.

Secondly, as proved in next sections, from formula (5-1) is derived equivalence between total energy of particle-field and mass of a system.

Thirdly, in nonrelativistic, static and week field formula (5-1) is converted to the Lagrangian integral formula in Riemannian space in which Einstein's principle of equivalence is admitted.

In non-relativistic, static and week field, $G_0 = U$ (gravitational potential) and $G_i = 0$, and ignoring terms of higher order more than $1/c^2$ is ignored, formula (5-1) yields

$$\begin{aligned} S &= -m_0 c\int \left[\delta_{\mu\nu}(1+2\alpha G_0 u^0)u^\mu u^\nu\right]^{\frac{1}{2}} ds = -m_0 c\int \left[\delta_{\mu\nu}(1+2\alpha G_0 u^0)dx^\mu dx^\nu\right]^{\frac{1}{2}} \\ &= -m_0 c\int \left[(1-\beta^2)(1+2\alpha G_0 u^0)\right]^{\frac{1}{2}} dt = -m_0 c\int \left[1-\beta^2+\frac{2}{c^2}U(1-\beta^2)\right]^{\frac{1}{2}} dt \\ &= -m_0 c\int \left[\left(1+\frac{2}{c^2}U\right)-\frac{V^2}{c^2}\right]^{\frac{1}{2}} dt = -m_0 c\int \left(g_{\mu\nu}dx^\mu dx^\nu\right)^{1/2} \qquad (5-2) \end{aligned}$$

where $g_{00} = 1 + 2U/c^2$, $g_{\alpha\beta} = \delta_{\alpha\beta}$ and $g_{0i} = 0$. The formula (5-2) coincides with Lagrangian integral formula defined in non-relativistic, static and week field in GR. Consequently, the new Lagrangian integral formula, within application limitation in which equivalence principle is valid, coincides with the Lagrangian integral formula in Minkowski space defined in infinitesimal spatiotemporal region in GR. Thus, within just this region of non-relativistic, static and week field is satisfied approximately Einstein's equivalence principle that can eliminate gravitational field in any point of space-time.

Fourthly, The formula (5-1), under approximation condition $r \gg r_0$ *(electron radius,* $e^2/m_0 c^2$*), is transformed into Lagrangian for motion of a particle in Maxwell's electrodynamics.*

The formula (5-1) can be transformed as follows.

$$S=-m_0c\int dt\left(1-\frac{V^2}{c^2}\right)^{\frac{1}{2}}\left(1+2\alpha A_\lambda u^\lambda\right)^{\frac{1}{2}} \qquad (5-3)$$

In case of $2\alpha A_\lambda u^\lambda \ll 1$, if one expands formula (5-3) as Taylor series, formula (5-3) can be written as follows,

$$S\approx -m_0c\int dt\left(1-\frac{V^2}{c^2}\right)^{\frac{1}{2}}-m_0c\alpha\int A_\lambda \dot{x}^\lambda\, dt$$

When $\alpha=e/m_0c^2$, the following form is obtained

$$dS\approx -m_0c\int dt\left(1-\frac{V^2}{c^2}\right)^{\frac{1}{2}}-\frac{e}{c}\int A_\lambda dx^\lambda \qquad (5-4)$$

And then, what is physical meaning implied by the approximate condition $2\alpha A_\lambda u^\lambda \ll 1$? In static and weak field, $u^0\approx 1$ and $2\alpha A_\lambda u^\lambda \approx 2\alpha\varphi u^0 \approx 2\alpha\varphi$, where φ is Coulomb potential. Therefore, $2\alpha A_\lambda u^\lambda$ arrives at

$$2\alpha\varphi=2\frac{e}{m_0c^2}\cdot\frac{e}{r}=\frac{2e^2}{m_0c^2}\cdot\frac{1}{r}=\frac{r_0}{r} \qquad (5-5)$$

where $r_0=2e^2/m_0c^2$ is electron radius (≈10-13cm). Consequently, the approximate condition transformed into Lagrangian for motion of a particle in Maxwell's electrodynamics is corresponded to the case where interaction distance between particles is much farther than electron radius r_0. In fact every experiments put in the ground of Maxwell theory were conducted in $r\gg r_0$, i.e., macroscopic region and then, in the process systemizing and generalizing experimental results by inductive method was built Maxwell's electrodynamics. The new electrodynamics is valid in not only region farther than r_0 but also neighborhood of r_0 or even one smaller than r_0.

Fifthly, formula (5-1) is essential to clarify, in quantum electrodynamics, cause of divergence occurring in approximation of higher order and its natural vanishment (convergence of higher order approximation), and explain, in gravitational theory, three effect verified already by experiments, red shift of light, deflection of light and shift of Mercury's perihelion (considered in next sections).

From formula (5-1), let us obtain the motion equation of a particle in the field

$$\delta S=-m_0c\delta\int\left(g_{\mu\nu}u^\mu u^\nu\right)^{1/2}ds=0 \qquad (5-6)$$

$$\delta\int\left(g_{\mu\nu}u^\mu u^\nu\right)^{1/2}ds=$$

$$=\int\frac{ds}{2\left(g_{\mu\nu}u^\mu u^\nu\right)^{1/2}}\left[u^\mu u^\nu\left(\frac{\partial g_{\mu\nu}}{\partial x^\lambda}\cdot\delta x^\lambda+\frac{\partial g_{\mu\nu}}{\partial u^\lambda}\cdot\frac{d\delta x^\lambda}{ds}\right)+2g_{\mu\nu}\frac{dx^\mu}{ds}\cdot\frac{d\delta x^\nu}{ds}\right]=0$$

Now, by the partial integral of the second term and the third term is obtained

$$\delta\int\left(g_{\lambda\sigma}u^\lambda u^\sigma\right)^{1/2}ds=\int ds\delta x^\lambda\cdot\left[\frac{1}{2}\cdot\frac{\partial g_{\mu\nu}}{\partial x^\lambda}u^\mu u^\nu-\frac{1}{2}\cdot\frac{d}{ds}\left(\frac{\partial g_{\mu\nu}}{\partial u^\lambda}u^\mu u^\nu\right)-\frac{d(g_{\lambda\nu}u^\nu)}{ds}\right]$$

$$=\int\left[\frac{1}{2}\cdot\frac{\partial(2\alpha K_\sigma)}{\partial x^\lambda}u^\sigma u^\mu u^\nu\delta_{\mu\nu}-\frac{1}{2}\frac{d}{ds}\left(2\alpha\delta_{\mu\nu}K_\lambda u^\mu u^\nu\right)-\frac{du_\lambda}{ds}\right]\delta x^\lambda ds=0$$

where

$$g_{\mu\nu}u^\mu u^\nu=1,\quad \delta_{\mu\nu}u^\mu u^\nu=\frac{1}{1+2\alpha K_\lambda u^\lambda}$$

and then if one rearranges the above formula, the result is

$$m_0c\frac{du_\lambda}{ds}=\frac{\alpha m_0c}{1+2\alpha K_\mu u^\mu}C_{\lambda\sigma}u^\sigma-K_\lambda\frac{d}{ds}\left(\frac{\alpha m_0c}{1+2\alpha K_\mu u^\mu}\right) \qquad (5-7)$$

where $C_{\lambda\sigma} = \frac{\partial K_\sigma}{\partial x^\lambda} - \frac{\partial K_\lambda}{\partial x^\sigma}$

Let us consider the formula (5-7) for motion of a particle about the cases of electromagnetic field and gravitational field, separately. The motion equation of particle in electromagnetic field is as follows:

$$m_0 c \frac{du_\lambda}{ds} = \frac{1}{c} \cdot \frac{e}{1 + 2\frac{e}{m_0 c^2} A_\mu u^\mu} F_{\lambda\sigma} u^\sigma - \frac{1}{c} A_\lambda \frac{d}{ds} \left(\frac{e}{1 + 2\frac{e}{m_0 c^2} A_\mu u^\mu} \right) \qquad (5-8)$$

Now, let us introduce the so-called *effective electric charge,* $\bar{e}$, dependent on the interaction of particle and electromagnetic field

$$\bar{e} = \frac{e}{1 + 2\frac{e}{m_0 c^2} A_\mu u^\mu} \qquad (5-9)$$

Then formula (5-8) leads to

$$m_0 c \frac{du_\lambda}{ds} = \frac{1}{c} \bar{e} F_{\lambda\sigma} u^\sigma - \frac{1}{c} A_\lambda \frac{d}{ds} (\bar{e}) \qquad (5-10)$$

The motion equation of particle in gravitational field can be written in the following form:

$$m_0 c \frac{du_\lambda}{ds} = \frac{1}{c} \cdot \frac{m_0}{1 + 2\frac{1}{c^2} G_\mu u^\mu} \cdot R_{\lambda\sigma} u^\sigma - \frac{1}{c} G_\lambda \frac{d}{ds} \left(\frac{m_0}{1 + 2\frac{1}{c^2} G_\mu u^\mu} \right) \qquad (5-11)$$

As in formula (5-9), if one introduces effective gravitational mass $\bar{m}_{0g}$, the formula (5-11) arrives at

$$m_0 \frac{du_\lambda}{ds} = \frac{1}{c} \bar{m}_{0g} R_{\lambda\sigma} u^\sigma - \frac{1}{c} G_\lambda \frac{d}{ds} (\bar{m}_{0g}) \qquad (5-12)$$

where

$$\bar{m}_{0g} = \frac{m_0}{1 + 2\frac{1}{c^2} G_\mu u^\mu} \quad , \qquad R_{\lambda\sigma} = \frac{\partial G_\sigma}{\partial x^\lambda} - \frac{\partial G_\lambda}{\partial x^\sigma}$$

and G_μ is gravitational potential.

From momentum $P_\lambda = -\partial L / \partial u^\lambda$ follows

$$P_\lambda = m_0 c u_\lambda + \frac{a}{c(1 + 2\alpha K_\mu u^\mu)} K_\lambda \qquad (5-13)$$

where a is e in case of electromagnetic field and m_0 in case of gravitational field. The form of u_λ is as follows:

$$u_\lambda = g_{\lambda\sigma} u^\sigma =$$

$$= \delta_{\lambda\sigma} (1 + 2\alpha K_\mu u^\mu) \frac{dx^\sigma}{cdt[(1 - \beta^2)(1 + 2\alpha K_\mu u^\mu)]^{\frac{1}{2}}}$$

$$= \frac{\dot{x}_\lambda (1 + 2\alpha K_\mu u^\mu)^{\frac{1}{2}}}{c\sqrt{1 - \beta^2}} \qquad (5-14)$$

Now let us introduce effective inertial mass $\bar{m}_0$

$$\bar{m}_0 = m_0 (1 + 2\alpha K_\mu u^\mu)^{\frac{1}{2}} \qquad (5-15)$$

and effective source $\bar{a}$

$$\bar{a} = \frac{a}{1 + 2\alpha K_\mu u^\mu} \qquad (5-16)$$

(in case of gravitation $a = m_0$, $\bar{a} = \bar{m}_{0g}$ and in case of electromagnetic field, $a = e$, $\bar{a} = \bar{e}$). Thus, formula (5-13) can be written as follows:

$$P_i = \frac{\bar{m}_0 V_i}{\sqrt{1-\beta^2}} + \frac{\bar{a}}{c} K_i \tag{5-17}$$

$$cP_0 = E = \frac{\bar{m}_0 c^2}{\sqrt{1-\beta^2}} + \frac{\bar{a}}{c} K_0 \tag{5-18}$$

Next, let us derive the formula for energy of a system, a key to establish new non-linear quantum electrodynamics on the basis of new classical theory. First of all, for the future work on quantum electrodynamics, we confine source a to electric charge e. From $g_{\lambda\sigma} = \delta_{\lambda\sigma}(1 + 2\alpha K_\mu u^\mu)$ and $g_{\lambda\sigma} g^{\lambda\sigma} = 1$, we have

$$g^{\lambda\sigma} = \frac{\delta_{\lambda\sigma}}{1 + 2\alpha A_\mu u^\mu} \tag{5-19}$$

and

$$g^{\lambda\sigma} u_\lambda u_\sigma = \frac{1}{1 + 2\alpha A_\mu u^\mu} (u_0 u_0 - u_i u_i) = 1$$

$$u_0 u_0 - u_i u_i = 1 + 2\alpha A_\mu u^\mu$$

Consequently, if one uses formula (5-13), (5-17) and (5-18), we obtain the following equation:

$$\left(\frac{E - \bar{e}\varphi}{c^2}\right)^2 - \left(\mathbf{P} - \frac{\bar{e}}{c}\mathbf{A}\right)^2 = \bar{m}_0^2 c^2 \tag{5-20}$$

$$E = \left[\bar{m}_0^2 c^4 + c^2 \left(\mathbf{P} - \frac{\bar{e}}{c}\mathbf{A}\right)^2\right]^{1/2} + \bar{e}\varphi \tag{5-21}$$

Finally, let us obtain Hamilton-Jacobi equation of a particle in the field. This equation is obtained by exchanging momentum $\mathbf{P}$ with $\partial S/\partial \mathbf{r}$ and E with $-\partial S/\partial t$. Therefore, from formula (5-20) is derived

$$\left(\mathrm{grad}S - \frac{\bar{a}}{c}\mathbf{K}\right)^2 - \frac{1}{c^2}\left(\frac{\partial S}{\partial t} + \bar{a} K_0\right)^2 + \bar{m}_0 c^2 = 0 \tag{5-22}$$

The formula (5-22) is similar to that of former theory. The difference are that m_0 and e regarded as constant in Maxwell theory were replaced by effective mass $\bar{m}_0$ and effective electric charge $\bar{e}$, which is very important and significant in the future.

Sect. 6 Lagrangian for field

This section is devoted to get the equation of field. The Lagrangian for the motion of a particle treated in sect. 5 does not obey principle of linear superposition. Accordingly, in general, the equation of field also becomes a non-linear equation not subject to principle of linear superposition. Of course, the field does not become the arithmetical sum of fields created by individual particles. But, unfortunately, in the paper, owing to mathematical poverty of authors, was not obtained non-linear differential equation, field equation in the most general form (nonlinear differential field equation in most general form) not subject to principle of superposition, whereas we get the approximate equation of field valid within some application limitation.

First of all, let us find new Lagrangian for field by generalizing Maxwell's equation of field to the equation of field established in four dimensional non-Euclidean space and next consider application limitation of the resultant field equation. Generalizing Maxwell's Lagrangian for field to non-Euclidean space, the result is

$$S_f = -\frac{1}{16\pi c}\int F_{ik} F^{ik} \sqrt{-g}\, d\Omega \tag{6-1}$$

where as understood easily, F_{ik} is asymmetry tensor and in terms of curved surface geometry, the form of F_{ik} coincides with that of Euclidean space. The only difference is integral volume multiplied by $\sqrt{-g}$. Therefore, the total Lagrangian integral formula can be written as follows:

$$S = -m_0 c \int \left(g_{\mu\nu} u^\mu u^\nu\right)^{1/2} ds - \frac{1}{16\pi c} \int F_{ik} F^{ik} \sqrt{-g} d\Omega \qquad (6-2)$$

or

$$S = -m_0 c \int \left(g_{\mu\nu} x^\mu x^\nu\right)^{1/2} dt - \frac{1}{16\pi c} \int F_{ik} F^{ik} \sqrt{-g} d\Omega$$

where $g_{\mu\nu} = \delta_{\mu\nu}\left(1 + 2\alpha A_\lambda u^\lambda\right)$ and $d\Omega = cdtdxdydz$. If one uses

$$F^{ik} = g^{i\lambda} F_\lambda^k = g^{i\lambda} g^{k\sigma} F_{\lambda\sigma} = -\frac{1}{\sqrt{-g}} F_{ik}$$

and regards the motion of a particle as being given, by variation principle is derived the equation of field

$$\frac{c}{4\pi} \frac{\partial F_{ik}}{\partial x^k} = -\frac{1}{1 + 2\alpha A_\mu u^\mu} V^i \delta(\mathbf{r} - \mathbf{r}_0) \qquad (6-3)$$

or

$$\frac{\partial F_{ik}}{\partial x^k} = -\frac{4\pi}{c} \bar{e} V^i \delta(\mathbf{r} - \mathbf{r}_0) \qquad (6-4)$$

In fact, equation (6-4) should include the non-linear component term destroying principle of linear superposition in left-hand side. However, now that we did not find this component, we can ignore the component, supposing that it is extremely small. That is why equation (6-4) is obviously the approximate equation.

Now let us consider application limitation of the equation. If one takes four-dimensional divergence on both sides of equation (6-4), the left-hand side of equation becomes zero and right-hand side leads to

$$\frac{\partial}{\partial x^i}\left(\bar{e} V^i\right) = \frac{\partial \bar{e}}{\partial x^i} V^i = \frac{d\bar{e}}{dt} \qquad (6-5)$$

by asymmetry of F_{ik}. The formula (6-5) is similar to the law of charge conservation which holds in case of $\bar{e} = e$. But in general effective charge is function of space-time, and then as $d\bar{e}/dt \neq 0$, conservation of effective charge is invalid. Therefore, only in case of

$$\frac{d\bar{e}}{dt} = 0 \qquad (6-6)$$

equation (6-4) is valid. That is to say, the application conditions of (6-4) is determined by formula (6-6) and they are as follows:

Firstly, when a charge is constant (i.e. in case of a free particle), the formula (6-6) is valid and then equation (6-4) holds.

Secondly, in the system of static particles is also satisfied formula (6-4). Actually in the system of static particles, as interaction term $A_\mu u^\mu$ in the form relevant to effective charge is constant, formula (6-6) holds.

Thirdly, for average electromagnetic field produced by the system of a particle in which $\overline{\frac{d\bar{e}}{dt}} = 0$ *is satisfied, the formula (6-4) is valid (where* $\overline{\frac{d\bar{e}}{dt}} = 0$ *is time-average of effective charge).* In case where electric charges moving within finite area possess finite momentum, this motion comes to have stationary characteristics and then we only can consider average electro-magnetic field produced by them. This field depends only on spatial coordinate but not on time coordinate. In this case, (6-4) becomes

$$\frac{\partial \overline{F_{ik}}}{\partial x^k} = -\frac{4\pi}{c} \overline{\bar{e} V^i} \delta(r - r_0) \qquad (6-7)$$

Fourthly, for field produced by charge moving in external field so that $\left|\frac{d\bar{e}}{dt}\right| \ll 1$ *is satisfied,* $\frac{d\bar{e}}{dt}$ *can be enough ignored and then formula (6-4) can hold approximately.* Now, let us consider this approximate condition

$$\frac{d\bar{e}}{dt} \to 2\alpha\left(1+2\alpha A_\mu u^\mu\right)^{-2}\frac{d}{dt}\left(eA_\mu u^\mu\right) \approx 2\frac{e^2}{mc^2}\frac{d\varphi}{dt} \qquad (6-8)$$

where e^2/mc^2 (about 10^{-17}) is the extremely small quantity and then except when field changes very rapidly, $\frac{d\bar{e}}{dt}$ can be enough ignored. Consequently, under the condition $\left|\frac{d\bar{e}}{dt}\right| \ll 1$, the equation (6-4) has enough validity.

For gravitational field, the field equation is also formally equal to the equation of electromagnetic field.

$$\frac{c}{4\pi}\frac{\partial R_{ik}}{\partial x^k} = -\frac{1}{1+2\alpha_{(g)}G_\mu u^\mu}V^i\delta(r-r_0) \qquad (6-9)$$

$$R_{ik} = \frac{\partial G_k}{\partial x^i} - \frac{\partial G_i}{\partial x^k}$$

The equation (6-9) includes Poisson equation for static gravitational field and this yields main theoretical results in the static field, obtained by Einstein.

Sect. 7 Energy-momentum tensor of particle-field

The total Lagrangian for matter and field can be written as follows:

$$S = \frac{1}{c}\int d\Omega\sqrt{-g}\Lambda = \int d\Omega\left(L_m + L_f\right) =$$

$$= \frac{1}{c}\int d\Omega\sqrt{-g}\left[-\frac{m_0 c}{\sqrt{-g}}\left(g_{\mu\nu}V^\mu V^\nu\right)^{\frac{1}{2}}\delta(\mathbf{r}-\mathbf{r}_0) - \frac{1}{16\pi}C_{ik}C^{ik}\right] \qquad (7-1)$$

where $d\Omega = cdtdxdydz$, L_m Lagrangian of matter, L_f Lagrangian of field, C_{ik} is F_{ik} in case of electromagnetic field and R_{ik} in case of gravitational field. If one introduces infinitesimal transformation $x'^i = x^i + \delta x^i$ and then uses the condition $\delta S = 0$, the well-known formula is obtained

$$\frac{\partial\sqrt{-g}T_i^k}{\partial x^k} = 0 \qquad (7-2)$$

$$T_{ik} = \frac{2}{\sqrt{-g}}\left[\frac{\partial}{\partial x^l}\left(\frac{\sqrt{-g}\Lambda}{\partial\frac{\partial g^{ik}}{\partial x^l}}\right) - \frac{\partial\sqrt{-g}\Lambda}{\partial g^{ik}}\right] = T_{ik}^{(m)} + T_{ik}^{(f)}$$

$$T_{ik}^{(m)} = \frac{1}{\sqrt{-g}}m_0 c u_i u_k \frac{ds}{dt}\delta(\mathbf{r}-\mathbf{r}_0) \qquad (7-3)$$

$$T_{ik}^{(f)} = -\frac{1}{4\pi}\left(C_{i\lambda}C_k^\lambda - \frac{1}{4}C_{lm}C^{lm}g_{ik}\right) \qquad (7-4)$$

where T_{ik} is total energy-momentum tensor and $T_{ik}^{(m)}$ energy-momentum tensor of particle and $T_{ik}^{(f)}$ energy-momentum tensor of field. When obtain formula (7-3) and formula (7-4), the followings were used:

$$C_{i\lambda} = -C_{\lambda i}, \qquad C^{kl} = g^{k\lambda}C_\lambda^l = g^{k\lambda}g^{l\sigma}C_{\lambda\sigma}, \qquad C^{kl} = \frac{-1}{\sqrt{-g}}C_{ki} \qquad (7-5)$$

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu} \qquad (7-6)$$

Now let us prove the conservation of energy-momentum in details. First of all, divergence of the energy-momentum tensor of a particle is

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}\left(\sqrt{-g}T_i^{k(m)}\right)=\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}\left[\sqrt{-g}\frac{1}{\sqrt{-g}}m_0 c u_i u^k\frac{ds}{dt}\delta(\mathbf{r}-\mathbf{r}_0)\right]=$$

$$=\frac{1}{\sqrt{-g}}m_0 c\left[\frac{\partial u_i}{\partial x^k}u_k\frac{ds}{dt}\delta(\mathbf{r}-\mathbf{r}_0)+\frac{\partial}{\partial x^k}\left(V^k\delta(\mathbf{r}-\mathbf{r}_0)\right)u_i\right]=\frac{1}{\sqrt{-g}}m_0 c\frac{du_i}{dt}\delta(\mathbf{r}-\mathbf{r}_0)$$

$$(7-7)$$

Next, divergence of energy-momentum tensor of field is

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}\left[\sqrt{-g}\left(-\frac{1}{4\pi}\right)\left(C_{il}C^{kl}-\frac{1}{4}C_{lm}C^{lm}\delta_i^k\right)\right]=$$

$$=-\frac{1}{4\pi}\cdot\frac{1}{\sqrt{-g}}\left[\left(\frac{\partial C_{il}}{\partial x^k}\sqrt{-g}C^{kl}+C_{il}\frac{\partial\left(\sqrt{-g}C^{kl}\right)}{\partial x^k}-\frac{1}{2}\frac{\partial C_{lm}}{\partial x^i}\sqrt{-g}C^{lm}\right)\right]$$

If one uses the identity

$$\frac{\partial C_{lm}}{\partial x^i}=-\frac{\partial C_{mi}}{\partial x^l}-\frac{\partial C_{il}}{\partial x^m}$$

and rearrange the above formula, first term and third term are cancelled. Hence, the following term only remains.

$$-\frac{1}{4\pi}\cdot\frac{1}{\sqrt{-g}}C_{il}\frac{\partial\left(\sqrt{-g}C^{kl}\right)}{\partial x^k}=\frac{1}{4\pi}\cdot\frac{1}{\sqrt{-g}}C_{il}\frac{\partial\left(\sqrt{-g}C^{lk}\right)}{\partial x^k}$$

Therefore, divergence of energy-momentum tensor of field, allowing for formula (7-5) and formula (6-3), is as follows:

$$\nabla_k T_i^{k(f)}\frac{1}{4\pi}\cdot\frac{1}{\sqrt{-g}}C_{il}\frac{\partial\left(\sqrt{-g}C^{lk}\right)}{\partial x^k}=-\frac{1}{c\sqrt{-g}}C_{il}\bar{e}V^l\delta(\mathbf{r}-\mathbf{r}_0)\qquad(7-8)$$

Using formula (7-2), (7-7), (7-8), (5-10) and (6-6), divergence of the total energy-momentum t ensor of field and particle can be written as

$$\frac{\partial\left(\sqrt{-g}T_i^k\right)}{\partial x^k}=\frac{\partial\left(\sqrt{-g}T_i^{k(m)}\right)}{\partial x^k}+\frac{\partial\left(\sqrt{-g}T_i^{k(f)}\right)}{\partial x^k}=$$

$$=m_0 c\frac{du_i}{dt}\delta(\boldsymbol{r}-\boldsymbol{r}_0)-C_{ik}\frac{\bar{e}}{c}V^k\delta(\boldsymbol{r}-\boldsymbol{r}_0)=0\qquad(7-9)$$

Thus, in case where the effective charge agrees with static and stationary condition (6-6), i.e. $\frac{d\bar{e}}{dt}=0$, $\frac{\overline{d\bar{e}}}{dt}=0$, and $\left|\frac{d\bar{e}}{dt}\right|\approx 0$, formula (7-9) yields conservation law of energy-momentum.

Sect. 8 Electromagnetic-gravitational isotopic vector space and unified Lagrangian

In the sect. 6 and 7 was treated the electromagnetic field and gravitational field, separately. In this section, we obtain unified Lagrangian by subjecting electromagnetic field and gravitational field to one metric of space-time. First of all, we, in order to describe unified Lagrangian in a simple and easy mathematical form, introduce the electromagnetic-gravitational isotopic vector space which is essential to understand physical meaning of integral formula.

When electromagnetic field and gravitational field exist together and act on a charged particle, the metric of space-time is given as follows:

$$g_{\lambda\sigma} = \delta_{\lambda\sigma}\left(1 + 2\alpha^{(E)}A_K u^K + 2\alpha^{(g)}G_K u^K\right) \qquad (8-1)$$

where clearly $\alpha^{(E)}$ contains electric charge e and $\alpha^{(g)}$ contains mass m. Now instead of taking product of source of fields (e, m) and fields (A_K, G_K) to be separate sum as like (8-1), let us write product as the following simple form, i.e. $\alpha\left(\alpha^{(E)},\ \alpha^{(g)}\right) \cdot K^i\left(G^i, A^i\right)$, where α and K^i should be considered as vectors of the electromagnetic-gravitational isotopic vector space.

When a charged particle moves, there exist naturally current of charge and current of mass together and these current, as both sides of motion of particle, are subject to position and velocity of a particle. Likewise, electromagnetic field and gravitational field, as fields of two forms created by a particle, are no more separate and independent and then the total energy of a particle and its field is equivalent to inertial mass and subject to it. In this regard, let us define electromagnetic-gravitational isotopic vector space, based upon the idea that charge and mass are both properties of a particle, and electromagnetic field and gravitational field are both sides of unified field.

Easily speaking, isotropic vector space is two-dimensional space in which $\left(e, A_i, \alpha^{(E)}\right)$ and $\left(m, G_i, \alpha^{(g)}\right)$ are orthogonal each other. Let us carry the more tangible consideration about isotropic vector space.

1. The unit isotopic vector of electric charge, e, and electromagnetic field, A_i, are the same, i.e.

$$\hat{e} \cdot \hat{A}_i = eA_i, \quad \hat{\alpha}^{(E)} \cdot \hat{A}_i = \frac{\hat{e}}{mc^2} \cdot \hat{A}_i = \frac{e}{mc^2} A_i \qquad (8-2)$$

where ^ denote isotopic vector.

2. The unit isotopic vector of mass m and gravitational field G_i produced by it are the same.

$$\hat{m} \cdot \hat{G}_i = mG_i, \quad \hat{\alpha}^{(g)} \cdot \hat{G}_i = \frac{\hat{m}}{mc^2} \cdot \hat{G}_i = \frac{1}{c^2} G_i \qquad (8-3)$$

From formula (8-3) follows an important conclusion that the self-interaction of mass and self-field created by it is always positive value and accordingly, with radiation of gravitational wave by radiation damping, matter comes to lose energy. GR does not yield this conclusion. (See sect. 3)

3. The unit isotopic vector of electric charge $\hat{e}$ and electromagnetic field $\hat{A}_i$ are orthogonal to the unit isotopic vector of mass $\hat{m}$ and gravitational field $\hat{G}_i$.

$$\hat{e} \cdot \hat{m} = 0, \qquad \hat{e} \cdot \hat{G}_i = 0, \qquad \hat{m} \cdot \hat{A}_i = 0, \qquad \hat{A}_i \cdot \hat{G}_i = 0 \qquad (8-4)$$

This reaches the conclusion that electromagnetic field acts only on the charge and gravitational field acts only on mass and then exchange of photon is possible only between charges and exchange of graviton is possible only between masses.

4. The product of mass $\hat{m}$ and external gravitational field $\hat{G}_i$ acting on it is always negative value

$$\hat{m} \cdot \hat{G}_i = -mG_i \qquad (8-5)$$

This follows from the fact that gravitational force is always attractive. In this space, source of field is expressed as a single source, $\hat{a}(\hat{e}, \hat{m})$ and $\hat{\alpha}\left(\hat{\alpha}^{(E)}, \hat{\alpha}^{(g)}\right)$, and field also becomes a unified field $\hat{K}_i\left(\hat{A}_i, \hat{G}_i\right)$.

The electromagnetic-gravitational isotopic vector space makes it possible to denote Lagrangian of particle-field in a unified form and in very easy, clear form

$$S = -m_0 c \int \left(g_{\mu\nu} u^\mu u^\nu\right)^{1/2} ds - \frac{1}{16\pi c} \int \hat{C}_{ik} \hat{C}^{ik} \sqrt{-g}\, d\Omega \qquad (8-6)$$

where $g_{\mu\nu} = \delta_{\mu\nu}\left(1 + 2\hat{\alpha}\hat{K}_\lambda u^\lambda\right)$ and

$$\hat{\alpha} \to \left(\frac{\hat{e}}{mc^2}, \frac{\hat{m}}{mc^2}\right)$$

$$\hat{K}_i \to \left(\hat{A}_i, \hat{G}_i\right)$$

$$\hat{C}_{ik} = \frac{\partial \hat{K}_k}{\partial x^i} - \frac{\partial \hat{K}_i}{\partial x^k}$$

From this, the motion equation of particle (5-8) and (5-11) can be written as

$$m_0 c\frac{du_\lambda}{ds}=\frac{1}{c}\cdot\frac{m_0 c\hat{a}}{1+2\hat{\alpha}\hat{K}_i u^i}\cdot\hat{C}_{\lambda\sigma}u^\sigma-\hat{K}_\lambda\frac{d}{ds}\left(\frac{m_0 c\hat{a}}{1+2\hat{\alpha}\hat{K}_i u^i}\right) \qquad (8-7)$$

On the other hand, the field equation in isotropic vector space is

$$\frac{1}{4\pi}\frac{\partial\hat{C}_{ik}}{\partial x^k}=-\frac{1}{c}\cdot\frac{\hat{e}+\hat{m}}{1+2\hat{\alpha}\hat{K}_\mu u^\mu}V^i\delta(\mathbf{r}-\mathbf{r}_0) \qquad (8-8)$$

If one writes equation (8-8) separately according to components of isotopic vectors, the formula (6-3) and (6-10) are expressed as follow:

$$\frac{1}{4\pi}\frac{\partial\hat{F}_{ik}}{\partial x^k}=-\frac{1}{c}\cdot\frac{\hat{e}}{1+2\hat{\alpha}\hat{K}_\mu u^\mu}V^i\delta(\mathbf{r}-\mathbf{r}_0) \qquad (8-9)$$

$$\frac{1}{4\pi}\frac{\partial\hat{R}_{ik}}{\partial x^k}=-\frac{1}{c}\cdot\frac{\hat{m}}{1+2\hat{\alpha}\hat{K}_\mu u^\mu}V^i\delta(\mathbf{r}-\mathbf{r}_0) \qquad (8-10)$$

where

$$\hat{R}_{ik}=\frac{\partial\hat{G}_k}{\partial x^i}-\frac{\partial\hat{G}_i}{\partial x^k}$$

The energy and momentum tensor of electromagnetic-gravitational field is

$$T_{ik}^{(f)}=-\frac{1}{4\pi}\left(\hat{C}_{i\lambda}\hat{C}_k^\lambda-\frac{1}{4}\hat{C}_{lm}\hat{C}^{lm}g_{ik}\right) \qquad (8-11)$$

or

$$T_i^{k(f)}=-\frac{1}{4\pi}\left(\hat{C}_{i\lambda}\hat{C}^{k\lambda}-\frac{1}{4}\hat{C}_{lm}\hat{C}^{lm}\delta_i^k\right) \qquad (8-12)$$

3. Physical Analysis for Main Functions Characterizing Particle-Field

In this chapter, we introduce KR space, a non-Euclidean space and, based upon it, define starting postulate 5 postponed in sect. 4 and then further give the physical analysis of main functions.

So, for evolution of our theory, why do we have to be forced to receive so complicated and intricate non-Euclidean KR space? The whole history of physics proved that space and time were a form of existence of matter and geometry of space-time was determined by essential content of matter. Actually as well known in history of physics, in Newton's classical mechanics were introduced the relativity principle of Galileo and three-dimensional spatial geometry. But later, the starting idea of Einstein, especially the idea of invariance of light velocity, led to a revolution in our understanding of matter and then birth of four-dimensional space-time. And in 1916, from the principle of equivalence of Einstein's GR was introduced the non-Euclidean four-dimensional Riemann space as a realistic space. Likewise, the new starting idea which the total energy of particle-its field should be m_0c^2 and the principle of correspondence give naturally birth to KR space and the Lagrangian integral formula is defined in it.

Sect. 9 KR space

We know that Euclidean space is corresponded to any point (or infinitesimal space region) of Riemann space $g'_{\mu\nu}(x)$ and this correspondence is expressed as follows:

$$\delta_{\mu\nu} = g'_{\mu\nu}(x = x_0 = \text{constant}) \qquad (9-1)$$

where $g'_{\mu\nu}$ is Riemann metric tensor and $\delta_{\mu\nu}$ is Euclidean metric tensor. On the other hand, Riemann space is corresponded to any point of Finsler space $g_{\mu\nu}(x, \dot{x})$.

$$g'_{\mu\nu}(x) = g_{\mu\nu}(x, \dot{x} = \dot{x}_0 = constant) \qquad (9-2)$$

where $g_{\mu\nu}$ is Finsler metric tensor and $g'_{\mu\nu}$ Riemann metric tensor. In a word, Riemannian space includes Euclidean space and Finsler space includes Riemann space as a special form.

But non-Euclidean space discussed in this paper is neither Riemann space nor Finsler space. The Lagrangian and equations of particle and field seen and derived before were defined in a new non-Euclidian space. We rewrite the metric tensor considered in formula (6-1)

$$g_{\lambda\sigma} = \delta_{\lambda\sigma}\left(1 + 2\alpha K_\mu u^\mu\right) \qquad (9-3)$$

that is, $g_{\lambda\sigma} = g_{\lambda\sigma}(x_i, u_i)$ and the field function, K_λ, of formula (9-3) to be studied sect. 11 and sect. 16 has the following form:

$$K_\lambda \sim \frac{\underline{K_\lambda}}{\left(1 + 2\alpha K_\mu u^\mu\right)} = \underline{K_\lambda} g^{\mu\nu}\delta_{\mu\nu} \qquad (9-4)$$

where $\delta_{\mu\nu}$ is Minkowski metric tensor, $\underline{K_\lambda}$ the field function obtained in Minkowski space (one in Maxwell's covariant theory) and in case of electromagnetic field becomes $\underline{K_\lambda} = e\dot{x}_\lambda/r$. And in formula (9-4), $g^{\mu\nu}$ has the form

$$g^{\mu\nu} = \delta^{\mu\nu}\frac{1}{1 + 2\alpha K_\mu u^\mu}$$

On the other hand, in formula (9-3) we have

$$u^i = \frac{dx}{cdt(g_{lk}\dot{x}^l\dot{x}^k)^{1/2}} = u^i(\dot{x}, g) \qquad (9-5)$$

$$K_i = K_i(x, g)$$

Consequently, from formula (9-3), (9-4), (9-5) follows

$$g_{\lambda\sigma} = g_{\lambda\sigma}(x_i, \dot{x}_i, g) \qquad (9-6)$$

(※function g included in the right-hand side of formula (9-5) and (9-6) reflects the implicit character of function)

As shown in formula (9-6), space metric has a new character dependent on metric itself in addition to coordinates and velocity. In the formula (9-6) of metric tensor, because K_λ and u_λ is functions dependent on metric, g, implicit function of metric has the characteristic of absolute implicit function which cannot be, by any means, transformed into explicit function. In this connection, we define a new non-Euclidean space called "KR space". We obtain Finsler metric tensor $g_{\lambda\sigma}(x,\dot{x})$, fixing metric of KR space, $g_{\lambda\sigma}(x,\dot{x},g_{ik})$, by a Euclidean constant metric, η_{ik}

$$g_{\lambda\sigma}(x,\dot{x},g_{ik}=\eta_{ik})=g_{\lambda\sigma}(x,\dot{x}) \qquad (9-7)$$

Consequently, KR space is meant by the space whose metric function, as function of metric itself, owns implicit character of function and that Finsler space is corresponded to every points $(x,\dot{x})$ of KR space. Namely, when $g_{ik}=\eta_{ik}$, formula (9-6) naturally results in

$$g_{\lambda\sigma}=\delta_{\lambda\sigma}\left(1+2\alpha\underline{K_i}\underline{u^i}\right) \qquad (9-8)$$

$$\underline{u_i}=u_i(\dot{x}), \quad \underline{K_i}=K_i(x)$$

where $\underline{K_i}$ and $\underline{u_i}$ are four dimensional functions in Maxwell's theory, defined in Minkowski space.

- **The normalization of metric**

Because of the character of absolute implicit function of metric in KR space, it is impossible to say its realistic meaning and essence of metric, necessary for measurement. To do this, we introduce an idea of normalization of metric. g_{ik} can be fixed by arbitrary constant metric in every point $(x,\dot{x})$ of KR space, and so Finsler spaces corresponded to $(x,\dot{x})$ is also different.

We define the *normalization of metric* or *normalized metric* as obtaining the metric of Finsler space, fixing g_{ik} by Minkowski metric δ_{ik} in any point $(x,\dot{x},g)$ of KR space.

From this definition, we have

$$\bar{g}_{\lambda\sigma}=g_{\lambda\sigma}(x,\dot{x},g_{ik}=\delta_{ik})=g_{\lambda\sigma}(x,\dot{x}) \qquad (9-9)$$

where $\bar{g}_{\lambda\sigma}$ is the normalized metric. The metric obtained from formula (9-9) is obviously Finsler metric and has the realistic physical meaning in every point $(x_i,\dot{x}_i)$. (KR space has not been thoroughly studied mathematically and so in this paper we also did not offer the rigorous mathematical definition and analysis of KR space. Therefore, we think that there can or will be many faults in our description for this space) The normalization of metric is the key to arguing the normalization of implicit function in the next sections.

Sect. 10 The normalization of absolute implicit function and its physical meaning

As already seen, functions characterizing field and particle in KR space, from the character of implicit function of metric, also become implicit ones. The implicit function defined in KR space has the character that cannot transform into explicit function by any manner of means. These implicit functions are sometimes called *absolute implicit functions*. The absolute implicit function has the following general form

$$F=F\big(x,F(x,g),f(x,\dot{x},g)\big), \qquad g=g(x,\dot{x},F) \qquad (10-1)$$

where $f(x,\dot{x},g)$ is all functions dependent on metric g. For example, $F(x,g)$ may be either electromagnetic field potential, $A_\lambda(x,g)$, or gravitational field potential, $G_\lambda(x,g)$, and $f(x,\dot{x},g)$ be four-dimensional velocity in KR space, $u_\lambda(x,\dot{x},g)$.

Generally, if coordinates of space-time is given, the value of a function is uniquely defined. But in KR space, owing to the implicit character of the function, even though coordinates are given, the value of the function is not uniquely determined and so it is impossible to consider the real physical meaning of the function. In this regards, we will first see how to define implicit functions and give real physical meaning to implicit function in order to get measurable physical quantities.

The real physical meaning of implicit function, F, is determined by normalized function, $\bar{F}$. We, again, represent the starting postulate 5, postponed in sect. 4; Postulate 5: In unified theory of fields, every physical quantity is expressed as implicit function (correctly speaking, absolute implicit functions) and the functions that have real physical meaning are determined as explicit ones uniquely corresponded to Finsler space according to rule of normalization.

The normalization of implicit function is, in essential, related to normalization of metric tensor. It is referred to the fact that a function includes metric as a variable and metric also includes the function itself as a variable. From this fact follows the implicit character of function.

Now, we define rules of normalization.

1) As for implicit function, F, characterizing fields or interaction between particle and field in KR space, we fix g_{ik} which becomes variable of $F(x, g)$ and $f(x, \dot{x}, g)$ by Minkowski metric $\delta_{\mu\nu}$, and obtain $\bar{F}$ corresponded to Finsler space called "normalized function", which is defined as "normalization of implicit function", F.

$$\bar{F} = F\big(x, F(x, g|_\delta), f(x, \dot{x}, g|_\delta)\big) = F(x, \dot{x}) \qquad (10-2)$$

This rule is key to removing infiniteness of physical quantities and getting finite quantities

The normalization of four dimensional velocity is defined as follows: As for implicit function, u, which denotes four-dimensional velocity in KR space, we fix g_{ik} which be variable of u by Minkowski metric $\delta_{\mu\nu}$ and obtain $\bar{u}$ corresponded to Finsler space called *normalized four-dimensional velocity*, which is defined as normalization of implicit function, u, according to rule of normalization 1, namely,

$$\bar{u}_\lambda = u_\lambda(\dot{x}, g|_\delta) = u_\lambda(\dot{x}) \qquad (10-3)$$

$$\bar{g}_{\mu\nu} = \delta_{\mu\nu}\left(1 + 2\alpha\bar{K}_\lambda u^\lambda(\dot{x}, g|_\delta)\right) = \delta_{\mu\nu}\left(1 + 2\alpha\bar{K}_\lambda u^\lambda(\dot{x})\right) \qquad (10-4)$$

where the field function, $\bar{K}_\lambda$, already normalized according to rule 1 was used as it is.

2) For a function $\varepsilon(x, F)$, the normalized function $\bar{\varepsilon}$ is as follows:

$$\bar{\varepsilon} = \bar{\varepsilon}(x, \bar{F}) \qquad (10-5)$$

where $\bar{F}$ is normalized function of F.

3) In case where a function ε consists of series of $\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n$, normalization of ε is the same as the sum of normalization of individual terms, i.e.

$$\varepsilon = \bar{\varepsilon}_1 + \bar{\varepsilon}_2 + \cdots + \bar{\varepsilon}_n \qquad (10-6)$$

This idea and rules of normalization become one of important starting points and are essential to establish unified theory of fields. If so, what are true essence and significance of the introduction of implicit function and its normalization? Why should physical quantities be expressed as implicit function and what is its true meaning? If one first says conclusion, what physical quantity is expressed as implicit function is because of feedback of particle-field. Until now, in history of physics has been considered only that a particle, through field, acts on another particle or is acted on by the field created by other particle. But as well known, a particle acts on itself through field produced by the particle. That is to say, particle-field has the character of feedback.

Particle → field (field is created by a particle)

Field → particle (the field acts on the particle)

The result becomes $F = F\big(x, F(x, g), f(x, \dot{x}, g)\big)$, But in case of introducing this feedback to former theory (Maxwell-Lorentz covariant theory), it was well known that unavoidable divergence occurs. As we shall see later, this problem of divergence in KR space is very easily solved, referring to the fact that in KR space, the character of feedback of particle-field is reflected in metric and functions characterizing physical quantities, and so physical quantities come to be expressed as implicit function.

If so, why does feedback in theory of field appear at all? The modern "systems theory" has already clarified that feedback is an important aspect of the nature of matter. As far as external action is applied to matter, some effect surely occurs. Here external action means factors giving birth to effect. For example, in physics external action is meant by external force and in biology it is the change of external environment. According to systems theory, the effect occurred is applied again to external action giving birth to it. Furthermore, feedback brings about reaction, which is rooted in self-maintenance or self-resistance character of matter which is going to diminish effect by external action as possible as it can. For example, in case of Newtonian mechanics reaction occurs owing to inertia of an object and in case of electrodynamics there exists radiation damping by feedback of radiation wave. Further in case of biology, whenever temperature of surrounding increases, in organism arises endothermic reaction and whenever temperature of surrounding decreases, occurs exothermic reaction to maintain constant temperature of organism. Consequently, feedback is based on the nature of matter and implicit function is mathematical manifestation of feedback. From this, manifestation of implicit function is not artificially introduced but comes naturally from the nature of matter. This is a very important conclusion. On the other hand, matter has wave-particle dualism. In quantum mechanics, one introduced wave model and wave function, and defined real physical meaning of wave function as $|\psi|^2$ in order to describe characteristic of wave of matter. In a similar way to this, in our theory, in order to remove infinite quantities and clarify characteristic of particle without any contradiction, implicit function (inevitable result of feedback character in model of point particle) was introduced and real physical meaning of implicit function defined as the normalized implicit function $\bar{F}$. The description of wave property and particle property has interesting symmetry (table 1). Thus, in order to describe mathematically wave property of matter, just as introduction of complex wave function is indispensable, so for the consistent description of particle property should be introduced implicit function and for clarification of its physical meaning should be defined normalization of implicit function. As we will see in next sections, finiteness of physical quantities and unification of electromagnetic-gravitational field are obtained based upon this idea of normalization.

Table 1

	wave property	**particle property**
model	wave (realistically no exist)	point (realistically no exist)
mathematical description	complex wave function (unmeasurable quantity)	implicit function in KR space (unmeasurable quantity)
physical meaning	defined as $|\psi|^2$	defined as normalized function

4. Theory of Electromagnetic Field in KR Space (Nonlinear Theory of Electromagnetic Field)

In Physics, Maxwell's theory has been recognized as the complete and unique theory for electromagnetic field. The Maxwell's theory was never established so deductively like Einstein's GR. It is well known that Maxwell, by generalizing and systemizing experimental laws discovered at those days, established the well-defined and beautiful theory for electromagnetic field. Of course, it is too far gone to view that Maxwell's theory was built only by inductive method. In fact, Maxwell, generalizing Ampere law so that the law of charge conservation is in agreement with a generalized form, discovered displacement current and established an equation system of field with four-dimensional covariant forms and then gave the wonderful prediction for generation of electromagnetic wave. This course for building of theory can be viewed to be based on inductive method. But, mainly clinging to inductive method, Maxwell did not complete the theory so that, on the basis of such main principles of physics as law of energy-momentum conservation, the total energy of particle and field possesses finite value.

The experimental laws which underlie Maxwell's theory clearly were discovered and argued within weak field. The weak field is meant by the field which interactional energy of particles is much less than rest energy of a particle m_0c^2. In this case $e^2/m_0c^2 \ll 1$ is satisfied and is dealt with interaction between particle and field in distance father than electron radius $e^2/m_0c^2 \approx 10^{-13}cm$. Simply put, in Maxwell's theory was considered the field made by a macro-charge and electric current. But there is no special reason that experimental laws discovered within weak field should always be in agreement with those in strong field. Actually in scientific view, it is reasonable that new characteristics which cannot be found in weak field is manifested in strong field. But, it is impossible that one carries out all experiments in the whole area comprising not only weak field but also strong field. So, that one uses inductive method as well as deductive method based on main principles of physics to establish more generalized and consistent theory is essential for building of theory. In this regard, main difficulties of Maxwell's theory are rooted in concluding that the experimental facts obtained in weak field are universal truths in the whole region comprising weak field and strong field and making no advance into establishment of more generalized and consistent theory.

Sect. 11 The equation of electrostatic field and variation of Coulomb's law

From equation (6-3) and (6-4) can be easily obtained equation of electrostatic field

$$\nabla^2\varphi = -4\pi\frac{e}{1+2\alpha\varphi u^0}\delta(\mathbf{r}-\mathbf{r}_0) \qquad (11-1)$$

where $\alpha = e/m_0c^2$. This equation, from the character of electrostatic field, satisfies the condition $d\bar{e}/dt = 0$ and from this equation we can obtain the solution of potential of electrostatic field. Supposing that right-hand side of this equation, as density of effective charge, is known at spatial coordinate, $\mathbf{r}_0$, we can find the solution of the above equation by the well-known method

$$\varphi = \int \frac{e}{r(1+2\alpha\varphi u^0)}\delta(\boldsymbol{r}-\boldsymbol{r}_0)dV = \frac{e}{r(1+2\alpha\varphi(\boldsymbol{r}_0)u^0)} + C \qquad (11-2)$$

where $\varphi(\mathbf{r}_0)$ is the potential at spatial coordinate, $\mathbf{r}_0$, where charge is placed. $\varphi(\mathbf{r}_0)$ consists of two components, i.e. $\varphi^{ex}(\mathbf{r}_0)$ (the potential in external field) and $\varphi^{in}(\mathbf{r}_0)$ (the potential in self-field)

$$\varphi(\boldsymbol{r}_0) = \varphi^{ex}(\boldsymbol{r}_0, g) + \varphi^{in}(\boldsymbol{r}_0, g) \qquad (11-3)$$

From formula (11-3), (11-2) for $\varphi(\mathbf{r}_0, g)$, we get

$$\varphi(r, \mathbf{r}_0) = \frac{e}{r\left[\left(1+2\alpha\left(\varphi^{ex}(\mathbf{r}_0, g)+\varphi^{in}(\mathbf{r}_0, g)\right)u^0\right)\right]} + C \qquad (11-4)$$

where potential in external field $\varphi^{ex}(\mathbf{r}_0)$ is given automatically by the initial condition.

Now, let us find $\varphi^{in}(\mathbf{r}_0)$ from formula (11-4). The field produced by a charge, e, acting on the charge itself can be define as follows.

$$\varphi^{in}(\mathbf{r}_0) = \lim_{r\to 0}\varphi(r,\mathbf{r}_0) = \lim_{r\to 0}\frac{e}{r\left[\left(1+2\alpha\left(\varphi^{ex}(\mathbf{r}_0,g)+\varphi^{in}(\mathbf{r}_0,g)\right)u^0\right)\right]} + C \quad (11-5)$$

where $\left[\left(1+2\alpha\left(\varphi^{ex}(\mathbf{r}_0,g)+\varphi^{in}(\mathbf{r}_0,g)\right)u^0\right)\right]$ is metric component of KR space. Because of $g = g\left(x,\dot{x},\varphi^{in}(\mathbf{r_0},g)u^0\right)$, $\varphi^{in}(\mathbf{r_0})$ results in absolute implicit function. The normalization of implicit function φ^{in} subject to normalization rules (10-2) and (10-3) yields $\bar{\varphi}^{in}(\mathbf{r}_0) = \varphi^{in}(\mathbf{r}_0,\bar{g})$

$$\bar{\varphi}^{in}(\mathbf{r}_0) = \lim_{r\to 0}\varphi(r,\mathbf{r}_0,\bar{g}) = \lim_{r\to 0}\frac{e}{r\left[\left(1+2\alpha\left(\varphi^{ex}(\mathbf{r}_0,\bar{g})+\varphi^{in}(\mathbf{r}_0,\bar{g})\right)\underline{u}^0\right)\right]} + C$$

$$= \lim_{r\to 0}\frac{e}{r\left[\left(1+2\alpha\left(\underline{\varphi}^{ex}(\mathbf{r}_0)+\underline{\varphi}^{in}(\mathbf{r}_0)\right)\underline{u}^0\right)\right]} + C \qquad (11-6)$$

(See formulas (9-4), (9-5), (9-8)), where $\underline{\varphi}^{in}$ and $\underline{\varphi}^{ex}$ are potentials defined in Minkowski four-dimensional space-time in Maxwell's theory and $\underline{u}^0 = u^0(\dot{x},g|_\delta) = 1$. Thus, formula (11-6) gives

$$\bar{\varphi}^{in}(\mathbf{r}_0) = \lim_{r\to 0}\frac{e}{r\left[1+2\alpha\left(\frac{e}{r}+\underline{\varphi}^{ex}(\mathbf{r}_0)\right)\right]} + C = \frac{m_0c^2}{2e} + C$$

$$(11-7)$$

On the other hand, according to the starting postulate 1, the Lagrangian for the motion of free-charge should be the same as one in special theory of relativity (SR). Thus, formula (11-7) gives

$$\bar{\varphi}^{in}(\mathbf{r}_0) = \frac{m_0c^2}{2e} + C = 0$$

$$C = -\frac{m_0c^2}{2e} \qquad (11-8)$$

Therefore, electrostatic potential can be written as follows.

$$\begin{cases} \varphi = \dfrac{e}{r\left[\left(1+2\alpha\left(\bar{\varphi}^{ex}(\mathbf{r}_0)+\bar{\varphi}^{in}(\mathbf{r}_0)\right)\right)\right]} + C \\ \bar{\varphi}^{in}(\mathbf{r}_0) = 0 \\ C = 0 \end{cases} \qquad (11-9)$$

In formula (11-9), C constant, because field vanishes at infinity far from source charge, should become zero. It is remarkable that, in our theory, a constant of electrostatic potential is not such arbitrary value as in Maxwell's theory but one determined uniquely under some physical condition.

The formula (11-9) yields important conclusions.

1. As easily understandable in formula (11-9), the electrostatic potential depends on external field φ^{ex}, and so our theory does not agree with principle of superposition; that is, the field is not equal to the arithmetical sum of fields produced by individual charges.

2. In formula (11-9), as C constant is defined uniquely, one of difficulties of Maxwell's theory seen in sect. 2, inconsistency which the energy of system loses the physical meaning is very easily solved.

3. From formula (11-9) follows the conclusion according to which electron and positron cannot approach infinitely near.

The interactional energy of electron-positron can be written as follows.

$$E = -\frac{e^2}{r\left(1-2\dfrac{e^2}{m_0c^2}\cdot\dfrac{1}{r}\right)} \qquad (11-10)$$

In case that electron and positron approach the region of electron radius $r = 2e^2/m_0c^2 \approx 10^{-13}cm$, interactional energy, E diverges. In order that interactional energy possesses finite value, e^2 in denominator should vanish. From this is drawn a new conclusion that electron radius $2e^2/m_0c^2$ is the critical distance which electron and positron can approach and if electron and positron approaches this distance, annihilation of electron and positron and production of new particles should follow. Figuratively speaking, this is such a similar situation as the relation between mass and velocity in SR; when matter approaches the velocity of light, rest mass should become zero and then photon with rest mass of zero is predicted.

4. The potential formula (11-9) help us resolve the problem of divergence of the total energy of particle-field (it will be seen in sect. 13).

Sect. 12 The equation of electromagnetic field and finiteness of radiation damping

Here, we drive the equation of electromagnetic field and resolve the problem for infiniteness of radiation damping potential, i.e. a fatal flaw of Maxwell's theory. We rewrite equation of field (6-3) [1]

$$\frac{\partial F_{ik}}{\partial x^k} = -\frac{4\pi}{c}\bar{e}V^i\delta(\mathbf{r}-\mathbf{r}_0) \qquad (12-1)$$

The substitution of F_{ik} expressed by potentials into equation (12-1) gives

$$\frac{\partial^2 A_k}{\partial x^i \partial x^k} - \frac{\partial^2 A_i}{\partial x^k \partial x^k} = -\frac{4\pi}{c}\bar{e}V^i\delta(\mathbf{r}-\mathbf{r}_0) \qquad (12-2)$$

Taking four dimensional divergences on both sides of equation (12-2), the left-hand side from definition of F_{ik} becomes zero

$$\nabla_i\left(\frac{\partial F_{ik}}{\partial x^k}\right) = 0 \qquad (12-3)$$

On the other hand, the right-hand side becomes zero, provided that the approximate condition $d\bar{e}/dt \approx 0$ considered in sect. 6 holds. So, equation (12-2) is satisfied.

According to theory for equation of partial differentiation well known in mathematics, as for partial differentiation equations with 4 unknowns, if there exists one identity between unknowns, independent unknowns are 3 and then one equation can be arbitrarily chosen. In this regards, an additional condition is given as follows:

$$\frac{\partial A_i}{\partial x^i} = 0 \qquad (12-4)$$

In Maxwell's theory, in case where equation (12-4) is satisfied, one can take a gauge transformation $A_i' = A_i + \frac{\partial f}{\partial x^i}$, where f obeys the following wave equation.

$$\Delta f - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0 \qquad (12-5)$$

But as Gauge invariance is not satisfied in our theory, the condition like equation (12-5) cannot hold. In relation with the argument mentioned above, some books described as if the Lorentz condition (12-4) was drawn from gauge invariance. But, this is wrong. For example, Einstein's GR, from nonlinear character of equations, does not obey Gauge invariance, but by the theory of partial differentiation equation is obtained an additional condition similar to equation (12-4). In Riemannian space, the equation of gravitational field is written as follows.

$$R_{ik} - \frac{1}{2}g_{ik}R = \frac{8\pi k}{c^4}T_{ik} \qquad (12-6)$$

Taking four-dimensional divergences on both of sides, the left-hand side and right-hand side become zero

$$\nabla_i\left(R_{ik} - \frac{1}{2}g_{ik}R\right) = 0 \qquad (12-7)$$

Thus, among 10 unknowns, independent unknowns are 6, remained 4 unknowns can be arbitrarily chosen. So we can give an additional condition

$$g^{\mu\nu}\Gamma_{\mu\nu}^{\lambda} = 0 \qquad (12-8)$$

where $\Gamma_{\mu\nu}^{\lambda}$ is Christoffel symbol, which consists of the sum of partial differentiation with respect to $g_{\mu\nu}$.

From the weak field approximation, $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$ ($h_{\mu\nu}$ is potential of weak field) and $\psi_i^k = h_i^k - \frac{1}{2}\delta_i^k h$ are allowed, and formula (12-8) leads to

$$\frac{\partial \psi_i^k}{\partial x^k} = 0 \qquad (12-9)$$

The equation (12-9) is similar to equation (12-4) in Maxwell's theory. In GR, using the equation (12-9), one can obtain the linear partial differential equation similar to that in Maxwell's theory and derive a formula for radiation of gravitational wave from this equation. All of these show that equation (12-4) in our theory and an additional condition (12-9) in GR follow from character of field equation itself, not from gauge principle.

Now, let us return to the main subject and if one substitutes equation (12-4) into equation (12-2), then field equation (12-2) becomes

$$\Box A_i = -\frac{4\pi}{c}\bar{e}V^i\delta(\mathbf{r} - \mathbf{r}_0) \qquad (12-10)$$

If we finds the solution of the equation (12-10) according to the well-known method of solution, we have

$$\varphi = \frac{\bar{e}(t - R/c)}{R} + \varphi_0 \ , \qquad \mathbf{A} = \frac{1}{c}\cdot\frac{\bar{e}\mathbf{V}(t - R/c)}{R} + \mathbf{A}_0 \qquad (12-11)$$

where φ_0 and $\mathbf{A}_0$ mean external field and are not arbitrary. In formula (12-11) effective charge, source of field, as shown in formula (5-9), depends on the interaction Lagrangian of particle and field, $(A_\lambda u^\lambda)$ and then stands against principle of superposition. This fact is a key to solve the problem of radiation damping, recognized as knotty point, "greatest crisis in Maxwell's theory" [2].

In order to get potentials of radiation damping produced by a moving charge, the following approximation is used.

1) Suppose that the distance between a moving particle and a system of charges which create external field acting on it is much farther than the radius of electron $r_0 = e^2/m_0c^2$. In this case $\frac{e}{m_0c^2}A_\lambda u^\lambda \ll 1$ holds (see formula (5-5)). The approximate formula (5-4) is obtained from condition $\frac{e}{m_0c^2}A_\lambda u^\lambda \ll 1$

$$dS \approx -m_0c^2\int dt\left(1 - \frac{V^2}{c^2}\right) - \frac{e}{c}\int A_\lambda dx^\lambda \qquad (12-12)$$

where $A_\lambda = A_\lambda^{in} + A_\lambda^{ex}$ and A_λ^{in}, as a finite quantity, is supposed to be very small quantity (order $1/c^3$).

2) Suppose the equation of field produced by a moving particle satisfies enough the condition $|d\bar{e}/dt| \ll 1$ or formula (6-6). In this case, we can write equation (12-10) by the vector potential and scalar potential of the field, respectively

$$\Box \boldsymbol{A} = -\frac{4\pi}{c}\cdot\frac{e\mathbf{V}}{1 + 2\alpha(A_\lambda^{in} + A_\lambda^{ex})u^\lambda}\delta(\mathbf{r} - \mathbf{r}_0)$$

$$\Box\varphi = -4\pi\cdot\frac{e}{1 + 2\alpha(A_\lambda^{in} + A_\lambda^{ex})u^\lambda}\delta(\mathbf{r} - \mathbf{r}_0)$$

$$(12-13)$$

From the well-known method of solution are obtained the following solutions

$$\mathbf{A} = -\frac{1}{c}\cdot\frac{e\mathbf{V}\left(t - \frac{R}{c}\right)}{R\left[1 + 2\alpha\left(A_{\lambda(1)}^{in} + A_\lambda^{ex}\right)u^\lambda\right]} + \mathbf{C}_1 \qquad (12-14)$$

$$\varphi = \frac{e\left(t - \frac{R}{c}\right)}{R\left[1 + 2\alpha\left(A_{\lambda(1)}^{in} + A_\lambda^{ex}\right)u^\lambda\right]} + C_2 \qquad (12-15)$$

where A_λ^{ex} is the potential of external field and $A_{\lambda(1)}^{in}$ term of first order of expansion of field potential acting on particle itself in powers of R/c. Terms more than second order have more than $1/c^4$ which in our consideration was ignored.

Under the condition $V \ll c$, expansion of potential gives

$$\varphi = b\frac{e}{R} - \frac{1}{c}\cdot\frac{\partial e}{\partial t} + b\frac{e}{2c^2}\cdot\frac{\partial^2 R}{\partial t^2} - b\frac{e}{6c^3}\cdot\frac{\partial^3 R^2}{\partial t^3} = \varphi_{(1)} + \varphi_{(2)} + \varphi_{(3)} + \varphi_{(4)} \qquad (12-16)$$

$$\mathbf{A} = b\frac{1}{c}\cdot\frac{e\mathbf{V}}{R} - b\frac{e}{c^2}\cdot\frac{\partial \mathbf{V}}{\partial t} = \mathbf{A}_{(1)} + \mathbf{A}_{(2)} \qquad (12-17)$$

$$b = \frac{1}{1 + 2\alpha\left(A_{\lambda(1)}^{in} + A_\lambda^{ex}\right)u^\lambda}$$

where $\varphi_{(1)}$, $\varphi_{(2)}$, $\varphi_{(3)}$ and $\varphi_{(4)}$ are terms of first order, second order , third order and fourth order of a power series of φ in R/c and $\mathbf{A}_{(1)}$ and $\mathbf{A}_{(2)}$ are terms of first order and second order of a power series of $\mathbf{A}$ in R/c.

Next, if one lets R (radius of interaction) go to zero to get potentials of radiation damping, φ^{in} and $\mathbf{A}^{in}$ in formula (12-16) and (12-17) are found from

$$\varphi^{in} = \lim_{R\to 0}\varphi\,, \qquad \mathbf{A}^{in} = \lim_{R\to 0}\mathbf{A} \qquad (12-18)$$

where φ^{in} and $\mathbf{A}^{in}$, as $\varphi^{in} = \varphi^{in}(x,g)$ and $\mathbf{A}^{in} = \mathbf{A}^{in}(x,g)$, are implicit functions.

Now, if one, according to normalization rule 4 defined in sect. 10, normalizes φ^{in} and $\mathbf{A}^{in}$, the results are

$$\bar{\varphi}^{in} = \bar{\varphi}_{(1)}^{in} + \bar{\varphi}_{(2)}^{in} + \bar{\varphi}_{(3)}^{in} + \bar{\varphi}_{(4)}^{in} \qquad (12-19)$$

$$\bar{\boldsymbol{A}}^{in} = \bar{\boldsymbol{A}}_{(1)}^{in} + \bar{\boldsymbol{A}}_{(2)}^{in} \qquad (12-20)$$

With reference of potential formula (11-5), according to normalization rule 1, let us normalize $\varphi_{(1)}^{in}$ and $\mathbf{A}_{(1)}^{in}$ (divergent quantities). The normalized functions result in $\bar{\varphi}_{(1)}^{in} = \varphi(x,\bar{g})$, $\bar{\mathbf{A}}_{(1)}^{in} = \mathbf{A}(x,\bar{g})$ and $\bar{g}$ is the normalized metric tensor. Consequently, first order terms, $\varphi_{(1)}^{in}$ and $\mathbf{A}_{(1)}^{in}$ become

$$\bar{\varphi}_{(1)}^{in} = \lim_{R\to 0}\frac{e}{R\left[1 + 2\alpha\left(\frac{e}{R} - \frac{e}{R}\cdot\frac{V^2}{c^2}\right) + 2\alpha A_\lambda^{ex}u^\lambda\right]} + C_0 = \frac{1}{2\alpha(1-\beta^2)} + C_0 \qquad (12-21)$$

$$\bar{\mathbf{A}}_{(1)}^{in} = \lim_{R\to 0}\frac{1}{c}\cdot\frac{e\mathbf{V}}{R\left[1 + 2\alpha\left(\frac{e}{R} - \frac{e}{R}\cdot\frac{V^2}{c^2}\right) + 2\alpha A_\lambda^{ex}u^\lambda\right]} + \mathbf{C}_1$$

$$= \frac{1}{c}\cdot\frac{\mathbf{V}}{2\alpha(1-\beta^2)} + \mathbf{C}_1 \quad (12-22)$$

(see formula (9-8), (9-9)). Now, if we finds C_0 and $\mathbf{C}_1$ so that $\frac{1}{c}\bar{A}_{\lambda(1)}^{in}\dot{x}^\lambda = 0$, we have

$$\frac{1}{c}\bar{A}_{\lambda(1)}^{in}\dot{x}^\lambda = \varphi_{(1)}^{in} - \frac{1}{c}\mathbf{A}_{(1)}^{in}\mathbf{V} = \frac{(1-\beta^2)}{2\alpha(1-\beta^2)} + C_0 - \frac{\mathbf{V}}{c}\mathbf{C}_1 = 0$$

If one puts in $\mathbf{C}_1 = \mathbf{0}$ and $C_0 = -1/2\alpha$, formula (12-22) has

$$\frac{1}{c}\bar{A}_{\lambda(1)}^{in}\dot{x}^\lambda = 0 \qquad (12-23)$$

In the Lagrangian integral formula (12-1) for the motion of a charge, the term of first order of interaction with self-field, i.e. $\frac{e}{c}\bar{A}_{\lambda(1)}^{in}dx^\lambda$, vanishes. Thus, $\left(\bar{\varphi}_{(1)}^{in}, \bar{\mathbf{A}}_{(1)}^{in}\right)$ have not influence on the potential of radiation damping. $\bar{\varphi}_{(2)}^{in}$ vanishes because e in $\bar{\varphi}_{(2)}^{in}$ term is constant and then the differentiation of time is zero. Next, in $\bar{\varphi}_{(3)}^{in}$ term, according to normalization rule 3, $A_{\lambda(1)}^{in}$ in denominator of b should be replaced by already normalized potential $\bar{A}_{\lambda(1)}^{in}$ and $\bar{A}_{\lambda(1)}^{in}\underline{u}^\lambda$ becomes zero in terms of formula (12-

23). Accordingly, $\bar{\mathbf{E}}^{in}_{(3)}$ expressed by $\bar{\mathbf{E}}^{in}_{(3)} = \mathrm{grad}\bar{\varphi}^{in}_{(3)}$ becomes

$$\bar{\mathbf{E}}^{in}_{(3)} = \lim_{R\to 0} \mathrm{grad}\bar{\varphi}^{in}_{(3)} = b\frac{e}{2c^2}\ddot{\mathbf{n}} \qquad (12-24)$$

with

$$\dot{\mathbf{n}} = \frac{\partial}{\partial t}\frac{\mathbf{R}}{R}$$

where b has 1 from $\frac{e}{c}\bar{A}^{in}_{\lambda(1)}dx^{\lambda} = 0$ and $\mathbf{n}$ is unit vector of radius vector $\mathbf{R}$ from the charge to the given field point. Supposing time change of the unit vector is very slow, so we can put in $\bar{\mathbf{E}}^{in}_{(3)} = \mathbf{0}$. Finally, $\bar{\varphi}^{in}_{(4)}$ does not go to zero. In Maxwell's theory, $\varphi_{(4)}$, by gauge transformation, leads to zero. But in our theory as gauge principle is not allowed, $\varphi_{(4)}$ cannot be transformed to zero. b in $\bar{\varphi}^{in}_{(4)}$ term is equal to 1 in same way as in case of $\bar{\varphi}^{in}_{(3)}$, that is why, $\bar{\varphi}^{in}_{(4)}$ is the same as fourth term of power series in Maxwell's theory (The dependence of $\bar{\varphi}^{in}_{(4)}$ on external field is ignored in our consideration, because it includes high order term more than $1/c^4$).

Now, let us find electric force $\mathbf{E}^{in}_{(4)}$ from $\bar{\varphi}^{in}_{(4)}$

$$\mathbf{E}^{in}_{(4)} = -\mathrm{grad}\bar{\varphi}^{in}_{(4)} = \frac{e}{6c^3}\cdot\frac{\partial^3}{\partial t^3}(\nabla R^2) = \frac{e}{3c^3}\cdot\frac{\partial^3\mathbf{R}}{\partial t^3}$$

$$\mathbf{R} = \mathbf{R_0} - \mathbf{r}, \qquad \dot{\mathbf{R}} = -\dot{\mathbf{r}} = -\mathbf{V}, \qquad \ddot{\mathbf{R}} = -\dot{\mathbf{V}}, \qquad \dddot{\mathbf{R}} = -\ddot{\mathbf{V}}$$

where $\mathbf{R_0}$ is the distance from the reference point to the given field point and $\mathbf{r}$ from the reference point to the charge. Therefore, the result is

$$\mathbf{E}^{in}_{(4)} = -\frac{e}{3c^3}\ddot{\mathbf{V}} \qquad (12-25)$$

Next, let us get $\mathbf{E}^{in}_{(2)}$ from $\bar{\mathbf{A}}^{in}_{(2)}$. As the method is formally the same as finding $\mathbf{E}^{in}_{(4)}$, we write only result

$$\mathbf{E}^{in}_{(2)} = -\frac{1}{c}\frac{\partial\bar{\mathbf{A}}^{in}_{(2)}}{\partial t} = \frac{1}{c^3}e\ddot{\mathbf{V}} \qquad (12-26)$$

where normalization of $\bar{\mathbf{A}}^{in}_{(2)}$ was done in same way as in $\bar{\varphi}^{in}_{(2)}$. Thus, the force of radiation damping can be written as follows.

$$\mathbf{F}_R = e\mathbf{E}^{in}_{(4)} + e\mathbf{E}^{in}_{(2)} = \frac{2}{3c^3}e\ddot{\mathbf{V}} \qquad (12-27)$$

The result is similar to that of Maxwell's theory. But there is the essential difference in content. First, in our theory, infinite divergence vanishes naturally and without any contradiction. Second, in our theory was discussed radiation damping effect without gauge transformation, from the fact that gauge principle is not valid and gave the formula consistent to experiment of radiation damping.

We used an approximate method for description of sect. 11 and sect. 12. As for this, there will be, we are sure, readers who express dissatisfaction. In fact, the left-hand side of field equation is the same as in Maxwell's theory. We are sure that a completed nonlinear equation of field will be, in the future, obtained. But even though new solution for nonlinear equation is found, it has not influence on our description for natural elimination of infinite quantities with method of normalization. It is due to the fact that as far as in right -hand side of equation there exists current of effective charges, functions of potential field are always expressed as implicit function and our method according to which eliminates divergence using rules of normalization cannot be altered but applied as it is. On the other hand, in case of expanding potentials as a power series, under some condition $(V/c \approx 1)$, divergence is always manifested in term of first order of expansion. This shows that even if completed potential equation and solution is, in the future, founded, correction newly added to former solution will be reflected in higher order term, not first order term and then not give rise to divergence. Thus in our argument, as divergence manifested in first order term is naturally removed by normalization of implicit function, this method will be still valid for completed nonlinear equation to be founded in the future.

Sect. 13 Breaking of gauge symmetry and its physical meaning, equivalence of inertial mass and total energy of particle-field

(1) Breaking of gauge symmetry and its physical meaning,

In this sect., first of all, we intend to argue some problems related to breaking of gauge symmetry. As a result necessarily following from Lagrangian integral (5-1), our electrodynamics with non-linearity does not agree with gauge symmetry. Actually, modern theory of fields (except gravitation) is based upon gauge symmetry. Hence, breaking of gauge symmetry brings about great impact to not only classical theory of field but also quantum theory of field. But, careful study on physical meaning which gauge symmetry breaking involves provides the key to getting deeper and comprehensive, further new understanding of physical interaction and matter.

Now, let us grasp physical essence of gauge symmetry. In Maxwell's theory, the Lagrangian for the motion of a charge is invariant under gauge transformation. That is, when

$$A'_\lambda = A_\lambda - \frac{\partial f}{\partial x^\lambda}, \qquad (13-1)$$

a new additional term appears in Lagrangian integral formula.

$$\frac{e}{c}\cdot\frac{\partial f}{\partial x^\lambda}dx^\lambda = d\left(\frac{e}{c}f\right) \qquad (13-2)$$

As this term is perfect differentiation, it does not bring about any change in the equation of the motion of a charge. This result is drawn from the fact which charge is always a constant and the Lagrangian for interaction is linear. But, it is obvious that if a charge varies in time or the Lagrangian for interaction is nonlinear (for example, $\alpha\left(A_\lambda u^\lambda\right)^{1/2}$), formula (13-2) cannot hold. On the other hand, the field equation in Maxwell theory, as a linear partial differential equation satisfying gauge symmetry, obey the principle of linear superposition.

Summarizing arguments mentioned above, we arrive at the following conclusions.

First, charge, as a constant at any point of space and time, is always a conserved quantity. Accordingly, charge, during the motion, should not change and moreover be neither annihilated nor created.

Second, with principle of linear superposition, the field produced by total charge is the same as the sum of fields created by individual charges which comprise the system of charges. In this case, fields are independent and do not interfere each other, so we have

$$A^\lambda = A^\lambda_{(1)} + A^\lambda_{(2)} + \cdots + A^\lambda_{(n)} \qquad (13-3)$$

Third, the conservation of total charge of a system is the same as sum of conservations of individual charges. If a system of charges has discrete distribution, we have

$$\frac{\partial}{\partial t}\int \rho dV = \frac{\partial}{\partial t}\int\left(\sum_{i=1}^{n} e_i\delta(\mathbf{r}-\mathbf{r}_i)\right)dV$$

$$\frac{\partial}{\partial t}\int\left(\sum_{i=1}^{n} e_i\delta(\mathbf{r}-\mathbf{r}_i)\right)dV + \int\left(\sum_{i=1}^{n} e_iV_i\delta(\mathbf{r}-\mathbf{r}_i)\right)dS = 0 \qquad (13-4)$$

On the other hand, according to principle of linear superposition, formula (13-4) arrives at

$$\int\left(\frac{\partial e_1\delta(\mathbf{r}-\mathbf{r}_1)}{\partial t} + dive_1V_1\delta(\mathbf{r}-\mathbf{r}_1)\right)dV + \cdots$$

$$+\int\left(\frac{\partial e_n\delta(\mathbf{r}-\mathbf{r}_n)}{\partial t} + dive_nV_n\delta(\mathbf{r}-\mathbf{r}_n)\right)dV = 0 \qquad 13-5)$$

$$\begin{cases} \dfrac{\partial e_1 \delta(\boldsymbol{r}-\boldsymbol{r}_1)}{\partial t} + dive_1 V_1 \delta(\boldsymbol{r}-\boldsymbol{r}_1) = 0 \\ \quad \cdots \qquad\qquad \cdots \qquad\qquad \cdots \\ \dfrac{\partial e_n \delta(\boldsymbol{r}-\boldsymbol{r}_n)}{\partial t} + dive_n V_n \delta(\boldsymbol{r}-\boldsymbol{r}_n) = 0 \end{cases} \qquad (13-6)$$

These results follows from gauge principle and principle of linear superposition, but do not agree with experimental data in micro-world. Considering physical interaction accompanying creation and annihilation of particles, not individual charge but only total charge of a system is conserved. This shows that gauge principle and principle of linear superposition in Maxwell's theory stand entirely against experimental facts within very close distance between particles (accompanying creation and annihilation of particles).

In our theory, with the breaking of gauge principle, the problem mentioned above is easily solved. Here, effective charge which refers to charge in Maxwell's theory is not a constant but the function dependent on field and the interaction of charge and field is non-linear (Lagrangian for field to be found in the future should also be non-linear). (See sect. 5 of referfence [1]).

From this, we arrive at following conclusions.

First, as effective charge is function of space and time, there exists a critical distance (about 10^{-13}*cm, in case of annihilation of electron-positron) within which annihilation of particles and creation of new particles appears.* Unlike Maxwell's theory, our theory gives an explanation for annihilation of particles despite non-quantum theory (See sect. 11, sect. 21). Thus, charge is no more a quantity conserved individually.

Second, as field equation is non-linear, field is not arithmetical sum of fields produced by individual charges and it can be studied and considered only as field created by the total charge of a system. In this case, it is self-evident that the field equation of a system of charges cannot be separated into field equations of individual charges. The form of field equation can be written as follows.

$$(nonlinear\ \ equation\ \ of\ \ field)_\lambda \sim \sum_{i=1}^{n} \bar{e}_{(i)} V_{(i)}^{\lambda}\, \delta(\mathbf{r}-\mathbf{r}_i) \qquad (13-7)$$

Third, the conservation of total charge of a system does not mean conservations of individual charges which constitute the system and then in a close system which accompanies creation and annihilation of particles, the total charge of a system is only conserved. Furthermore, this shows that non-linear character of interaction and breaking of gauge symmetry is a basic factor which underlies creation and annihilation of particles.

The principle of linear superposition is a basic principle that underlies gauge symmetry. This principle, in a word, is used to "atomism", founded by Carteret and Newton in 16~17 century, according to which the whole is the sum of its constituents and understanding of the parts leads to understanding of the whole. But this idea, with appearance of modern systems theory, got serious criticism. According to systems theory, all substances and phenomena can be considered as a system in which elements (constituents of a system) are combined by structure or relation. This idea has been introduced to many individual sciences and interdisciplinary sciences. In this regard, breaking of principle of linear-superposition in our theory shows again validity of idea of systems theory. As for final conclusion related to the breaking of principle of linear-superposition and gauge symmetry, there is nothing to be surprise or afraid. This will be an advance in clarifying the essence of material world.

(2) Equivalence of inertial mass and total energy of particle-field.

First of all, let us calculate energy of field. From formula (7-4), the energy-momentum tensor of electromagnetic field is

$$T_i^k = -\frac{1}{4\pi}\left(F_{i\lambda}F^{k\lambda} - \frac{1}{4}F_{lm}F^{lm}\delta_i^k\right) \qquad (13-8)$$

From formula (13-8), the energy density of electrostatic field is

$$T_0^0 = -\frac{1}{8\pi} E_i E^i \qquad (13-9)$$

where i is spatial component. From formula (7-5) is obtained

$$E^i = -\frac{1}{\sqrt{-g}} E_i \qquad (13-10)$$

and then

$$T_0^0 = \frac{1}{8\pi} E_i E_i \frac{1}{\sqrt{-g}}$$

Hence, the energy of electrostatic field is

$$U = \int T_0^0 \sqrt{-g}\, dV = \frac{1}{8\pi} \int \frac{1}{\sqrt{-g}} (E_i)^2 \sqrt{-g} dV = -\frac{1}{8\pi} \int E_i \frac{\partial \varphi}{\partial x^i} dV$$

$$= -\frac{1}{8\pi} \int \partial_i (E_i \varphi) dV + \frac{1}{8\pi} \int \varphi \partial_i E_i$$

According to Gauss's theorem, the first integral is equal to the integral of $E_i \varphi$ over the surface bounding the volume of integration, but since the integral is taken over all space and the field is zero at infinity, this integral vanishes. Substituting $\partial_i E_i = 4\pi \int \bar{e} \delta(\mathbf{r} - \mathbf{r}_0) dV$ into the second integral, we find the following expression for the energy of a system of charges

$$U = \frac{1}{2} \int \varphi \bar{e} \delta(\mathbf{r} - \mathbf{r}_0) dV = \frac{1}{2} \bar{e} \varphi^{in} \qquad (13-11)$$

According to normalization rule 3, if φ^{in} is replaced by the normalized function $\bar{\varphi}^{in}$ which $\bar{\varphi}^{in} = 0$ (see formula (11-7), (11-8)). The formula (13-11) arrives at

$$\bar{U} = \frac{1}{2} \bar{e} \bar{\varphi}^{in} = \frac{1}{2} e \bar{\varphi}^{in} = 0 \qquad (13-12)$$

In formula (13-11) normalized effective charge becomes a constant charge. Namely, when one normalizes effective charge, $\bar{e}$, in $\bar{e} = e g^{\mu\nu} \delta_{\mu\nu}$, because $g^{\mu\nu}$ is replaced by $\delta^{\mu\nu}$, normalized effective charge becomes a constant charge.

Next, let us calculate the energy of a particle. Considering that the energy-momentum tensor of a particle is

$$T_0^0 = \frac{m_0}{\sqrt{-g}} u_0 \delta(\mathbf{r} - \mathbf{r}_0)$$

the energy of a particle is

$$T = \int \frac{m_0}{\sqrt{-g}} u_0 \delta(\mathbf{r} - \mathbf{r_0}) \sqrt{-g} dV = m_0 c^2 u_0 \qquad (13-13)$$

where u_0 is implicit function.

Normalization of $\bar{u}_0$ subject to normalization rule (10-4) yields

$$\bar{u}_0 = \bar{g}_{0\lambda} \bar{u}^\lambda = \delta_{0\lambda} (1 + 2\alpha \bar{\varphi}^{in}) \bar{u}^\lambda = \frac{(1 + 2\alpha \bar{\varphi}^{in})^{\frac{1}{2}}}{c\sqrt{1 - \beta^2}} = \frac{1}{c\sqrt{1 - \beta^2}} \qquad (13-23)$$

where $\bar{\varphi}^{in} = 0$ were used. (see formula (10-4)). So, the energy of particle is

$$T = \frac{m_0 c^2}{(1 - \beta^2)^{\frac{1}{2}}} \qquad (13-15)$$

Consequently, total energy of particle and field produced by it is

$$E = T + U = \frac{m_0 c^2}{(1 - \beta^2)^{\frac{1}{2}}} = mc^2 \qquad (13-16)$$

The formula (13-16) is, formally, the same as energy formula of particle in SR but different entirely in essential content. In SR, mc^2 is energy confined to particle only but mc^2 in formula (13-16) is energy which includes not only energy of particle but also energy of electrostatic field. This is the inevitable conclusion following from starting postulate 3 (sect. 4). Therefore, in case of discussing energy of field in our theory, the energy of static self-field (diverging in former theory) becomes naturally zero and accordingly only interactional energy with external field has real meaning. If one generalizes argument mentioned above to a system of multi-particles, infinite terms arising from self-field vanish naturally and only terms of interactional energy dependent on mutual distribution of particles remains.

5. Theory of Gravitational Field in KR Space

The Einstein's general theory of relativity (GR) established in 1916 has been recognized as one and only theory of gravitation. The validity of this theory was confirmed by marvelous experiments for its main theoretical results, i.e. three effects (deflection of light rays in the sun's gravitational field, shift of Mercury's perihelion, red shift of spectrum of light). But later among many scientists were presented doubts about whether the theoretical answers to these effects of gravitational field can be decisive ground on which verifies validity of GR. It is referred to facts that, except Einstein's GR, there are various kinds of theories with well-ordered logical system which gives theoretical answer to three effects. In the context, many scientists emphasize that theoretical description of three effects cannot be the touchstone or decisive factor which verifies uniqueness and rightness of GR and accordingly present experiments for gravitation are not enough to confirm the truth of GR.

We, refering to views presented by many scientists, put forward the following solution measures for finding out uniqueness and rightness of theory of gravitational field.

The first, with further improvement of modern experimental apparatus, is to select the best theory which explains wonderfully approximation of higher order about three effects among theories known until now.

The second is to discover new experiments, especially effects concerning strong field and give theoretical answer to it.

The third is to introduce more necessary and enough, physical and logical requirements to theory of gravitational field so that it leads to a consistant theory.

In fact, what realizes the solution measures of first and second are nearly impossible at present. They are related to facts that observable gravitational phenomena, until now, manifested in weak, static field and new improvement of experiment apparatus is also difficult to be realized in near future. On this context, third solution measure should be focused on study.

As well known, GR was build by deductive method. It is because observable gravitational effects are phenomena manifested in weak, static field and not sufficient so as to get necessary and enough data for generalized theory. At present also were not discovered very rich experimental data necessary and enough for building of theory. But if one put forward a new idea of the unification of electromagnetic field and gravitational field and introduce it to the theory of gravitation, this becomes presentation of new unprecedented requirement for building of the consistent theory of gravitation and will give a great shock to completion of theory of gravitation. First of all, experimental verification for the unification of electromagnetic field and gravitational field will lead to confirm, at the new angle, justness of the theory of gravitation. As a simple example, the experiment for the annihilation of particle and antiparticle argued in sect. 1 needs establishment of the unified conservation formula of gravitation-electromagnetic field. If theory of gravitation in the form of combination of electromagnetic field, makes it possible to provide a unified conservation formula of energy-momentum and describes all experiments well, it will be a great advance for confirmation of validity of theory of gravitation.

Next, establishment of the unification theory of gravitation and electromagnetic field, by indirect method, will open up new way to verify theory of gravitation. The theory of electromagnetic field gave, unlike theory of gravitation, wonderful answers to so many phenomena in not only static field but also non-static field and micro-world. That is why, we, regarding electrodynamics as a pivotal thing, comes to combine theory of gravitation with it and from this, function of gravitational field results in four dimensional vector, but not tensor. If the new theory of gravitation is established, as an integrated field, without any contradiction and without being separated from the theory of electromagnetic field which was verified under different and various situations, it leads to verify by indirect method that theory of gravitation has universal meaning in not only static field of macro-world but also area of micro-world.

Sect. 14 Motion of object in a spherically symmetric gravitational field and shift of Mercury perihelion

Referring to sect. 5 and sect. 6, we find Hamilton-Jacobi equation widely used in consideration of the motion of an object. The starting formula is

$$u_k u^k = 1 \qquad (14-1)$$

Using $P_k = \partial S/\partial x^k$, we find the following equation

$$g^{\alpha\beta}\left(\frac{\partial S}{\partial x^\alpha} - \hat{m}_{(g)}\hat{G}_\alpha\right)\left(\frac{\partial S}{\partial x^\beta} - \hat{m}_{(g)}\hat{G}_\beta\right) - m_0^2 c^2 = 0 \qquad (14-2)$$

where $\hat{m}_{(g)}$ is the effective gravitational mass and $\hat{G}_\alpha$ gravitational potential and

$$g^{\alpha\beta} = \delta^{\alpha\beta}\frac{1}{1 + 2\hat{\alpha}_{(m)}\hat{G}_\lambda u^\lambda}$$

If one, in equation (14-2), replaces **P** with $\partial S/\partial \mathbf{r}$ and E with $-\partial S/\partial t$ and then formula (14-2) arrives at

$$\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + \hat{m}_{(g)}\hat{G}_0\right)^2 - \left(\mathrm{grad}S - \frac{1}{c}\hat{m}_{(g)}\hat{G}\right)^2\right] - m_0^2 c^2\left(1 + 2\hat{\alpha}_{(m)}\hat{G}_\lambda u^\lambda\right) = 0 \qquad (14-3)$$

This formula is essential to discuss the motion of an object in gravitational field including the shift of Mercury's perihelion.

Now, let us consider the shift of Mercury's perihelion. Because equation (14-3) has the characteristic of spherical symmetry, it is easy to treat it in spherical coordinates. Then equation (14-3) leads to

$$\left[\frac{1}{c^2}\left(-E_0 + \hat{m}_{(g)}\hat{\varphi}\right)^2 - \left(\frac{\partial S_r}{\partial r}\right)^2 - \frac{M^2}{r^2}\right] - m_0^2 c^2\left(1 + 2\hat{\alpha}_{(m)}\hat{\varphi}u^0\right) = 0 \qquad (14-4)$$

where $E_0 = -\partial S/\partial t$ was used. A rearrangement of equation (14-4) yields.

$$\left(\frac{\partial S_r}{\partial r}\right)^2 = \frac{\left(E_0 - \hat{m}_{(g)}\hat{\varphi}\right)^2}{c^2} - \frac{M^2}{r^2} - m_0^2 c^2\left(1 + 2\hat{\alpha}_{(m)}\hat{\varphi}u^0\right)$$

$$S_r = \int dr\left[\frac{\left(E_0 - \hat{m}_{(g)}\hat{\varphi}\right)^2}{c^2} - \frac{M^2}{r^2} - m_0^2 c^2\left(1 + 2\hat{\alpha}_{(m)}\hat{\varphi}u^0\right)\right]^{\frac{1}{2}} \qquad (14-5)$$

The path of motion is determined from $\partial S/\partial M = \text{constant}$. We consider, in detail, terms of equation (14-5)

$$E_0 - \hat{m}_{(g)}\hat{\varphi} = m_0 c^2 u_0$$

$$E = m_0 c^2 u_0 = \frac{m_0 c^2\left(1 + 2\hat{\alpha}_{(m)}\hat{\varphi}u^0\right)^{\frac{1}{2}}}{\sqrt{1-\beta^2}} = \frac{\overline{m}_0 c^2}{(1-\beta^2)^{\frac{1}{2}}} \qquad (14-6)$$

where $\overline{m}_0$ is effective inertial mass. In the field produced by rest object, m', we have

$$1 + 2\hat{\alpha}_{(m)}\hat{\varphi}u^0 \approx 1 - 2\frac{k_{(g)}}{c^2}\cdot\frac{m'}{r\left(1 - 2k_{(g)}\frac{m'}{c^2 r}\right)}\cdot\frac{1}{\left[\left(1 - 2k_{(g)}\frac{m'}{c^2 r}\right)(1-\beta^2)\right]^{1/2}} \qquad (14-7)$$

where $k_{(g)}$ is the *gravitational constant*. Introducing $r_0 = 2\frac{k_{(g)}}{c^2}m'$ called the *gravitation radius* and allowing, in nonrelativistic approximation, $(1-\beta^2)^{-1/2} \approx 1 + \frac{1}{2}\beta^2$, formula (14-7) becomes

$$1 + 2\hat{\alpha}_{(m)}\hat{\varphi}u^0 \approx 1 - \frac{r_0}{r\left(1 - \frac{r_0}{r}\right)}\cdot\frac{1}{\left(1 - \frac{r_0}{r}\right)^{\frac{1}{2}}} + 0(V^2/c^4)$$

$$\approx 1-\frac{r_0}{r\left(1-\frac{r_0}{r}\right)^{\frac{3}{2}}}\approx 1-\frac{r_0}{r}\left(1+\frac{3}{2}\cdot\frac{r_0}{r}\right) \qquad (14-8)$$

where $\hat{\alpha}_{(m)}\hat{\varphi}=\frac{\widehat{m}}{mc^2}\cdot k_{(g)}\frac{\widehat{m}'}{r}=-\frac{1}{c}k_{(g)}\frac{\widehat{m}'}{r}$ in isotopic vector space was used. On the other hand, in formula (14-6) is $E=\frac{\overline{m}_0c^2}{(1-\beta^2)^{1/2}}\approx\overline{m}_0c^2+\frac{1}{2}\overline{m}_0v^2\approx\overline{m}_0c^2+\frac{1}{2}m_0v^2$ and then putting $T'=\frac{1}{2}m_0v^2$, the result is

$$E=T'+\overline{m}_0c^2 \qquad (14-9)$$

Using formula (14-8) and (14-9) is obtained formula for S_r

$$S_r=\int\left\{\frac{E^2}{c^2}-\frac{M^2}{r^2}-m_0^2c^2\left[1-\frac{r_0}{r}-\frac{3}{2}\left(\frac{r_0}{r}\right)\right]\right\}^{\frac{1}{2}}=$$

$$=\int\left\{\frac{1}{c^2}(T'^2+2T'\overline{m}_0c^2+\overline{m}_0^2c^4)-\frac{M^2}{r^2}-m_0^2c^2+m_0^2c^2\frac{r_0}{r}+m_0^2c^2\frac{3}{2}\left(\frac{r_0}{r}\right)^2\right\}^{\frac{1}{2}}dr$$

$$(14-10)$$

Now, using $\overline{m}_0^2c^2=m_0^2\left(1-\frac{r_0}{r}\right)c^2=m_0^2c^2-m_0^2c^2\frac{r_0}{r}$, $\overline{m}_0\approx m_0\left(1-\frac{r_0}{2r}\right)$, formula (14-10) leads to

$$S_r=\int\left[\left(\frac{T'^2}{c^2}+2T'\overline{m}_0\right)-\frac{1}{r^2}\left(M^2-\frac{3}{2}m_0^2c^2r_0^2\right)\right]^{\frac{1}{2}}dr$$

$$=\int\left[\left(2T'm_0+\frac{T'^2}{c^2}-m_0T'\frac{r_0}{r}\right)-\frac{1}{r^2}\left(M^2-\frac{3}{2}m_0^2c^2r_0^2\right)\right]^{\frac{1}{2}}dr \qquad (14-11)$$

Let us transform formula (14-11) as follows:

$$T'=E'-U,\quad E'=T'+U$$

And then arrangement of formula (14-11) gives

$$S_r=\int\left[\left(2E'\overline{m}_0+\frac{E'^2}{c^2}\right)-\frac{1}{r}\left(2m_0^2m'k_{(g)}-\frac{1}{2}k_{(g)}\frac{m_0^2m'}{r}r_0\right)-\frac{1}{r^2}\left(M^2-\frac{3}{2}m_0^2c^2r_0^2\right)\right]^{\frac{1}{2}}dr$$

$$(14-12)$$

Comparing formula (14-12) with formula of GR, second term in second bracket is different. But this difference is not essential. As already well known, in case of discussing the motion of an object in spherical symmetry field, corrections in first two brackets contribute only to the relation between energy and momentum and the change of parameters of Newton's trajectory (ellipse), which has no significance in consideration. Therefore, even though correction of second bracket are different from that of GR, it does not spoil the validity of our theory. The most important thing is that the term related to Mercury's perihelion in our theory is the same as that in GR which has been already experimentally verified.

Sect. 15 The equation of propagation of light, red shift of spectrum of light and deflection of light

It is well known that there exists formal similarity between Hamilton-Jacobi equation and equation of propagation of light. In our theory, it is also supposed that this similarity holds as it is. In Hamilton-Jacobi equation, the total momentum of a particle, four-dimensional momentum of a particle and the total energy of field of conservative force can be written as follows, respectively:

$$P_\alpha = \frac{\partial S}{\partial x_\alpha}, \qquad m_0 c u_\alpha = \frac{\partial S}{\partial x^\alpha} - \widehat{m}_{(g)}\widehat{G}_\alpha, \qquad P_0 = \frac{\partial S}{\partial t} \tag{15-1}$$

Referring to formula (5-22), we make the equation of propagation of light

$$g^{\alpha\beta}\left(\frac{\partial \Psi}{\partial x^\alpha} - h_\alpha\right)\left(\frac{\partial \Psi}{\partial x^\beta} - h_\beta\right) = 0 \tag{15-2}$$

where Ψ is *eikonal* and h_α is the term concerning interaction between light and gravitational field (This term is similiar to term of interaction of particle and field). Comparing equation (15-2) with eikonal equation considered in GR, the term of interaction between light and gravitational field is added. But what describe h_α in the strict mathematical viewpoint is a difficult problem. Moreover, on the left-hand side of equation (15-2), as $V/c = 1$ in denominator of u^0 appears, $g^{\alpha\beta}$ goes to zero and then equation (15-2) leads to indeterminate form (0/0). Accordingly, as for tangible calculation should be paid deep attention so that this indeterminate form is solved. Therefore, we, for simplification of consideration, intend to discuss equation (15-2) established in static gravitational field. In this case, $\omega_0 = -\partial\Psi/\partial t$ is proper frequency which is not changed at any point of space-time and $\partial\Psi/\partial x^i = \mathbf{n}_i\omega_0/c$ (i denotes spatial component) are components of wave vector. From this is obtained the simpler form of equation (15-2). In arbitrary point of space time, actually the changed frequency is

$$\omega = \frac{\partial \Psi}{\partial \tau} \quad (\tau;\ \text{proper time}) \tag{15-3}$$

This change is caused by interaction of light and gravitational field. So we have

$$\frac{\partial \Psi}{\partial t} = \frac{\partial \Psi}{\partial \tau} + c h_0 \tag{15-4}$$

In static gravitational field, gravitational vector potential is $G_i = 0$ and therefore,

$$h_i = 0 \tag{15-5}$$

Using formula (15-4) and (15-5), equation (15-2) arrives at

$$g^{00}\left(\frac{\partial \Psi}{\partial \tau}\right)^2 + g^{ij}\frac{\partial \Psi}{\partial x^i}\frac{\partial \Psi}{\partial x^j} = 0 \tag{15-6}$$

or

$$\left(1 + 2\hat{\alpha}_{(m)}\widehat{G}_0 u^0\right)\left[\left(\frac{\partial \Psi}{\partial \tau}\right)^2 - \frac{\partial \Psi}{\partial x^i}\frac{\partial \Psi}{\partial x^i}\right] = 0 \tag{15-7}$$

where x^i, x^j is spatial components of x^α.

(1) The red shift of spectrum of light

In static gravitational field, the following formula is obtained

$$\omega_0 = -\frac{\partial \Psi}{\partial t} = -c\frac{\partial \Psi}{\partial x^0} \tag{15-8}$$

From the character of static field, as formula (15-8) does not include explicitly world time, x_0, frequency ω_0, remains invariant during propagation of light. On the other hand,

$$\omega = -\frac{\partial \Psi}{\partial \tau} \tag{15-9}$$

is changed at any point of space. In order to see change of frequency, we can consider two infinitely near events taking place in a point of space. In this case, the line element between two events observed, because events occur in a point of space, allowing $(dx_i)^2 = dl^2 = 0$, can be written as follows

$$ds^2 = c^2 d\tau^2 = g_{00}(dx^0)^2 \tag{15-10}$$

where $g_{00} = 1 + 2\alpha_{(m)}\varphi u^0$. From this, we have

$$d\tau^2 = \frac{1}{c^2}\left(1 + 2\hat{\alpha}_{(m)}\frac{\hat{\varphi}}{c}\frac{dx^0}{d\tau}\right)(dx^0)^2$$

$$d\tau = \frac{1}{c}\left(1 + 2\hat{\alpha}_{(m)}\frac{\hat{\varphi}}{c}\frac{dx^0}{d\tau}\right)^{\frac{1}{2}}(dx^0) \qquad (15-11)$$

Consequently $d\tau$ is implicit function in KR space. Normalizing $d\tau$ by normalization rule 1, the result is

$$\overline{d\tau} = \overline{d\tau}(x, \dot{x}, \bar{g}) = \overline{d\tau}(x, \dot{x}) = \frac{1}{c}\left(1 + 2\hat{\alpha}_{(m)}\hat{\varphi}\right)^{\frac{1}{2}} dx^0 = \frac{1}{c}\left(1 + 2k(g)\frac{\widehat{m}}{mc^2}\cdot\frac{\widehat{m}'}{r}\right)^{\frac{1}{2}} dx^0 =$$

$$= \frac{1}{c}\left(1 - \frac{r_0}{r}\right)^{\frac{1}{2}} dx^0 \qquad (15-12)$$

where

$$\left.\frac{1}{c}\frac{dx^0}{d\tau}\right|_{g_{00}=\delta_{00}} = \left.\frac{1}{\sqrt{g_{00}}}\right|_{g_{00}=\delta_{00}} = 1$$

was used in the normalization of $d\tau$ and m is, in case of light, mass formally introduced and $r_0 = 2k(g)\frac{m'}{c^2}$ the gravitational radius.

Also, for ω we have

$$\omega = -\frac{\partial\psi}{\partial\bar{\tau}} = -\frac{\partial\psi}{\partial x^0}\frac{\partial x^0}{\partial\bar{\tau}} = \frac{c}{(1 - r_0/r)^{\frac{1}{2}}}\frac{\partial\psi}{\partial x^0}$$

When $r_0 \ll r$,

$$\omega = \omega_0\left(1 + \frac{1}{2}\cdot\frac{r_0}{r}\right) \qquad (15-13)$$

is found. The formula (15-13) is the same as formula concerning red shift of spectrum of light in GR.

(2) The deflection of light from rectilinear path

Let us consider formula (14-5) for S_r in detail. Denominator of u^0 in third term inside root of formula (14-5), i.e. $m_0^2c^2\left(1 + 2\hat{\alpha}_{(m)}\hat{\varphi}u^0\right)$, includes $(1 - V^2/c^2)^{1/2}$ and in case of light, u^0 would diverge. But as $m_0 = 0$, $m_0^2c^2u^0$ has indeterminate form of 0/0. Hence, a path of light rays cannot be easily obtained like in GR. In this regard, first of all, we transform formula (14-5) for the motion of a particle to an approximate form and solve the problem of indeterminate form of $m_0^2c^2u^0$, and then, by using similarity of the motions of particle and light, obtain the path of a light ray. To do this, we transform u^0 as follows (in this case the normalized u^0 is used)

$$u^0 = \frac{1}{\sqrt{1-\beta^2}\sqrt{1 - \frac{r_0}{r}}} = \frac{1}{1-\beta^2}\cdot\frac{(1-\beta^2)^{\frac{1}{2}}}{\left(1 - \frac{r_0}{r}\right)^{\frac{1}{2}}} \qquad (15-14)$$

where $b = (1-\beta^2)^{\frac{1}{2}}/\left(1 - \frac{r_0}{r}\right)^{\frac{1}{2}}$ has a very interesting character. In case of $V \approx c$, $r \approx r_0$, that is to say, when a particle moves in the vicinity of r_0 and so the velocity of the particle approaches the light velocity, $b \approx 1$ holds. On the other hand, when a particle moves far away from r_0 (i.e. $r_0 \ll r$) and the velocity of the particle is much smaller than the velocity of light (i.e. $V \ll c$), $b \approx 1$ holds. Therefore, the result is

$$b = (1 + \alpha_0), \quad \alpha_0 \ll 1 \qquad (15-15)$$

Assuming that $\frac{V}{c} \ll 1$, $\frac{r_0}{r} \ll 1$ and expanding b as a power series, we get

$$b \approx \left(1 - \frac{1}{2}\beta^2\right)\left(1 + \frac{1}{2}\frac{r_0}{r}\right) \approx \left[1 + \left(\frac{r_0}{r} - \frac{1}{2}\beta^2\right)\right] \qquad (15-16)$$

Considering formula (15-15) and (15-16), formula (15-14) leads to

$$u^0 = \frac{1}{1-\beta^2}(1+\alpha_0), \qquad \alpha_0 \ll 1 \tag{15-17}$$

and then we have

$$m_0^2 c^2 2\hat{\alpha}_{(m)}\hat{\varphi} u^0 = -m_0^2 c^2 \frac{r_0}{r} u^0 = -m_0^2 c^2 \frac{r_0}{r} \cdot \frac{1}{1-\beta^2}(1+\alpha_0)$$

$$= -\left(\frac{m_0 c^2}{\sqrt{1-\beta^2}}\right)^2 \frac{1}{c^2} \cdot \frac{r_0}{r}(1+\alpha_0) = -\left(\frac{E}{c}\right)^2 \frac{r_0}{r}(1+\alpha_0) \approx -\left(\frac{E}{c}\right)^2 \frac{r_0}{r} \tag{15-18}$$

where

$$E = \frac{m_0 c^2}{\sqrt{1-\beta^2}}$$

Substituting formula (15-18) into formula (14-5) for S_r, we obtain

$$S_r = \int \left[\frac{E^2}{c^2} - \frac{M^2}{r^2} + \frac{r_0}{r}\left(\frac{E}{c}\right)^2\right]^{1/2} dr \tag{15-19}$$

If one substitutes $\omega = -\partial\psi/\partial\tau$ for E and, allowing for formula (15-13) and $m_0 = 0$, introduces a new constant $\rho = cM/\omega_0$, the result is

$$\left(\frac{E}{c}\right)^2 \to \left(\frac{\omega_0}{c}\right)^2 \left(1 + \frac{r_0}{r}\right)$$

$$\psi_r = \frac{\omega_0}{c} \int \left[\left(1 + \frac{r_0}{r}\right) - \frac{\rho^2}{r} + \frac{r_0}{r}\left(1 + \frac{r_0}{r}\right)\right]^{\frac{1}{2}} dr \tag{15-20}$$

Ignoring term $\left(\frac{r_0}{r}\right)^2$ in formula (15-20), we have

$$\psi_r = \frac{\omega_0}{c} \int \left(1 + 2\frac{r_0}{r} - \frac{\rho^2}{r}\right)^{1/2} dr \tag{15-21}$$

This formula agrees with the formula for path of light ray in GR. Formula (15-21) yields formula for light deflection which has been already experimentally verified.

Sect. 16 Equivalence of inertial mass and total energy of particle-gravitational field

This topic is formally the same as in theory of electromagnetic field (see sect. 13.2). Taking the following transformation $e \to m,\ A_i \to G_i,\ F_{ik} \to R_{ik}$ and considering $\hat{m}k_{(g)}\frac{\hat{m}}{r} = k_{(g)}\frac{m^2}{r}$, we can get the formula for the equivalence of inertial mass and total energy of particle-field

$$E = T + U = \frac{m_0 c^2}{\sqrt{1-\beta^2}} = mc^2 \tag{16-1}$$

Our theory is of significance in relation to the fact that conservation formula of total energy-momentum of particle and gravitational field is derived on the basis of Noether's theorem and then the difficulties seen in sect. 3 are unraveled.

6. Quantum Electrodynamics in KR Space (Non-linear Quantum Electrodynamics)

The classical theory underlies quantum theory and accordingly as long as basis of classical theory is varied, quantum theory also should be, in view of new angle, rethought and rebuilt naturally. As our theory for electromagnetic field is formulated in KR space, quantum theory also should be set up newly.

Sect. 17 Modification of Dirac equation

First of all, let us rewrite formula (5-21) for the total energy of a particle in electrodynamics, discussed in sect 5

$$E = \left[\bar{m}_0 c^4 + c^2\left(\mathbf{P} - \frac{\bar{e}}{c}\mathbf{A}\right)^2\right]^{1/2} + \bar{e}\varphi \qquad (17-1)$$

where $\bar{m}_0$ is the effective inertial mass and $\bar{e}$ the effective charge (see formula (5-9), (5-15)).

Comparing formula (17-1) with formula for energy in Maxwell's theory, in formula (17-1) appear effective charge and effective inertial mass, instead of constant charge and constant mass in Maxwell's theory. Namely, if one replaces constant charge and constant mass in Maxwell's theory by effective charge and effective inertial mass, the formula for the total energy of a particle in KR space is obtained.

Now, for reformulating quantum electrodynamics, let us suppose that this relationship is held as it is. Consequentely, by replacing charge and mass in Dirac equation by effective charge and effective inertial mass, quantum mechanical equation in KR space is obtained. The result is

$$\left[\gamma^\mu\left(i\partial_\mu + \bar{e}A_\mu\right) - \bar{m}_0\right]\psi(x) = 0 \qquad (17-2)$$

And then interaction Harmiltonian H_s and Lagrangian are

$$H_s = \int L dx^3, \qquad L = \bar{e}\bar{\psi}\gamma_\lambda A^\lambda \psi \qquad (17-3)$$

Now, let us express $\bar{e}$ as a quantum mechanical operator. To do this, we change $eA_\lambda u^\lambda$ in denominator of $\bar{e}$ as follows;

$$\frac{2}{m_0 c^2} eA_\lambda u^\lambda = \frac{2}{m_0 c^2}\bar{e}\left(1 + \frac{2e}{m_0 c^2}A_\sigma u^\sigma\right)A_\lambda \frac{\dot{x}^\lambda}{\left(g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu\right)^{\frac{1}{2}}} =$$

$$= \frac{2}{m_0 c^2}\bar{e} g_{\lambda\sigma}\delta^{\lambda\sigma}\left(g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu\right)^{-\frac{1}{2}} A_\lambda \dot{x}^\lambda = \alpha_0 \bar{e} A_\lambda \dot{x}^\lambda \qquad (17-4)$$

where $\alpha_0(\dot{x}, g) = \frac{2}{m_0 c^2} g_{\lambda\sigma}\delta^{\lambda\sigma}\left(g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu\right)^{-\frac{1}{2}}$ (When one normaizes H_s, $g_{\lambda\sigma}\delta^{\lambda\sigma}$ becomes 1). If one expresses formula (17-4) as a quantum mechanical operator, the result is

$$\frac{2}{m_0 c^2} eA_\lambda u^\lambda \to \int \alpha_0 \bar{e}\bar{\psi}\gamma_\lambda A^\lambda \psi dx^3 \qquad (17-5)$$

Hence, formula (17-3) arrives at

$$L = \bar{e}\bar{\psi}\gamma_\lambda A^\lambda\psi = \frac{e\bar{\psi}\gamma_\lambda A^\lambda(x, g)\psi}{1 + \int dx^3 \alpha_0(\dot{x}, g)\bar{e}\bar{\psi}\gamma_\lambda A^\lambda(x, g)\psi}$$

$$= \frac{L_m}{1 + \int dx^3 \alpha_0(\dot{x}, g)L} \qquad (17-6)$$

$$\bar{e} = \frac{e}{1 + \int \alpha_0 \bar{e}\bar{\psi}\gamma_\lambda A^\lambda \psi dx^3}$$

where $L_m = e\bar{\psi}\gamma_\lambda A^\lambda \psi$. Therefore, L becomes an absolute implicit function and so do H_s in formula (17-3). The result is

$$H_s = \int dx^3 \frac{L_m}{1 + \int dx'^3 \alpha_0(\dot{x}, g) L} \qquad (17-7)$$

The implicit function, H_s becomes the key to solving problem of divergence.

Sect. 18 The normalization of S-matrix and its convergence

In this section, we define newly S-matrix in non-linear quantum mechanics and, by using normalization rule, normalize S-matrix and then see its convergence. If one starts with the theory of perturbation expansion of S-matrix treated in the traditional quantum electrodynamics, the nth-order term of the perturbation expansion of S-matrix is as follows:

$$S^{(n)} = \frac{(-1)^n}{n!} \int dx_1^4 \cdots dx_n^4 T\big(L(x_1) \cdots L(x_n)\big) \qquad (18-1)$$

where L is given by formula (17-6).

Now, in order to normalize $S^{(n)}$, let us express formula (18-1) as the concise form of implicit function. For this, first of all, we consider $H_s' = \int dx^3 \alpha_0 L$ which put in denominator of $\bar{e}$ in formula (17-6). According to the mean value theorem in mathematics, there must be t_0 satisfing

$$\int dt\, H_s' = T_0 H_s'|_{t=t_0} = T_0 \bar{H}_s' \qquad (18-2)$$

where $\bar{H}_s'$ is the average value of H_s', T_0 time interval between t_1 (before interaction occures) and t_2 (after interaction finishes), and T_0 is supposed to be finite. Next, in formula $H_s' = \int dx^3 \alpha_0 L$, as $\alpha_0 \ll 1$ holds, ignoring the difference of H_s' and $\bar{H}_s'$ does not give essential influence to final calculation of $S^{(n)}$. Hence, in calculation of $S^{(n)}$, $\bar{H}_s'$ is considered to be approximately same as H_s', Namely,

$$\bar{H}_s' = H_s'|_{t=t_0} \approx H_s' \qquad (18-3)$$

$$H_s' \approx \bar{H}_s' = \frac{1}{T_0} \int dx^4\, \alpha_0 L(x) \qquad (18-4)$$

Now, considering formula (17-6) and (17-7), formula (18-1) arrives at

$$S^{(n)} = \frac{(-1)^n}{n!} \bar{e}_0^n \int dx_1^4 \cdots dx_n^4 T\big(L_m(x_1) \cdots L_m(x_n)\big) \qquad (18-5)$$

where

$$L_m' = \bar{\psi}\gamma_\lambda A^\lambda \psi$$

and

$$\bar{e}_0^n = \frac{e^n}{\left[1 + V_{(n-1)} + (\alpha_0)^n \frac{1}{(T_0)^n} S^{(n)}\right]}$$

In denorminator of $\bar{e}_0^n$, $S^{(n)}$ is S-matrix of *n*th-order and $V_{(n-1)}$ is mixed product of arbitrary order of any different terms of $\frac{1}{T_0}(-i)\alpha_0 \int dx_j^4 L(x_j)$ from $j = 1$ to $j = \mathrm{n} - 1$ (As $g_{\mu\nu}$ terms of α_0 becomes 1 when one normalizes S-matrix, from the beginning, these are put in front of $S^{(n)}$ to avoid complication of calculation).

The formula (18-5) can be written in the following more concise form

$$S^{(n)} = \frac{(-i)^n}{n!} \cdot \frac{S_0^{(n)}}{1 + (\alpha_0)^n \frac{1}{(T_0)^n} S^{(n)} + V_{(n-1)}} \qquad (18-6)$$

where $S_0^{(n)} = e^n \int dx_1^4 \cdots dx_n^4 T\left(\bar{\psi}\gamma_\lambda A^\lambda \psi \cdots \bar{\psi}\gamma_\lambda A^\lambda \psi\right)$ is similiar in the traditional quantum electrodynamics. But from strict inquiring is found some difference. In formula for $S_0^{(n)}$, A^λ is, as mentioned in sect. 9, implicit function. The character of implicit function vanishes, when one normalizes it, and then A^λ leads to field potential in Maxwell's theory.

In the past, so called "method of renormalization" according to which removes terms of divergence occurred in perturbation expansion is, in a word, to remove artificially infinite quantities by separating it from physical quantities, harming the logical system of the theory. In other words, the traditional theory assumed artificially that finite quantities such as charge and mass with real physical meaning, verified experimentally in classical physics comes to have divergent character of "necessary form" (necessary for elimination of infinite quantities occurred) in the stage of quantum electrodynamics and then concluded that finite quantities with physical meaning could be obtained from eliminating this infinite (infinite of necessary form) by infinite occurred in the process of interaction with field could be obtained. But this argument stands against principle of correspondence, a main principle of physics. According to principle of correspondence new theory involves, as an approximate form, the former theory which underlies it. However, allowing theory of renormalization, mass and charge belonging to equation of Dirac are not mass and charge (finite quantity) in Maxwell's theory and Schrodinger equation.

In view of methodology, the traditional quantum electrodynamics, in order to solve problem of divergence, divided any formulas into finite part and divergent part and by adding divergent part to mass, charge, combination constant, etc. and reformulating those, obtained measurable quantities. This is just basic idea of so called *renormalization*. As well known, the simplest method by which separates finite part and infinite part in integral formula is to expand Taylor series in external momentum. For example Taylor expansion of $\Gamma(P^2)$ in the neighborhood of $P^2 = 0$ is as follows.

$$\Gamma(P^2) = a_0 + a_1 P^2 + \cdots + \frac{1}{n!} a_n (P^2)^n + \cdots + a_n = \frac{\partial^n}{\partial P^2} \Gamma(P^2)|_{P^2=0} \qquad (18-7)$$

where coefficients of a_n with $n \geq 1$ are finite and only a_0 diverges logarithmically. If one expresses the sum of all finite quantities as $\tilde{\Gamma}(S)$, the result is

$$\Gamma(S) = \Gamma(0) + \tilde{\Gamma}(S) \qquad (18-8)$$

where $\Gamma(0)$ is infinite quantity and $\tilde{\Gamma}(S)$ finite quantity. This situation is similar to case where divergent term occurs in Maxwell's theory. Actually in Maxwell's theory, in case of expanding potential of field as a series in powers of V/c and considering radiation damping, the first term e/R of expansion would diverge and the second term $\mathbf{A}'^{(2)}$ of expansion of vector potential expressed by partial differentiation becomes finite (see sect. 2). Such situation allows Maxwell's theory to subtract infinite quantity $\left(\lim_{R\to 0} e/R\right)$ regarding it as meaningless one or under an excuse according to which within small spatial area of $R \approx 10^{-1}\ cm$, not classical physics but quantum theory is essential, to "solve" such theoretical difficulty. What divergence in the traditional quantum electrodynamics occurs within small area (region of large momentum) and first term of expansion of Taylor series leads to infinite quantity is formally the same as the difficulty in Maxwell' s theory. This shows that divergence occurred in quantum electrodynamics is rooted in difficulty of classical electrodynamics.

Now in our theory, let us see how brief and concise the problem of divergence of *S*-matrix is solved without using method of renormalization which is complicated and artificial. At the first place, let us expand Taylor series of $S^{(n)}$ and put it into divergent term and measurable finite term. In this case, by expanding Taylor series of $S_0^{(n)}$ in the traditional scattering theory, included in $S^{(n)}$, we can obtain the sum of divergent term $S_{in}^{(n)}$ and finite term $S_f^{(n)}$.

$$S^{(n)} = S_{in}^{(n)} + S_f^{(n)} \qquad (18-9)$$

where

$$S_{in}^{(n)} = \frac{S_{0(in)}^{(n)}}{1 + (\alpha_0)^n \frac{1}{(T_0)^n}\left(S_{in}^{(n)} + S_f^{(n)}\right) + V_{(n-1)}} + C \qquad (18-10)$$

$$S_f^{(n)} = \frac{S_{0(f)}^{(n)}}{1 + (\alpha_0)^n \frac{1}{(T_0)^n}\left(S_{in}^{(n)} + S_f^{(n)}\right) + V_{(n-1)}} \qquad (18-11)$$

And C is a finite constant, $S_{0(in)}^{(n)}$, the sum of all terms divergent in *n*th-order term of the scattering matrix expansion in the traditional quantum electrodynamics and $S_{0(f)}^{(n)}$ convergent term (finite correction).

Next, by using normalization rule discussed in sect. 10, let us obtain measurable finite correction from $S^{(n)}$. According to normalization rule 4, the normalization of some quantity expanded in series becomes the sum of the normalizations of each physical quantities which constitute it. From this, the following form is obtained

$$\bar{S}^{(n)} = \bar{S}_{in}^{(n)} + \bar{S}_f^{(n)} \qquad (18-12)$$

First of all, we find $\bar{S}_{in}^{(n)}$. If one, in terms of normalization rule 1, transforms $g_{\mu\nu}$ (metric of KR space) into $\delta^{\mu\nu}$ (Minkowski metric) in implicit function $\bar{S}_{in}^{(n)}$, $S_{in}^{(n)}$ and $S_f^{(n)}$ in the denominator of formula (18-11) leads to $S_{0(in)}^{(n)}$ and $S_{0(f)}^{(n)}$ defined in the traditional quantum electrodynamics. And $V_{(n-1)}$ is replaced by already normalized quantity, $\bar{V}_{(n-1)}$. Actually, the normalization of scattering matrix is applied in turn from lower order terms and accordingly $\bar{V}_{(n-1)}$ is considered as already normalized term. Namely, the result is

$$g_{\mu\nu} \to \delta_{\mu\nu}, \qquad g_{\mu\nu}\delta^{\mu\nu} \to 1$$

$$\left(g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}\right)^{-1/2} \to (1-\beta^2)^{-1/2}$$

$$S_{in}^{(n)} \to S_{0(in)}^{(n)} \qquad (18-3)$$

$$S_f^{(n)} \to S_{0(f)}^{(n)}$$

$$V_{(n-1)} \to \bar{V}_{(n-1)}$$

In the approximate calculation, $(1-\beta^2)^{-1/2} \approx 1$ is used. Hence, formula (18-10) arrives at

$$\bar{S}_{in}^{(n)} = \frac{S_{0(in)}^{(n)}}{1 + \left(\frac{\alpha_0}{T_0}\right)^n \left(S_{0(in)}^{(n)} + S_{0(f)}^{(n)}\right) + \bar{V}_{(n-1)}} + C \qquad (18-14)$$

If one divides numerator and denominator by $S_{0(in)}^{(n)}$ and taking into consideration the fact that $S_{0(in)}^{(n)}$ is an infinite quantity, formula (18-14) leads to

$$\bar{S}_{in}^{(n)} = \left(\frac{\alpha_0}{T_0}\right)^{-n} + C \qquad (18-15)$$

By choosing properly integral constant C, formula (18-15) arrives at

$$\bar{S}_{in}^{(n)} = 0 \qquad (18-16)$$

Next, let us obtain $\bar{S}_f$. According to normalization rule 1 and 3, the following transformation is done for $S_f^{(n)}$ in formula (18-11).

$$g_{\mu\nu} \to \delta_{\mu\nu}, \qquad S_f^{(n)} \to S_{0(f)}^{(n)}, \qquad S_{in}^{(n)} \to \bar{S}_{in}^{(n)} = 0, \qquad V_{(n-1)} \to \bar{V}_{(n-1)} \qquad (18-17)$$

Therefore, formula (18-11) leads to

$$\bar{S}_f^{(n)} = \frac{S_{0(f)}^{(n)}}{1 + \left(\frac{\alpha_0}{T_0}\right)^n S_{0(f)}^{(n)} + \bar{V}_{(n-1)}} \approx S_{0(f)}^{(n)} \qquad (18-18)$$

Taking together formula (18-16) and (18-18), the result is

$$\bar{S}^{(n)} = \bar{S}_{in}^{(n)} + \bar{S}_f^{(n)} = \bar{S}_f^{(n)} \approx \bar{S}_{0(f)}^{(n)} \qquad (18-19)$$

The method discussed above can be applied to all terms of *n*th order formula of scattering matrix expansion and then, as the trivial result, be obtained measurable finite quantity with physical meaning. Here, the calculation for individual terms of S-matrix is omitted because it is no more necessary.

Synthesizing all arguments mentioned above arrives at the following conclusions.

First, from scattering matrix of non-linear quantum electrodynamics, we obtained finite corrections with the procedure which infinite quantities were removed naturally and by themselves. From this we can make the correct theoretical analysis for shift of Coulomb's law, Lamb shift and anomalous magnetic moment of electron.

Second, our theory was built on the basic of the new classical theory of field established in KR space. The experimental verification of theoretical results mentioned above is essential to confirm validity of new classical theory of field which non-linear quantum electrodynamics is rooted in.

7. Unified Action Integral Formula of Electromagnetics-Gravitation

We eventually arrive at the final conclusion for unification of electromagnetics-gravitation that we aimed and kept searching so much. Here, we intend to discuss, in the unified form, electromagnetic field and gravitational field which have been, until now, regarded as separate fields.

If so, what is meant by unification of electromagnetic-gravitation? What is its real meaning? This is a serious problem with historical controversy and a large number of scholars has different views about this. The unification of electromagnetic-gravitation involves the following essential meaning.

First, unification of electromagnetic-gravitation is unification of conservation laws of energy-momentum. Accordingly in this case the total energy of electromagnetic-gravitation should be subject to inseparable unified conservation law of energy-momentum, not be conserved individually and separately.

Second, unification of electromagnetic-gravitation is mutual dependency and physical connection between sources of fields. In Maxwell's theory and GR, charge and mass-sources of fields are independent of one another and has mutually no relation. However, in our theory effective gravitational mass $\overline{m}_g$ which is in position of source of gravitational field is dependent on electromagnetic field potential and in case of electromagnetic field effective charge, $\bar{e}$, dependent on gravitational potential.

Third, unification of electromagnetic-gravitation is the mutual relation between two fields. In our unified theory two fields are subject to a metric of space-time.

If so, why should electromagnetic field and gravitational field be unified? What is that reason? It, in a word, is rooted in the starting point of our theory based upon experimental fact according to which the total energy of particle and two fields is the same as mc^2. From this is derived the conclusion that two fields are no more independent and separate and then mc^2 becomes common denominator combining two fields into a metric. Furthermore, this idea underlies the foundation for building unified theory of all fields, including nuclear field. Actually, fields produced by a particle mean all fields, not some specific field, including not only electromagnetic-gravitational field but also nuclear field. The annihilation of proton and antiproton shows clearly that energy of nuclear field made by proton and antiproton is also, as in annihilation of electron-positron, converted into some part of energy of photon. Hence, in unified theory of three fields to be constructed in the future, three different fields should be embodied by the metric of KR space or other new metric of space, and in this case evolution of theory must be based upon the fact that the total energy of particle-fields is mc^2.

Sect. 19 The motion equation of particle and the equation of field in unified theory of field.

Let us rewrite Lagrangian integral formula shown in sect. 8

$$S = -m_0 c \int \left(g_{\lambda\sigma} u^\lambda u^\sigma\right)^{\frac{1}{2}} ds - \frac{1}{16\pi c} \int \hat{C}_{\lambda\sigma} \hat{C}^{\lambda\sigma} \sqrt{-g}\, d\Omega \qquad (19-1)$$

The formula (19-1) yields the following motion equation of a particle.

$$m_0 c \frac{du_\lambda}{ds} = \frac{1}{c} \hat{\alpha} \hat{C}_{\lambda\sigma} u^\sigma - \frac{1}{c} \widehat{K}_\lambda \frac{d\hat{\alpha}}{ds} \qquad (19-2)$$

where $\hat{C}_{\lambda\sigma} = \hat{F}_{\lambda\sigma} + \hat{R}_{\lambda\sigma}$, $\hat{\alpha} = \hat{\alpha}_{(E)} + \hat{\alpha}_{(g)}$ (see sect. 8)

$$\hat{\alpha} = \frac{\hat{e} + \widehat{m}_{0(g)}}{1 + 2\alpha \widehat{K}_\sigma u^\sigma} = \hat{\bar{e}} + \widehat{\overline{m}}_{(g)}$$

$$\widehat{K}_\lambda = \hat{A}_\lambda + \hat{G}_\lambda$$

If one uses rules of isotopic vector space, formula (19-2) yields

$$m_0c^2\frac{du_\lambda}{ds}=\left(\bar{e}F_{\lambda\sigma}-\bar{m}_{0(g)}R_{\lambda\sigma}\right)u^\sigma$$
$$-\left[eA_\lambda\frac{d}{ds}\left(\frac{1}{1+2\hat{\alpha}\hat{K}_\sigma u^\sigma}\right)-m_{0(g)}G_\lambda\frac{d}{ds}\left(\frac{1}{1+2\hat{\alpha}\hat{K}_\sigma u^\sigma}\right)\right]\qquad(19-3)$$

Next, the equation of field can be written as follows

$$\frac{1}{4\pi}\frac{\partial\hat{C}_{ik}}{\partial x^k}=-\frac{1}{c}\hat{\alpha}V^i\delta(\mathbf{r}-\mathbf{r}_0)\qquad(19-4)$$

As $\hat{e}$ and $\hat{m}$, in isotopic vector space, are orthogonal each other, equation (19-4) can be written as follows.

$$\frac{1}{4\pi}\frac{\partial\hat{F}_{ik}}{\partial x^k}=-\frac{1}{c}\cdot\frac{\hat{e}V^i\delta(\mathbf{r}-\mathbf{r}_0)}{1+2\hat{\alpha}\hat{K}_\sigma u^\sigma}\qquad(19-5)$$

$$\frac{1}{4\pi}\frac{\partial\hat{R}_{ik}}{\partial x^k}=-\frac{1}{c}\cdot\frac{\hat{m}V^i\delta(\mathbf{r}-\mathbf{r}_0)}{1+2\hat{\alpha}\hat{K}_\sigma u^\sigma}\qquad(19-6)$$

As equation (19-5) and (19-6) include the mixed term, i.e. $2\hat{\alpha}\hat{K}_\lambda u^\lambda=2\alpha_{(E)}A_\lambda u^\lambda-2\alpha_{(g)}G_\lambda u^\lambda$ related to source of field in the denominators of the right-hand sides, electromagnetic potential and gravitational potential are no more independent of each other, that is, equation (19-5) and (19-6) show the inseparable unification of two fields. The procedure finding electromagnetic potential is the same as in sect. 6. If there is a unique difference, it is that denominators of $\bar{e}$ and $\bar{m}_{(g)}$ include the mixed term $2\hat{\alpha}\hat{K}_\lambda u^\lambda$ (the mixed term of electromagnetic-gravitation). The resultant four-dimensional potentials of two fields can be written as follows: In case of electromagnetic field, potential is

$$\hat{A}_i=\frac{1}{c}\cdot\frac{\hat{e}V^i}{r\left(1+2\frac{\hat{e}}{m_0c^2}\hat{A}_\lambda^{ex}u^\lambda+2\frac{\hat{m}}{m_0c^2}\hat{G}_\lambda^{ex}u^\lambda\right)}\qquad(19-7)$$

where we took on

$$\varphi_{(E)}^{in}=A_0^{in}=0\qquad(19-8)$$

for scalar potential of electrostatic field produced by charge itself, as shown in sect 11. In equation (19-7), if both sides are multiplied by $\hat{e}$, considering that unit vectors of $\hat{A}_i$ and $\hat{e}$ are the same in isotopic vector space, the left-hand side is eA_i and the right-hand side e^2. The result is

$$A_i=\frac{1}{c}\cdot\frac{eV^i}{r\left(1-2\frac{1}{c^2}G_\lambda^{ex}u^\lambda+2\frac{e}{m_0c^2}A_\lambda^{ex}u^\lambda\right)}\qquad(19-9)$$

In case of gravitational field, potential is

$$\hat{G}_i=\frac{1}{c}\cdot\frac{\hat{m}V^i}{r\left(1+2\frac{\hat{m}}{m_0c^2}\hat{G}_\lambda^{ex}u^\lambda+2\frac{\hat{e}}{m_0c^2}\hat{A}_\lambda^{ex}u^\lambda\right)}\qquad(19-10)$$

In formula (19-10) if both sides are multiplied by $\hat{m}'$ (m' is mass of another particle placed in field produced by m). From the definition of isotopic vector space

$$\hat{m}'\cdot\hat{m}=-m'm,\qquad\hat{e}\cdot\hat{A}_\lambda^{ex}=eA_\lambda^{ex},\qquad\hat{m}\cdot\hat{G}_\lambda^{ex}=-mG_\lambda^{ex}\qquad(19-11)$$

and in isotopic vector space unit vectors of $\hat{m}'$ and $\hat{G}_i$ are always vectors with opposite direction and so, $\hat{m}'\hat{G}_i$ has negative value, we find

$$G_i=\frac{1}{c}\cdot\frac{mV^i}{r\left(1-2\frac{1}{c^2}G_\lambda^{ex}u^\lambda+2\frac{e}{m_0c^2}A_\lambda^{ex}u^\lambda\right)}\qquad(19-12)$$

where $m_0 = m$

In case of scalar potential of static gravitational field, like in case of electrostatic field, the following formula holds.

$$G_0^{in} = \varphi_{(g)}^{in} = 0 \qquad (19-13)$$

The equation (19-9) and (19-12) show the obvious unification of electromagnetic field and gravitational field.

Sect. 20 Equivalence of total energy of particle-fields and inertial mass

Here, we show that the total energy of particle and electromagnetic-gravitational field produced by it is the same as inertial mass multiplied by c^2. The energy-momentum tensor of matter is given as follows (see sect. 7)

$$T_{\mu\nu}^{(m)} = \frac{1}{\sqrt{-g}} m_0 c u_\mu u_\nu \delta(\mathbf{r} - \mathbf{r}_0) \frac{ds}{dt} \qquad (20-1)$$

and the energy-momentum tensor is

$$T_{\mu\nu}^{(f)} = -\frac{1}{4\pi}\left(\hat{C}_{\mu\nu}\hat{C}_\nu^\lambda - \frac{1}{4}\hat{C}_{lm}\hat{C}^{lm}\right) g_{\mu\nu} \qquad (20-2)$$

Referring to the argument in sect. 13 for the equivalence of total energy of particle-electromagnetic field and inertial mass, one can get easily the formula for the equivalence of total energy of particle–fields (electromagnetic-gravitational field) and inertial mass. In this case, the formula for the total energy of particle and fields can be finally written as follows.

$$U = m_0 c^2 + \frac{1}{2}\int \left(\hat{G}_0 + \hat{A}_0\right)\left(\hat{\bar{e}} + \hat{\bar{m}}_{(g)}\right)\delta(\mathbf{r} - \mathbf{r}_0) = m_0 c^2 + \frac{1}{2}\left(G_0^{in}\bar{m}_{(g)} + A_0^{in}\bar{e}\right)$$

$$(20-3)$$

where G_0^{in} and A_0^{in} are scalar potentials which are given by particle and acts on particle itself. These potentials, as shown in formula (19-9) and (19-10), leads to zero. On the other hand, the normalization of $\bar{m}_{(g)}$ and $\bar{e}$ yields constant mass, m_0 and constant charge, e. Hence, in formula (20-3) only remains $m_0 c^2$. This is owing to the fact that G_0^{in}, $\bar{m}_{(g)}$, A_0^{in} and $\bar{e}$ are defined by normalized quantities following from rules of normalization. Thus, the total energy of particle and fields is

$$U = m_0 c^2 \qquad (20-4)$$

This formula is formally equal to Einstein's formula for energy. But as for its real meaning, the two formulas are quite different. In Einstein's theory U is the energy confined to particle only, but in our case U is the total energy of particle-fields and measured mass m_0 is equivalent with the total energy.

8. Experimental Verification for Unified Theory of Electromagnetic-Gravitational field

Here, we first show that theoretical results obtained from our theory gives, without any inconsistency, good solutions to already known experimental facts. Next, we present new theoretical results (i.e. new theoretical predictions) which require experimental verification concerning mutual dependency of two fields.

Sect. 21 Consistent theoretical solution to already found experimental facts

As well known, practice (experiment in the physics) is the criterion which decides whether or not theory is truth and basic factor that ensures viability of the theory. But, under the pretense of constructing theory in agreement with experiment, one recognizes inconsistency and insufficiency of the theory, but if one removes artificially or neglect inconsistency of the theory for conformity with experiment, nobody can say that the theory achieved the good agreement with experiment. Consequently, what build a theory as the consistent-closed theory is the basic premise for giving right solution to experiment. On this context, problems on whether Maxwell's theory and quantum electrodynamics gave satisfactory solution to already found experiments arise. Explicit answer follows. No!

(1) Annihilation of particles and production of photon.

As already argued in sect 1, the former consideration of pair annihilation of electron-positron (in general view, pair annihilation of particle-antiparticle) does not agree with conservation law of energy and total energy of electromagnetic-gravitational field results in loss of meaning. About this was already enough argued in chapter 1. Hence, traditional theory (quantum electrodynamics) fails to give the satisfactory solution to experimental facts associated with annihilation of particles.

Our theory found the unified Lagrangian of two fields and clarified finite character of the total energy of particle-fields and equivalence of inertial mass and total energy of particle-fields. Only this logic makes it possible to give right solution to experiment of pair annihilation of particle-antiparticle.

According to the traditional classical theory of field, electron and positron can approach infinitesimally till distance of interaction becomes zero. This stands against the actual experimental data. In our theory, electron and positron cannot approach closer than $r_0 = e^2/m_0c^2$ $(\approx 10^{-1}\ cm)$. Here, r_0 is the critical distance to allow approach of electron and positron. At just this critical distance r_0 is obtained a conclusion according to which electron-positron annihilates and photon produces, which is completely in agreement with experiment.

Now, let us see this in detail. The energy of particle shown in formula (5-18) is

$$E = \frac{m_0c^2\left(1 + 2\alpha_{(E)}A_\lambda u^\lambda\right)^{\frac{1}{2}}}{\sqrt{1-\beta^2}} + \frac{e\varphi}{\left(1 + 2\alpha_{(E)}A_\lambda u^\lambda\right)} \qquad (21-1)$$

and then allowing for

$$\varphi u^0 \gg A_i u^i, \qquad 2\alpha_{(E)}\varphi u^0 \approx 2\alpha_{(E)}\varphi \qquad (21-2)$$

where i denotes spatial components of space-time. In case of interaction of particle-antiparticle formula (21-1) yields

$$E = \frac{m_0c^2\left(1 - \frac{r_0}{r}\right)^{1/2}}{\sqrt{1-\beta^2}} - \frac{e\varphi}{\left(1 - \frac{r_0}{r}\right)} \qquad (21-3)$$

where

$$r_0 = 2\frac{e^2}{m_0c^2}$$

When a particle and an antiparticle approach r_0, the denominator of term relevant to interactional energy (second term of the right-hand side of formula (21-3)) becomes zero. In order for interactional energy to have finite value, charge, e, should be zero. Namely, at $r = r_0$ charge vanishes.

Let us consider the term relevant to particle in formula (21-3). At $r = r_0$, the numerator becomes zero. In order for the energy of particle to have finite value which is not zero, $\beta^2 = 1$ should holds in the denominator, that is, the particle has the velocity of light $(V = c)$. And m_0 should also vanish. This fact results from the requirement of SR that a free particle with rest mass cannot reach velocity of light. (Actually at $r = r_0$, $e = 0$ is allowed and then, owing to absence of interaction of particles, a particle becomes a free particle). This agrees with the experimental fact about annihilation of particles and production of photon.

Next, let us consider the capture of electron by proton. In case of proton-antiproton, the critical distance is $R_0 = e^2/M_0c^2$, where M_0 is the rest mass of proton and $M_0 = 1836\ m_0$ (m_0; rest mass of electron), accordingly, $r_0 \gg R_0 (r_0 = 1836 R_0)$. In this connection, if electron approaches proton, what will result in? As far as electron reaches $r = r_0$, life time of electron ends and from formula (21-3) follows $e = 0,\ m_0 = 0,\ \beta^2 = 0$.

Notes: we can think that in formula (21-3) an electron is not captured by a proton but converted into photon, i.e. $e = 0,\ m_0 = 0,\ \beta^2 = 1 (V = c)$. But this conversion cannot be made. It is because this conversion stands against the conservation law of charge. The proton maintains life time of existence as far as antiparticle does not arrive at $r = R_0$. Thus electron is annihilated to be captured by proton and then a system of proton-electron is converted into neutron. This also was verified experimentally.

But from the traditional theory does not follow the possibility of capture of an electron by a proton. According to Maxwell's theory, proton and electron can approach, irrespective of difference of mass, near infinitely and from this leads to the conclusion that at $r = 0$, charge of electron and proton becomes zero simultaneously. Moreover, in this case is not drawn the conclusion that mass of electron becomes zero. Actually annihilation of a charged particle is meant by the simultaneous vanishment of charge and mass. But the traditional theory does not yield this conclusion.

The annihilation and occurrence of particles are singular physical effects appearing in strong field, namely of near distance between particles (particle radius). In the theory of field experimental verification of physical effects manifested in strong field is of decisive significance to confirm validity of the theory. Actually in Einstein's GR also existence of Black Hole (Black Star) relevant to gravitational radius $\left(R_{(g)} = 2k_{(g)}\, m/c^2\right)$ or gravitational singular point was predicted but not yet discovered. By the way, in electromagnetic interaction the annihilation and production of particles have already been observed through the experiment. With regard to our theory the consistent solution to the annihilation and production of particles is also of decisive significance to verify validity of this theory.

(2) The radiation damping effect

It have already been verified by experiment that in electromagnetic field accelerated charge radiates electromagnetic wave and necessarily is acted upon by damping force. But all attempts to explain radiation damping effect within framework of Maxwell's theory lead to a knotty point (see sect. 2). Our theory, as already shown in sect. 12, gives the consistent solution to radiation damping.

(3) The decay of a system of particles and deficiency of mass (difficulty of gauge symmetry)

Let us consider decay of a system of particles into two particles. In this case, the result is

$$Mc^2 = m_1c^2 + m_2c^2 + \varepsilon \qquad (21-4)$$

where m_1 and m_2 are masses of collapsed particles and ε energy radiated outside. Therefore, from formula (21-4) follows

$$M > m_1 + m_2 \qquad (21-5)$$

But, in the view of Maxwell's theory, the explanation for this experiment arrives at difficulty. As shown in sect. 2, according to principle of gauge symmetry, electric potential of subparticles which constitute a system can be changed into arbitrary value and energy of a system transformed to zero or negative value by the proper gauge transformation. This shows that gauge principle does not agree with formula (21-4) and (21-5), verified strictly by many experiments.

In our theory, gauge principle is not, from Lagrangian integral formula, allowed and as a result, electric potential is defined so that it has a unique value. Hence, our theory explains formula (21-4) and (21-5) without inconstancy.

(4) The main experimental results of quantum electrodynamics.
As well known, Lamb shift and anomalous magnetic of electron were verified by many experiments. But renormalization method, a main methodology in the traditional quantum electrodynamics is artificial and harms logical system of theory (see sect. 18).

In our theory, the consistent solutions to experiments mentioned above is of important significance to validity of our theory.

(5) The experimental results in gravitational field (shift of Mercury's perihelion, deflection of light ray, red shift of light spectrum).
As mentioned in sect. 3, Einstein's GR involves unavoidable difficulties and so, the theoretical solution of GR for three effects of gravitation is also not satisfactory.

Our theory for gravitation, based upon good solutions to difficulties of GR, is combined with theory of electromagnetic field whose validity have already been confirmed in macroworld as well as microworld and can also successfully explains three effects of gravitation. This is of essential significance in verifying validity of theory of gravitation.

Sect. 22 New theoretical results to be verified by experiments (theoretical predictions)

In this section, we consider the theoretical results, obtained from unified theory of fields, to be verified by experiments.

(1) Variation of electromagnetic field by gravitational field.
Until now on, in physics electromagnetic field and gravitational field have been discussed separately and with no relation between them. But in our theory electromagnetic field and gravitational field are no more independent.

Now, let us consider electrostatic field (scalar potential) created by a charge placed in the static, constant gravitational field. From formula (19-9) follows

$$\varphi_{(E)} = \frac{e}{r\left(1+\frac{2}{mc^2}\hat{m}\hat{G}_0 u^0\right)} \approx \frac{e}{r\left(1-\frac{2}{c^2}G_0\right)} = \frac{e}{r\left(1-\frac{2}{c^2}\varphi_{(g)}\right)} \qquad (22-1)$$

where G_0 or $\varphi_{(g)}$ is gravitational potential which has positive value. Then, formula (22-1) gives

$$\varphi_{(E)} = \frac{\varphi'_{(E)}}{1-\frac{2}{c^2}\varphi_{(g)}} \qquad (22-2)$$

where $\varphi'_{(E)} = e/r$ (electrostatic potential in Maxwell' s theory)

Intensity of electric field results in

$$\mathbf{E} = -\mathrm{grad}\varphi_{(E)} = -\frac{\mathrm{grad}\varphi'_{(E)}}{1-\frac{2}{c^2}\varphi_{(g)}} \qquad (22-3)$$

If $\frac{2}{c^2}\varphi_{(g)} \ll 1$ is allowed, formula (22-3) yields

$$\mathbf{E} = -\mathrm{grad}\varphi'_{(E)}\left(1+\frac{2}{c^2}\varphi_{(g)}\right) \qquad (22-4)$$

Putting $\mathbf{E}_0 = -\mathrm{grad}\varphi'_{(E)}$ which is the intensity of electric field in Maxwell' s theory, the result is

$$\mathbf{E} = \mathbf{E}_0\left(1+\frac{2}{c^2}\varphi_{(g)}\right) \qquad (22-5)$$

In case of magnetic field also is obtained the following similar formula.

$$\mathbf{H} = \mathbf{H}_0\left(1 + \frac{2}{c^2}\varphi_{(g)}\right) \qquad (22-6)$$

Consequently, the intensities of electric field and magnetic field are increased by $2\varphi_{(g)}/c^2$ in static gravitational field when $\frac{2}{c^2}\varphi_{(g)} \ll 1$.

(2) Variation of gravitation by electromagnetic field.

As shown in formula (19-12) also is varied gravitational force which acts on charge placed in electromagnetic field. It is named *variation of gravitation by electromagnetic field.*

Now, let us estimate roughly to how much gravitation is varied in electric field. For this, we consider a system which consists of two plates A and B. applying voltage to the two plates and making electric potential of one plate B become zero, one can consider the plate, A only. For better consideration we present following assumption; firstly, the plates are metal material which consists of identical atoms only; secondly, the ions and free electrons which constitutes metal material interact individually with external field and are considered as point particles.

Now, let us consider the variation of effective gravitational mass of plates A, B. First of all, total mass of A in the absence of external electric field is

$$\bar{M} = Nm_+ + nm_e \qquad (22-7)$$

where N is total number of ions and n total number of free electrons, m_+ mass of a positive ion and m_e mass of a free electron.

Next, in case of being external electric field (when electric source is supplied), let us calculate total effective mass of plate A, referring to $\bar{m}_{0g}$ of formula (5-12). The effective mass of a positive ion is

$$\bar{m}'_+ = \frac{m_+}{1 + \dfrac{2q}{m_+c^2}\varphi} \approx m_+(1 - 2\alpha_+\varphi) \qquad (22-8)$$

and the effective mass of a free electron,

$$\bar{m}'_e \approx m_e(1 + 2\alpha_e\varphi) \qquad (22-9)$$

where

$$\alpha_+ = \frac{q}{m_+c^2} = \frac{eZ}{m_+c^2}$$

Z is valence and $\alpha_e = |e|/m_ec^2$. Thus, total effective mass of plate A is

$$\bar{M}' = N\bar{m}'_+ + n'\bar{m}'_e \qquad (22-10)$$

where n' is total number of electrons varied by external electric field. Consequently, the variation of total effective mass of plate A.

$$\Delta\bar{M} = \bar{M}' - \bar{M} = N(\bar{m}'_+ - m_+) + n(\bar{m}'_e - m_e) + \Delta n\frac{2e}{c^2}\varphi + \Delta nm_e \qquad (22-11)$$

where $\Delta n = n' - n$ and Δnm_e is the variation of mass occurred by movement of free electrons (This term has no significance).

If one calculates formula (22-11) and arranges, the result is

$$\Delta\bar{M} = -NZ\frac{2e}{c^2}\varphi + n'\frac{2e}{c^2}\varphi + \Delta nm_e = \frac{2e}{c^2}\varphi(n' - NZ) + \Delta nm_e \qquad (22-12)$$

On the other hand, $Q = |e|(NZ - n')$ is the total charge of plate A and can be written as

$$Q = |e|(NZ - n') = c_0u \qquad (22-13)$$

where c_0 is the electric capacity of condenser consisting of the two plates and u the voltage between the plates A and B.

In case where plate A is charged with positive value, the following formula is obtained.

$$\Delta n = -\frac{c_0u}{|e|} \qquad (22-14)$$

Therefore, the variation of total effective mass of plate A is

$$\Delta\bar{M} = -2\frac{c_0 u^2}{c^2} - c_0 u\frac{m_e}{c} \quad (22-15)$$

where $\varphi = u$ was used. In the static gravitational field, (earth gravitational field) the variation of gravitational force is proportional to variation of effective gravitational mass and accordingly, the result is

$$\Delta F = \Delta\bar{M}g \quad (22-16)$$

where g is gravitational acceleration.

The variation of gravitational force by electric field further suggests necessity of study relevant to possibility of anti-gravitation and teleportation. Total effective gravitational mass of plate A can be written as (see formula 22-10).

$$\bar{M} = \frac{Nm_+}{1 + \frac{2eZ}{m_+c^2}\varphi} + \frac{nm_e}{1 - \frac{2e}{m_ec^2}\varphi} \quad (22-17)$$

Now, in formula (22-17) we suppose that the following condition is allowed

$$\left|\frac{2e}{m_+c^2}\varphi\right| \approx 1, \quad \left|\frac{2e}{m_ec^2}\varphi\right| > 1 \quad (22-18)$$

In this case as second term of the right-hand side of formula (22-18) has the negative value and much larger value than first term, $\bar{M}$ has negative value and consequently, arrives at the conclusion that a system of the two plates results in rising upward by anti-gravitation. But in terms of non-quantum theory or classical physics, condition (22-18) cannot be allowed. It is relevant to what if condition (22-18) holds, the effective inertial mass has complex value and formula (5-18) for energy of matter results in loss of meaning. But considering in the view of quantum theory the situation is, to some degree, changed. In terms of quantum theory can be considered some probability that as if particles happen to come out of potential well by tunnel effect associated with uncertainty of energy, it may be in state satisfying condition (22-18). Of course, this is nothing but a hypothesis. This problem needs further deeper theoretical study associated with experimental verification. If anti-gravitation subject to condition (22-18) is actually manifested, it goes without saying that there will make the great revolution in practical regions.

(3) Nonlinear effects of interaction.

Maxwell's theory and the traditional quantum electrodynamics are linear theories which are subject to principle of superposition. But our theory is nonlinear, which bring about nonlinear character of interaction (see sect. 5 and sect. 17). Many effects concerning this nonlinearity should be studied and verified by experiments.

Here, let us consider briefly effects of nonlinear interaction. In classical theory of field, when $A_\lambda = \sum_{i=1}^{n} A_{\lambda(i)}$, $\mathbf{E} = \sum_{i=1}^{n} \mathbf{E}_{(i)}$ are given, the electric force which external electric field acts on the effective charge is as follows:

$$\mathbf{F} = \bar{e}\mathbf{E} = \frac{e\left(\mathbf{E}_{(1)} + \cdots + \mathbf{E}_{(n)}\right)}{\left[1 + 2\alpha\left(\sum_{i=1}^{n} A_{\lambda(i)}\right)u^\lambda\right]} \quad (22-19)$$

For simplicity, we will neglect the fact that $\mathbf{E}_{(i)}$ is dependent on other external electric fields in formula (22-19). Expanding formula (22-19) as a power series with $2\alpha\left(\sum_{i=1}^{n} A_{\lambda(i)}\right)u^\lambda \ll 1$ and taking to terms of first order of $2\alpha\left(\sum_{i=1}^{n} A_{\lambda(i)}\right)u^\lambda$, the result is

$$\mathbf{F} \approx e\sum_{i=1}^{n} \mathbf{E}_{(i)} - 2e\alpha\sum_{i=1}^{n} \mathbf{E}_{(i)} A_{\lambda(i)}u^\lambda \quad (22-20)$$

or

$$\mathbf{F} \approx e\left(\sum_{i=1}^{n} \mathbf{E}_{(1)} - 2\alpha \mathbf{E}_{(1)} \sum_{i=1}^{n} A_{\lambda(i)} u^{\lambda}\right) + \cdots + e\left(\sum_{i=1}^{n} \mathbf{E}_{(n)} - 2\alpha \mathbf{E}_{(n)} \sum_{i=1}^{n} A_{\lambda(i)} u^{\lambda}\right) \quad (22-21)$$

From above formula, in quantum theory of field, interactional Hamitonian H_{ξ} becomes

$$H_{\xi} \sim \bar{e} \int dx^4 \bar{\psi} \gamma_{\lambda} A^{\lambda} \psi \approx e\left(\int dx^4 \bar{\psi} \gamma_{\lambda} A^{\lambda} \psi\right)\left(1 - 2\alpha \int dx^4 \bar{\psi} \gamma_{\lambda} A^{\lambda} \psi\right)$$

or

$$H_{\xi} \sim \left[\int e dx^4 \bar{\psi} \gamma_{\lambda} A^{\lambda}_{(1)} \psi - 2\alpha \int dx^4 \bar{\psi} \gamma_{\lambda} A^{\lambda}_{(1)} \psi \left(\sum_{i=1}^{n} \int dx^4 \bar{\psi} \gamma_{\lambda} A^{\lambda}_{(i)} \psi\right)\right] + \cdots$$
$$+ \left[\int e dx^4 \bar{\psi} \gamma_{\lambda} A^{\lambda}_{(n)} \psi - 2\alpha \int dx^4 \bar{\psi} \gamma_{\lambda} A^{\lambda}_{(n)} \psi \left(\sum_{i=1}^{n} \int dx^4 \bar{\psi} \gamma_{\lambda} A^{\lambda}_{(i)} \psi\right)\right] \quad (22-22)$$

Second terms in each bracket of formula (22-21) and (22-21) are interference terms by interrelation of fields. The experimental verification of this interference effects is essential to confirm validity of this theory. The closer distance of particle is, the more notably this effect is manifested (especially, in neighborhood of electron radius r_0).

Next, let us consider briefly interaction between nucleus and electron. In this case electrostatic field of nucleus is regarded as classical field and motion of electron only is considered in view of quantum theory. In our theory occurs the nonlinear term because the effective charge of electron is dependent on field of nucleus and effective charge of nucleus is dependent on the field created by an electron. From formula (11-9) and (5-9) electrostatic field potential of nucleus and effective charge of electron can be written as follows.

$$A^{0}_{(X)} = \varphi_p = \frac{Ze}{4\pi r\left(1 + 2\alpha_p \varphi_e u^0\right)} \quad (22-23)$$

$$\bar{e} = \frac{e}{1 + 2\alpha_e \varphi_p u^0} \quad (22-24)$$

where $\alpha_e = e/mc^2$, $\alpha_p = Ze/Mc^2$ and φ_p electrostatic potential of nucleus, φ_e electrostatic potential of electron. Therefore, potential energy of electron is as follows.

$$U = \bar{e}\varphi_p \approx \frac{Ze^2}{4\pi r\left(1 + 2\alpha_e \varphi_p\right)\left(1 + 2\alpha_p \varphi_e\right)} \approx \frac{Ze^2}{4\pi r\left(1 - Z\frac{r_0}{r}\right)} \quad (22-25)$$

where r_0 is radius of electron, $u^0 \approx 1$ and $\alpha_p \ll \alpha_e$.

In case where scattering of electron and radiation reaction, etc. are considered, it is important to get new effects produced by term $(1 - Zr_0/r)$ and verify it through experiments. Here, we don't give the detailed calculation about this. On the other hand, Formula (22-25) yields singular point in $r = Zr_0$. As showed in discussion about formula (21-3), at just this point is drawn the conclusion that capture of electron by proton occurs. This follows from requirement according to which scattering matrix of particle should have finite value even in singular point.

Until now we discussed the several main things of the unified theory of field and quantum electrodynamics. In modern physics Maxwell's theory, Einstein's GR and quantum electrodynamics have been recognized as "incarnation of absolute cult". The criterion that judges validity of new theory does not consist in cult of ready-made theory but objective experiments and, based upon it, strict logical rules and basic principles for building of consistent theory.

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