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THE PHILOSOPHY OF MATHEMATICS

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ABSTRACT

What is mathematics? Why does it exist? Is it consistent? Is it complete? We answer these questions as well as resolve Russell's Paradox and debunk Godel's Incompleteness Theorem.

WHY MATHEMATICS EXISTS?

The brain is a product of evolution. All structures in the brain helped our ancestors to survive at one point. Newborns demonstrate numerosity and spatial reasoning[1,2], which are the basis for mathematics. This means that the building blocks for mathematical reasoning are innate, which means that we are born with these building blocks. Since all structures in the brain are evolutionary adaptations, the building blocks of mathematics must have helped out ancestors to survive at one point. This makes sense because our ancestors who could not estimate quantities or reason about space would perish in the wild. The ones that made it are the ones that could, bestowing upon us the faculties for advanced mathematics. Thus, the basis for mathematical reasoning are evolutionary adaptations, and advanced mathematics is a byproduct of these evolutionary adaptations.

IS MATHEMATICS CONSISTENT?

Consistent means containing no logical contradictions. We first show that mathematics is consistent with reality. By the anthropic principle, if our immediate environment is not intelligible, then we would not evolve intellect. Here, we extend the contrapositive of this statement. If we evolved the basis for mathematical reasoning, then our immediate environment abides by laws comprehensible by mathematical reasoning. Thus, mathematics is always consistent with the environment in which we evolved. If there is a corner of the universe that abides by no physical law, then this corner would render our mathematical reasoning useless, and be unintelligible. Nothing intelligent will evolve out of there either. More formally, mathematics is as consistent as the observable universe. If the observable universe contains no contradiction, then neither does mathematics. Next, we debunk Russell's Paradox as an inconsistency in mathematics.

Russell's Paradox

Consider the set of all sets that are not members of themselves. Such a set appears to be a member of itself if and only if it is not a member of itself.

Debunking Argument

Mathematics is neurons sizzling away to reason precisely about entities using symbols. Recent neuroscientific evidence suggest that these symbols are likely single neurons that fire if and only if given the symbolized entity as a stimulus[3]. If so, a set is a neuron in the brain that fires when the collection it represents is required for reasoning. We hereunto refer to this as the collection maps to this neuron. Consider a set that is not a member of itself. This is a neuron that does not map to itself. This is fine as this is usually the case. Let us call such a neuron as a neuron x. Now, consider a neuron y that all of neuron x's map to. Clearly, this neuron y is separate from the neuron x's that y represents. The confusion now lies in whether y can map to itself, as by our definition, neuron y is also a neuron x. For the brain specifically, if neuron y maps to itself, the concept of neuron y alone is then equal to the entirety of neuron x's excluding y. We clearly have no difficulty differentiating the two, and these concepts are thus separate, requiring different neurons. If we were to reason about neuron y, there most likely exists a neuron z that y maps to, and all x's map to y. On the other hand, in the C language, if y is a pointer to all pointers, does y point to itself? Well, memory address 1 can store value 1. Either you interpret 1 as a value, or as a pointer in itself, storing value 1 at address 1 results in no inconsistency in reality. In the particular architecture of the C language, y can point to itself. Thus, the answer depends on the wiring. Either way results in no logical inconsistency in reality. The apparent inconsistency in Russell's Paradox is linguistic. It arises from the faulty definition of "all". In the brain, neuron y is excluded from this "all" as a neuron cannot map to itself. In the C language, pointer y can point to itself because the C language allows it. Thus, the answer depends on the definition of "all." More importantly, the problem is trivial because reality remains consistent however "all" is defined. When it is, and properly so, the mathematics about it is consistent.

IS MATHEMATICS COMPLETE?

Godel tried to prove that mathematics is incomplete. Complete means that we can prove anything that is true. Unfortunately, Godel suffered from a similar kind of confusion as Russell.

Godel's Incompleteness Theory

Now let's consider "This statement is unprovable." If it is provable, then we are proving a falsehood, which is extremely unpleasant and is generally assumed to be impossible. The only alternative left is that this statement is unprovable. Therefore, it is in fact both true and unprovable. Our system of reasoning is incomplete, because some truths are unprovable.[4]

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Debunking Argument

Here, "this statement" is a symbol for "this statement is unprovable," Thus, "this statement is unprovable" is saying that it is impossible to prove "this statement is unprovable." If we substitute the symbol "this statement" back in, then we have "this statement" is unprovable. Well, we just created a symbol that is by definition unprovable. The act of creating a symbol assumes there exists something in reality that shares the property of this symbol. Thus, the creation of this symbol assumes there exists statements that are unprovable, and this is therefore an assumption that mathematics is incomplete. This is a circular argument. We are assuming mathematics is incomplete to prove that mathematics is incomplete. This is, by the way, no proof of contradiction, which is assuming something to be true only to prove that it is false, or vice versa. This shows that we cannot introduce in any argument a symbol that represents something we are uncertain to exist. The act of symbolizing assumes existence, and decouples the argument from reality. The assumption, be it true or false, renders any argument that is not a proof by contradiction, be it mathematics or otherwise, invalid. Thus, bringing in Zeus who always speaks true to prove a point because Zeus says so is an invalid argument. On the other hand, Turing's assumption of existence of an oracle for the halting problem is to disprove the existence of such an oracle. This is then a proper proof by contradiction. We use it to show that mathematics is incomplete.

Mathematics is not complete because there exists a statement in the halting problem that is true but unprovable. Turing showed that for any program f that may determine if programs halt, a program g can pass itself and its input to f and do the opposite of what f does. A contradiction results. Thus, there exists a program that is impossible to determine from its description if it halts or not. It halts, or it does not. Therein lies a true statement. However, we do not know which is true, let alone proving it. This program can then be translated into a mathematical statement by the Curry-Howard Correspondence. Mathematical is thus incomplete.

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