The Standard Model reformulated in terms of a derivative-free analogue of the Lagrangian density

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Abstract

1 Motivation

By design, the Lagrangian density of the Standard Model ($\mathcal{L}$) is that functional whose invariance with respect to variation in each of its component fields generates the empirically established equations of motion. This paper introduces *another* (non-local, zero dimensional) scalar functional ($\mathcal{O}$) that, like $\mathcal{L}$, also yields the empirically observed dynamics of physical quantum fields, but which, unlike $\mathcal{L}$, does not make recourse to non-physical (gauge and ghost) modes of freedom.

2 Space-time anti-derivatives

Consider a complex spinor $\Lambda = \big(\lambda_1 + i\lambda_2, \quad \lambda_3 + i\lambda_4\big)$ with a $U(1)$ degenerate mapping onto the past null cone with vertex at the origin of x:

$$x_\mu = \Lambda^* \sigma_\mu \Lambda$$

$$d\Lambda = \prod_{i=1}^{4} d\lambda_i = 2\pi\delta(t^2 - r^2)d^4x = 2\pi\frac{d^3\mathbf{r}}{r}$$

It can be shown that, for all $k^2 \neq 0$

$$\int_{-\infty}^{0} e^{-ik_\nu x^\nu} d\Lambda = \frac{1}{k^2} \tag{1}$$

and

$$\int_{-\infty}^{0} ix_\mu e^{-ik_\nu x^\nu} d\Lambda = \frac{2k_\mu}{k^4} \tag{2}$$

The product of two Λ cones maps onto space-time with a $U(1)_L \times SU(2) \times U(1)_R$ degeneracy:

$$x_\mu = a_\mu + b_\mu = (\Lambda_a^*, \Lambda_b^*)[\gamma_\mu \otimes \mathbf{1}](\Lambda_a, \Lambda_b)$$

$$\int\int e^{-ik_\nu x^\nu} d\Lambda_a d\Lambda_b = \frac{1}{k^4} \tag{3}$$

$$d\Lambda_a d\Lambda_b = 4\pi^2 d^4x$$

So, for an arbitrary field ψ:

$$\int \psi d\Lambda = (\partial_\nu \partial^\nu)^{-1}\psi \ , \quad \int \psi x_\mu d\Lambda = (\partial_\nu \partial^\nu)^{-2}\partial_\mu \psi \ , \quad \int\int \psi d\Lambda_a d\Lambda_b = (\partial_\nu \partial^\nu)^{-2}\psi$$

3 The electroweak standard model reconstrued

We define dimensionless antisymmetric tensors whose 1st derivatives yield fixed-gauge boson vector fields:

$$\partial^\nu \mathcal{A}_{\mu\nu} \equiv A_\mu$$
$$\partial^\nu \mathcal{W}^\pm_{\mu\nu} \equiv W^\pm_\mu$$
$$\partial^\nu \mathcal{Z}_{\mu\nu} \equiv Z_\mu$$

We define $SU(2)_L \times U(1)_Y$ gauge transformations:

$$e^{i\Phi_L} \equiv \exp\left[i \begin{pmatrix} g'\mathcal{Z}^{\mu\nu} & g\mathcal{W}^{\mu\nu}_+ \\ g\mathcal{W}^{\mu\nu}_- & e\mathcal{A}^{\mu\nu} - g'\mathcal{Z}^{\mu\nu} \end{pmatrix} \sigma^L_\mu \sigma^L_\nu \right] \ , \qquad e^{i\Phi_R} \equiv \exp\left[ie\mathcal{A}^{\mu\nu}\sigma^R_\mu \sigma^R_\nu \right]$$

and gauge-transformed fermion fields:

$$\psi_L \equiv e^{i\Phi_L} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \ , \qquad \psi_R \equiv e^{i\Phi_R} e_R$$

We define a dimensionless scalar field, analogous to $\mathcal{L}$:

$$\mathcal{O} \equiv \mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} + \mathcal{W}^\pm_{\mu\nu}\mathcal{W}^{\mu\nu,\mp} + \mathcal{Z}_{\mu\nu}\mathcal{Z}^{\mu\nu} + \int \phi^2 \left[g^2 \mathcal{W}^\pm_{\mu\nu}\mathcal{W}^{\mu\nu,\pm} + (g^2 + g'^2)\mathcal{Z}_{\mu\nu}\mathcal{Z}^{\mu\nu} \right] d\Lambda$$
$$+ \int\int \phi^4 d\Lambda_a d\Lambda_b$$
$$+ \int ix^\alpha \bar{\psi}_{L,f} e^{i\Phi_L} \sigma^L_\alpha e^{-i\Phi_L} \psi_{L,f} d\Lambda + 2\lambda_f \int\int \phi \bar{\psi}_{L,f}\psi_{R,f} d\Lambda_a d\Lambda_b$$
$$+ \int ix^\alpha \bar{\psi}_{R,f} e^{i\Phi_R} \sigma^R_\alpha e^{-i\Phi_R} \psi_{R,f} d\Lambda + 2\lambda_f \int\int \phi \bar{\psi}_{R,f}\psi_{L,f} d\Lambda_a d\Lambda_b$$

f is the fermion family index, $\phi = (0, v + h)$ denotes the Higgs scalar $SU(2)$ doublet, $m_{e,f} = \lambda_f v$, $\sigma^L_0 = \sigma^R_0$, $\sigma^L_i = -\sigma^R_i$ and the neutrino Majorana mass terms have been omitted.

We now posit invariance of $\mathcal{O}$ with respect to variations in each of its component fields. Since $\mathcal{O}$ contains no derivatives, the equations are considerably simpler than the analogous Euler-Lagrange constraints on $\mathcal{L}$. For example,

$$\frac{\partial \mathcal{O}}{\partial \bar{\psi}_L} = 0$$

yields the Dirac equation and

$$\frac{\partial \mathcal{O}}{\partial \mathcal{A}_{\mu\nu}} = 0$$

yields the e-m source equation.
As well as yielding the mass of the Higgs boson,

$$\frac{\partial \mathcal{O}}{\partial \phi} = 0$$

predicts that the v.e.v. of the Higgs field is given by the sum of contributions from all fermions in the universe.