

ABC conjecture is in the Ambiguity in which case of $\varepsilon > 0$

Zhang Tianshu

Zhanjiang city, Guangdong province, China
chinazhangtianshu@126.com
china.zhangtianshu@tom.com

Abstract

Since there are infinitely many consecutive satisfactory values of ε to enable $A+B=C$ satisfying $C > (\text{rad}(A, B, C))^{1+\varepsilon}$, thus the author uses a representative equality, namely $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(1, 2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$, and that first let ε equal a value near the greater end of the infinitely many consecutive satisfactory values to prove the *ABC* conjecture; again let ε equal a value near the smaller end to negate the *ABC* conjecture. This shows that the *ABC* conjecture is in the ambiguity in which case of $\varepsilon > 0$.

AMS subject classification: 11D75, 11A51, 11D88

Keywords: *ABC* conjecture, illustrate with example, tenable, untenable

1. Introduction

ABC conjecture was proposed by Joseph Oesterle and David Masser in 1985.

The conjecture states that if A , B and C are three co-prime positive integers, satisfying $A+B=C$, then for any real number $\varepsilon > 0$, there is merely at most a finite number of solutions to the inequality $C > (\text{rad}(A, B, C))^{1+\varepsilon}$, where $\text{rad}(A, B, C)$ denotes the product of all distinct prime divisors of A , B and C . Yet it is still both unproved and un-negated a conjecture hitherto.

2. For *ABC* conjecture the proof and negation coexist

As everyone knows, whether who wants to prove the *ABC* conjecture or negate it, all in all, that is a very difficult thing, also is impossible in reality.

Such it is so, the author has to find an equality such that the difference of C minus $\text{rad}(A, B, C)$ is small as far as possible. Self-evidently, not only the way of doing is simple and convenient, but also it implies that once proved the equality, actually proved likewise other equalities that are covered by it.

So let A or B to equal 1, and the rest one equals O^2-1 , then C is equal to O^2 according to $A+B=C$, where O expresses an odd number ≥ 3 .

Then, the equality $A+B=C$ satisfying $C > (\text{rad}(A, B, C))^{1+\varepsilon}$ is changed into the equality $1+(O^2-1)=O^2$ satisfying $O^2 > (\text{rad}(1, O^2-1, O^2))^{1+\varepsilon}$ in the case that regards ε as an infinitesimal real number > 0 .

If O is a positive prime P , then the equality $1+(O^2-1)=O^2$ satisfying $O^2 > (\text{rad}(1, O^2-1, O^2))^{1+\varepsilon}$ is turned into the equality $1+(P^2-1)=P^2$ satisfying $P^2 > (\text{rad}(1, P^2-1, P^2))^{1+\varepsilon}$. In the case that regards ε as an infinitesimal real number > 0 , $P^2 > (\text{rad}(1, P^2-1, P^2))^{1+\varepsilon}$ approximates to $P > (\text{rad}(P^2-1))^{1+\varepsilon}$. When $P \geq 7$, see also APPENDIX at the back of this article, for reference only.

Thus it can be seen, the equality $1+(P^2-1)=P^2$ satisfying $P > (\text{rad}(P^2-1))^{1+\varepsilon}$ by and large, seemingly should last forever in the case that regards ε as an infinitesimal real number > 0 , although the densities of satisfactory primes are getting sparser and sparser along with which the values of P are getting greater and greater, but there are infinitely many primes after all.

To say nothing of the conjecture including all positive integers, then

presumably satisfactory positive integers must be even more.

Well then, let the equality $1+(O^2-1) = O^2$ be endowed with certain peculiar values, enable it to turn into a representative equality, and that let us use the representative equality, both can prove and can negative the *ABC* conjecture.

From $O^2-1=(O+1)(O-1)$, we know that $O+1$ and $O-1$ are positive even numbers. Further let $O+1$ to equal 2^N with $N \geq 2$, then not only 2 is a common prime factor of $O+1$ and $O-1$, but also 2 is the unique prime factor of $O+1$.

From $O+1=2^N$, get $O=2^N-1$, $O-1=2^N-2$, $O^2=(2^N-1)^2$ and $O^2-1=2^N(2^N-2)$, so the equality $1+(O^2-1)=O^2$ satisfying $O^2 > (\text{rad}(1, O^2-1, O^2))^{1+\varepsilon}$ is transformed into equality $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(1, 2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ i.e. $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ in the case that regards ε as an infinitesimal real number > 0 .

Since $N \geq 2$, thus there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$. Also the symbol between $(2^N-1)^2$ and $[\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ is alterable, and illustrate with example as follows.

Let $N=2$, then it has $(2^2-1)^2=9$, and $[\text{rad}(2^2(2^2-2), (2^2-1)^2)]^{1+\varepsilon}=(2 \times 3)^{1+\varepsilon}$, evidently $(2^2-1)^2 > [\text{rad}(2^2(2^2-2), (2^2-1)^2)]^{1+\varepsilon}$ where $\varepsilon < \log_6 9 - 1$.

In the inequality, if $\varepsilon > \log_6 9 - 1$, then it has $(2^2-1)^2 < [\text{rad}(2^2(2^2-2), (2^2-1)^2)]^{1+\varepsilon}$; if $\varepsilon = \log_6 9 - 1$, then it has $(2^2-1)^2 = [\text{rad}(2^2(2^2-2), (2^2-1)^2)]^{1+\varepsilon}$.

By this token, after N is endowed with a concrete positive integer, different valuations of ε decide large or small of $[\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ in comparison with $(2^N-1)^2$.

As thus, suppose that $(2^N-1)^2=[\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$, then it has $1+\varepsilon=\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2$, and there is $\varepsilon=[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$.

So if $\varepsilon=[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, then $(2^N-1)^2=[\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$;

If $0<\varepsilon<[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, then $(2^N-1)^2>[\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$,

and that there are infinitely many real numbers of ε between 0 and

$[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$;

If $\varepsilon >[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, then $(2^N-1)^2 < [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$,

of course, there are infinitely many real numbers of ε in the case too.

Hereinafter, we will divide the range of values of ε into four parts as compared with requirements of the conjecture, and from this decide the take or the abandonment for each part.

Firstly, when $\varepsilon=0$, there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 >[\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ where $N \geq 2$. It is obvious that this case has nothing to do with the conjecture, because $\varepsilon=0$ is inconformity to its requirement.

Secondly, when $0<\varepsilon<[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2>[\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ where $N \geq 2$. Namely there are infinitely many pairs of N and ε to satisfy infinitely many such equalities plus inequalities in the case monogamously.

Thirdly, when $\varepsilon=[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, there is only one equality $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2=[\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ where $N \geq 2$.

This case has nothing to do with the conjecture clearly, because $(2^N-1)^2=[\text{rad}$

$(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ is inconformity to its requirement.

Fourthly, when $\varepsilon > [\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 < [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ where $N \geq 2$. This case has nothing to do with the conjecture either, because $(2^N-1)^2 < [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ is inconformity to its requirement.

Thus it can be seen, whether anybody wants to prove the ABC conjecture or negate the ABC conjecture, he/she can only comes from aforesaid second case, i.e. when $0 < \varepsilon < [\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$ to consider it. Below, list $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ according to front 14 values of N, where $0 < \varepsilon < [\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, but values of ε within each of inequalities are incomplete alike to values of other ε .

$$N, 2^N(2^N-2), (2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}, 1+2^N(2^N-2)=(2^N-1)^2$$

2,	8,	$9 > (2*3)^{1+\varepsilon} = 6^{1+\varepsilon},$	$1+8=9$
3,	48,	$49 > (2*3*7)^{1+\varepsilon} = 42^{1+\varepsilon},$	$1+48=49$
4,	224,	$225 > (2*3*5*7)^{1+\varepsilon} = 210^{1+\varepsilon},$	$1+224=225$
5,	960,	$961 > (2*3*5*31)^{1+\varepsilon} = 930^{1+\varepsilon},$	$1+960=961$
6,	3968,	$3969 > (2*3*7*31)^{1+\varepsilon} = 1302^{1+\varepsilon},$	$1+3968=3969$
7,	16128,	$16129 > (2*3*7*127)^{1+\varepsilon} = 5334^{1+\varepsilon},$	$1+16128=16129$
8,	65024,	$65025 > (2*3*5*17*127)^{1+\varepsilon} = 64770^{1+\varepsilon},$	$1+65024=65025$
9,	261120,	$261121 > (2*3*5*7*17*73)^{1+\varepsilon} = 260610^{1+\varepsilon},$	$1+261120=261121$
10,	1046528,	$1046529 > (2*3*7*11*31*73)^{1+\varepsilon} = 1045506^{1+\varepsilon},$	$1+1046528=1046529$
11,	4190208,	$4190209 > (2*3*11*23*31*89)^{1+\varepsilon} = 4188162^{1+\varepsilon},$	$1+4190208=4190209$
12,	16769024,	$16769025 > (2*3*5*23*89*91)^{1+\varepsilon} = 5588310^{1+\varepsilon},$	$1+16769024=16769025$
13,	67092480,	$67092481 > (2*3*5*7*13*8191)^{1+\varepsilon} = 22361430^{1+\varepsilon},$	$1+67092480=67092481$
14,	268402688,	$268402689 > (2*3*43*127*8191)^{1+\varepsilon} = 268386306^{1+\varepsilon},$	$1+268402688=268402689$
15,	1073676288,	$1073676289 > (6*7*31*43*127*151)^{1+\varepsilon} = 1073643522^{1+\varepsilon},$	$1+1073676288=1073676289$
...

From listed above inequalities and predicting inequalities extend infinitely, be not difficult to make out that values of ε are getting smaller and smaller

up to infinitesimal along with which values of N are getting greater and greater up to infinite.

When $0 < \epsilon < [\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2] - 1$, if successive valuations of ϵ begin with some point near to $[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2] - 1$, then the conjecture can be proved; if successive valuations of ϵ begin with some point near to 0, then the conjecture will be negated. Nobis, be necessary to prove respectively two such aspects mentioned just, ut infra.

3. Proving ABC conjecture

Prove the *ABC* conjecture, obviously this implies that anyone is unable to find a fixed value of ϵ , such that there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\epsilon}$ where $N \geq 2$. Namely for any real number $\epsilon > 0$, there are merely finitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\epsilon}$ in the case that regards ϵ as a fixed value.

Since $N \geq 2$, on the one hand, values of N are getting more and more up to infinite many along with which values of N are getting greater and greater up to infinite, so form infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$.

On the other hand, begin with a greater suited value of ϵ in correspondence with a value of N , then ϵ is getting smaller and smaller successively up to infinitesimal along with which N is getting greater and greater successively up to infinite. As thus, pairs of ϵ plus N are getting more and more up to infinitely many, accordingly form infinitely many equalities like

$1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ monogamously.

Since N and ε appear in pairs within equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$, thus start with any given value of ε , when lessen successively values of ε to reach any very tiny fixed value ε_x in finite field, N in correspondence with ε_x is too a finite-large natural number in finite field, so there are unquestionably finitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_x}$.

Further speak with emphasis, begin with any given pair of N and ε , although natural numbers of N are getting greater and greater successively and corresponding real numbers of ε are getting smaller and smaller successively to form more and more equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$, but since forever cannot reach greatest natural number and forever cannot reach smallest positive real number, therefore, for any tiny fixed ε_x in finite field, there are only finitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_x}$.

On balance, 1 , $2^N(2^N-2)$ and $(2^N-1)^2$ are three co-prime positive integers satisfying $1+2^N(2^N-2)=(2^N-1)^2$, for any fixed real number $\varepsilon > 0$, there is merely at most a finite number of solutions to $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$.

Now that inside the range of finite many consecutive values of ε , satisfactory wee integer 2 within the equality causes only finite many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(1, 2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$, then, not excepting each and every integer > 2 .

Consequently, the *ABC* conjecture is proven to be tenable.

4. Negating *ABC* conjecture

Negate the *ABC* conjecture, undoubtedly this implies that so long as you find a value of ε between 0 and $[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$ such that there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon}$ where $N \geq 2$, then you can achieve the goal surely.

For the half that there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$, this is out of question. The problem is to confirm a satisfactory real number.

Now that there are infinitely many positive real numbers of ε between 0 and $[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$, then the positive real number which and 0 border on each other is certainly the smallest positive real number.

Suppose that we name the smallest positive real number “ ε_0 ”, then, there is not a real number between 0 and ε_0 . Then again, there are infinitely many positive real numbers between ε_0 and $[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$.

On supposition that any real number near $[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$ between ε_0 and $[\log_{\text{rad}(2^N(2^N-2), (2^N-1)^2)}(2^N-1)^2]-1$ is ε_x , then there are still infinitely many positive real numbers between ε_0 and ε_x .

Consequently, if N is endowed with infinite many values, then there are infinitely many values of ε between ε_0 and ε_x too, enable them one-to-one pairing, such that $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(1, 2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_0}$ where $N \geq 2$.

In other words, when $\varepsilon = \varepsilon_0$ and $N \geq 2$, there are infinitely many equalities like

$1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_0}$.

Additionally, begin with ε_0 , name orderly-increasing and orderly-adjacent real numbers “ $\varepsilon_0, \varepsilon_1, \varepsilon_2 \dots \varepsilon_y$ ”, where y is a concrete natural number which consists of Arabic numerals.

Without doubt, for real number ε_y , there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_y}$ where $N \geq 2$, because infinitely many values of ε between ε_x and ε_y and infinitely many values of N in pairs form infinitely many inequalities $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_y}$.

Because of this, begin with any fixed value ε_x , let ε decrease successively, and N in correspondence with ε pair to form equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_y}$, yet values of ε forever can not be decreased to ε_y from ε_x .

That is to say, when $\varepsilon = \varepsilon_y$, there are infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_y}$ where $N \geq 2$.

By now, let the representative equality as compared with the definition of the conjecture as follows.

Firstly, three terms $1, 2^N(2^N-2)$ and $(2^N-1)^2$ within the representative equality are co-prime positive integers.

Secondly, $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > [\text{rad}(2^N(2^N-2), (2^N-1)^2)]^{1+\varepsilon_0}$ are completely in conformity with the requirements of the conjecture.

By this token, if regard ε_0 as a fixed real number, then the *ABC* conjecture

has to be negated by infinitely many equalities like $1+2^N(2^N-2)=(2^N-1)^2$ satisfying $(2^N-1)^2 > (\text{rad}(1, 2^N(2^N-2), (2^N-1)^2))^{1+\varepsilon}$ where $N \geq 2$ and $\varepsilon = \varepsilon_0 > 0$, or $\varepsilon = \varepsilon_1, \varepsilon_2 \dots \varepsilon_y$ and $y \geq 1$.

That is to say, that the *ABC* conjecture is untenable. As thus, the *ABC* conjecture can only be regarded as a fallacy or a defective expression.

Can ε_0 or ε_y be a fixed real number? At present, we only know that ε_0 or ε_y has the designation and the fixed location, therein ε_0 neighbors 0. In addition to this, it can compare out large-small between any real number and one of them. When we regard 0 as a fixed real number, if the positive real number ε_0 which exclusively neighbors 0 is not a fixed real number, seemingly such an inference is unreasonable.

5. The eventual statement

What cause can lead up to both prove and negate the *ABC* conjecture? In my opinion, the key to the settlement of the question lies in mathematical circles, whether they can admit ε_0 as a fixed real number.

If ε_0 is admitted as a fixed real number, thereupon the *ABC* conjecture is negated either, according to the disproof of preceding fourth section.

If ε_0 can not be admitted as a fixed real number, then the *ABC* conjecture is tenable too, according to the proof of preceding third section.

In this article, the author has analyzed merely two aspects which the *ABC* conjecture is both proved and negated, for reference only.

APPENDIX: Foremost some primes P , P^2-1 and $\text{rad}(P^2-1)$ in equality $1+(P^2-1)=P^2$ satisfying $P^2 > (\text{rad}(1, P^2-1, P^2))^{1+\varepsilon}$ i.e. satisfying $P > (\text{rad}(P^2-1))^{1+\varepsilon}$ by and large are listed below, where limits of real number ε which satisfy each inequality are incompletely alike to any other.

P ,	P^2-1 ,	$\text{rad}(P^2-1)$
7,	48,	$2*3=6$
17,	288,	$2*3=6$
31,	960,	$2*3*5=30$
97,	9408,	$2*3*7=42$
127,	16128,	$2*3*7=42$
251,	63000,	$2*3*5*7=210$
449,	201600,	$2*3*5*7=210$
487,	237168,	$2*3*61=366$
577,	332928,	$2*3*17=102$
1151,	1324800,	$2*3*5*23=690$
1249,	1560000,	$2*3*5*13=390$
1567,	2455488,	$2*3*7*29=1218$
1999,	3996000,	$2*3*5*37=1110$
2663,	7091568,	$2*3*11*37=2442$
4801,	23049600,	$2*3*5*7=210$
4999,	24990000,	$2*3*5*7*17=3570$
7937,	62995968,	$2*3*7*31=1302$
8191,	67092480,	$2*3*5*7*13=2730$
12799,	163814400,	$2*3*5*79=2370$
13121,	172160640,	$2*3*5*41=1230$
13183,	173791488,	$2*3*13*103=8034$
15551,	241833600,	$2*3*5*311=9330$
31249,	976500000,	$2*3*5*7*31=6510$
31751,	1008126000,	$2*3*5*7*127=26670$
32257,	1040514048,	$2*3*7*127=5334$
33857,	1146296448,	$2*3*11*19*23=28842$
35153,	1235733408,	$2*3*7*13*31=16926$
39367,	1549760688,	$2*3*7*19*37=29526$
65537,	4295098368,	$2*3*11*331=21846$
79201,	6272798400,	$2*3*5*11*199=65670$
81919,	6710722560,	$2*3*5*37*41=45510$
85751,	7353234000,	$2*3*5*7*397=83370$
115249,	13282332000,	$2*3*5*7*461=96810$
117127,	13718734128,	$2*3*11*241=15906$

124001,	15376248000,	$2^3 \cdot 5^3 \cdot 31 \cdot 83 = 77190$
126001,	15876252000,	$2^3 \cdot 5^7 \cdot 7 \cdot 251 = 52710$
131071,	17179607040,	$2^3 \cdot 5^5 \cdot 17 \cdot 257 = 131070$
153089,	23436241920,	$2^3 \cdot 5^7 \cdot 7 \cdot 13 \cdot 23 = 62790$
160001,	25600320000,	$2^3 \cdot 5^9 \cdot 2963 = 88890$
161839,	26191861920,	$2^3 \cdot 5^7 \cdot 7 \cdot 17 \cdot 37 = 132090$
165887,	27518496768,	$2^3 \cdot 7^3 \cdot 17 \cdot 41 = 29274$
196831,	38742442560,	$2^3 \cdot 5^6 \cdot 6151 = 184530$
215297,	46352798208,	$2^3 \cdot 29 \cdot 443 = 77082$
281249,	79101000000,	$2^3 \cdot 5^5 \cdot 11 \cdot 17 \cdot 47 = 263670$
442367,	195688562688,	$2^3 \cdot 29^2 \cdot 263 = 45762$
474337,	224995589568,	$2^3 \cdot 61 \cdot 487 = 178242$
511757,	261895227048,	$2^3 \cdot 7^3 \cdot 13 \cdot 373 = 203658$
524287,	274876858368,	$2^3 \cdot 7^3 \cdot 19 \cdot 73 = 58254$
538001,	289445076000,	$2^3 \cdot 5^4 \cdot 41 \cdot 269 = 330870$
665857,	443365544448,	$2^3 \cdot 17^3 \cdot 577 = 58854$
715823,	512402567328,	$2^3 \cdot 71 \cdot 1657 = 705882$
902501,	814508055000,	$2^3 \cdot 5^5 \cdot 19 \cdot 619 = 352830$
911249,	830374740000,	$2^3 \cdot 5^5 \cdot 13 \cdot 337 = 131430$
988417,	976968165888,	$2^3 \cdot 11 \cdot 13 \cdot 19 \cdot 37 = 603174$
1039681,	1080936581760,	$2^3 \cdot 5^7 \cdot 7 \cdot 19 \cdot 103 = 410970$
1062881,	1129716020160,	$2^3 \cdot 5^7 \cdot 7 \cdot 13 \cdot 73 = 199290$
1102249,	1214952858000,	$2^3 \cdot 5^7 \cdot 7 \cdot 4409 = 925890$
1179649,	1391571763200,	$2^3 \cdot 5^5 \cdot 23593 = 707790$
1229311,	1511205534720,	$2^3 \cdot 5^7 \cdot 7 \cdot 29 \cdot 157 = 956130$
1246589,	1553984134920,	$2^3 \cdot 5^7 \cdot 7 \cdot 19 \cdot 211 = 841890$
1272833,	1620103845888,	$2^3 \cdot 11 \cdot 97 \cdot 113 = 723426$
...

References

- [1] Wolfram Mathworld, *abc conjecture*, <http://mathworld.wolfram.com/abcConjecture.html>
- [2] WIKIPEDIA, *abc conjecture*, https://en.wikipedia.org/wiki/Abc_conjecture
- [3] Stewart, C. L.; Yu, Kunrui (2001), "On the abc conjecture, II". *Duke Mathematical Journal* 108 (1): 169–181, doi:10.1215/S0012-7094-01-10815-6.