zeta odd-numbers

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Abstract

I tried to find a new expression for zeta odd-numbers, but it seems to have remained a general expression.

However, it may be a new expression and will be published here.

key words
zeta odd-numbers, new expression

1 introduction

$$\zeta(3)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}+\sum_{n=1}^{\infty}\frac{1}{(2n)^3}=\frac{1}{8}\sum_{n=1}^{\infty}\frac{1}{n^3}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\frac{1}{8}\zeta(3)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{1}$$

$$\zeta(3)=\frac{8}{7}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{2}$$

from Eq.(2)

$$\zeta(3)=\frac{2^3}{2^3-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} \tag{3}$$

$$\zeta(3)\approx\frac{2^3}{2^3-1}\sum_{n=1}^{10000}\frac{1}{(2n-1)^3} \tag{4}$$

$$\approx 1.142857142857142857142857142 8[1.0517997896396450013....] \tag{5}$$

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$$= 1.2020569024453085729142857142256114405920202856 4... \tag{6}$$

$$\zeta(3) = 1.2020569031596... \tag{7}$$

Do the same

$$\zeta(5) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} + \sum_{n=1}^{\infty} \frac{1}{(2n)^5} = \frac{1}{32} \sum_{n=1}^{\infty} \frac{1}{n^5} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{1}{32}\zeta(5) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \tag{8}$$

$$\zeta(5) = \frac{2^5}{2^5-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \tag{9}$$

$$\approx \frac{32}{31} \sum_{n=1}^{10000} \frac{1}{(2n-1)^5} \approx 1.032258064516129032258064516129032258[1.004523762795139615352 26...] \tag{10}$$

$$= 1.0369277551433699255249135950840030465376730708911 03... \tag{11}$$

$$\zeta(5) = 1.036927755143369926331... \tag{12}$$

do the same

$$\zeta(7) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} + \sum_{n=1}^{\infty} \frac{1}{(2n)^7} = \frac{1}{128} \sum_{n=1}^{\infty} \frac{1}{n^7} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{1}{128}\zeta(7) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \tag{13}$$

$$\zeta(7) = \frac{2^7}{2^7-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \tag{14}$$

$$\approx \frac{128}{127} \sum_{n=1}^{10000} \frac{1}{(2n-1)^7} \approx 1.007874015748031496062992125984 25[1.000471548652376554755111 63...] \tag{15}$$

$$= 1.0083492773819228268397975483512073398201167479716865954 97... \tag{16}$$

$$\zeta(7) = 1.008349277381922826839797549849796759599863560565238706417... \tag{17}$$

do the same

$$\zeta(9)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}+\sum_{n=1}^{\infty}\frac{1}{(2n)^9}=\frac{1}{512}\sum_{n=1}^{\infty}\frac{1}{n^9}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}=\frac{1}{512}\zeta(9)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9} \tag{18}$$

$$\zeta(9)=\frac{2^9}{2^9-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9}=\frac{512}{511}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^9} \tag{19}$$

$$\approx\frac{512}{511}\sum_{n=1}^{10000}\frac{1}{(2n-1)^9}\approx 1.00195694716242661448140900195694[1.00000513451838437725928179005425 05...] \tag{20}$$

$$=1.002008392826082214417852769232404892051595618903725788276... \tag{21}$$

$$\zeta(9)=1.002008392826082214417852769232412060485605851394888756548... \tag{22}$$

do the same

$$\zeta(11)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{11}}=\frac{1}{2048}\sum_{n=1}^{\infty}\frac{1}{n^{11}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}=\frac{1}{2048}\zeta(11)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}} \tag{23}$$

$$\zeta(11)=\frac{2^{11}}{2^{11}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}}=\frac{2048}{2047}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{11}} \tag{24}$$

$$\approx\frac{2048}{2047}\sum_{n=1}^{10000}\frac{1}{(2n-1)^{11}}\approx 1.00048851978505129457743038593063019[1.000005666051090109351398 2...] \tag{25}$$

$$=1.000494188604119464558702253835447194036760299214... \tag{26}$$

$$\zeta(11)=1.000494188604119464558702 28... \tag{27}$$

do the same

$$\zeta(13)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{13}}=\frac{1}{8192}\sum_{n=1}^{\infty}\frac{1}{n^{13}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}=\frac{1}{8192}\zeta(13)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}} \tag{28}$$

$$\zeta(13)=\frac{2^{13}}{2^{13}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}}=\frac{8192}{8191}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{13}} \tag{29}$$

$$\approx \frac{8192}{8191}\sum_{n=1}^{10000}\frac{1}{(2n-1)^{13}} \approx 1.00012208521548040532291539494 5672[1.00000062805542180231946 34...] \tag{30}$$

$$= 1.00012271334757848914675182185331 6305... \tag{31}$$

$$\zeta(13) = 1.000122713347578489146751 8... \tag{32}$$

do the same

$$\zeta(15) = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{15}} = \frac{1}{32768}\sum_{n=1}^{\infty}\frac{1}{n^{15}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}} = \frac{1}{32768}\zeta(11)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}} \tag{33}$$

$$\zeta(13) = \frac{2^{15}}{2^{15}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}} = \frac{32768}{32767}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{15}} \tag{34}$$

$$\approx \frac{32768}{32767}\sum_{n=1}^{10000}\frac{1}{(2n-1)^{15}} \approx 1.00003051850947599719229712820825 83[1.00000006972470312928809233...] \tag{35}$$

$$= 1.00003058823630702049355172794091 6854... \tag{36}$$

$$\zeta(15) = 1.0000305882363070204935517285... \tag{37}$$

do the same

$$\zeta(17) = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{17}} = \frac{1}{131072}\sum_{n=1}^{\infty}\frac{1}{n^{17}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}} = \frac{1}{131072}\zeta(11)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}} \tag{38}$$

$$\zeta(17) = \frac{2^{17}}{2^{17}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}} = \frac{131072}{131071}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{17}} \tag{39}$$

$$\approx \frac{131072}{131071}\sum_{n=1}^{10000}\frac{1}{(2n-1)^{17}} \approx 1.0000076294527393550060654149[1.000000007744839455869605736267...] \tag{40}$$

$$= 1.0000076371976378997622736002650939... \tag{41}$$

$$\zeta(17) = 1.0000076371976378997622736002935630292130 8... \tag{42}$$

do the same

$$\zeta(19)=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}+\sum_{n=1}^{\infty}\frac{1}{(2n)^{19}}=\frac{1}{524288}\sum_{n=1}^{\infty}\frac{1}{n^{19}}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}=\frac{1}{524288}\zeta(19)+\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}} \tag{43}$$

$$\zeta(19)=\frac{2^{19}}{2^{19}-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}}=\frac{524288}{524287}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{19}} \tag{44}$$

$$\approx\frac{524288}{524287}\sum_{n=1}^{1000}\frac{1}{(2n-1)^{19}}\approx 1.0000019073522707982459988517 7[1.0000000008604441145228910749615316...] \tag{45}$$

$$=1.00000190821271655393892565695386204406942342790011013981 3... \tag{46}$$

$$\zeta(19)=1.000001908212716553938925656957795101353258571144838630235... \tag{47}$$

make official

$$\zeta(2m-1)=\frac{2^{2m-1}}{2^{2m-1}-1}\sum_{n=1}^{\infty}\frac{1}{(2m-1)^{2m-1}} \tag{48}$$

m is a positive integer greater than or equal to 2

2 Postscript

These calculations were performed with WolframAlpha.

References

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I was finally crazy because of the curse of Riemann.

Please raise the prize money to my little son and daughter who are still young.