Euler's wavefunction

Jean Louis Van Belle, Drs, MA*Ec*, BA*Ec*, B*Phil*
Email: jeanlouisvanbelle@outlook.com
22 October 2018

Abstract: This paper is the 5th in a series of explorations to see if simple geometric and physical interpretations of the quantum-mechanical wavefunction could possibly make sense. It acknowledges the usual objections to naïve interpretations head-on, but it also challenges these objections by presenting some heuristic arguments on how the basic axioms of quantum mechanics may be subject to some interpretation themselves.

The arguments in this paper are what they are: heuristic. They do, therefore, not provide any mathematical proof. This is to be expected, as we are discussing *interpretations* of the wavefunction only: we surely do not want to challenge the math ! Hence, one should not expect formal proofs: thought experiments were the initial inspiration for quantum mechanics, and they still play the same role in contemporary physics.

The paper focuses on two of the usual objections to geometric or physical interpretations of the wavefunction:

1. The superposition of wavefunctions is done in the complex space and, hence, the assumption of a real-valued envelope for the wavefunction is, therefore, not acceptable.
2. The wavefunction for spin-1/2 particles cannot represent any real object because of its 720-degree symmetry in space. Real objects have the same spatial symmetry as space itself, which is 360 degrees. Hence, physical interpretations of the wavefunction are nonsensical.

The author hopes that this paper might contribute to a less dogmatic interpretation of the quantum-mechanical mathematical framework. If anything, the ideas presented in this paper – which is, in essence, a detailed discussion on why some visualizations make more sense than others – might contribute to a better *didactic* model for teaching quantum mechanics.

Contents

Introduction 1
Real or complex amplitudes? 2
Theoretical spin-zero particles versus real spin-1/2 particles 3
The reality of the complex conjugate of a wavefunction 5
360° and 720° symmetries: what is real? 8
References 11

Euler's wavefunction[1]

Introduction

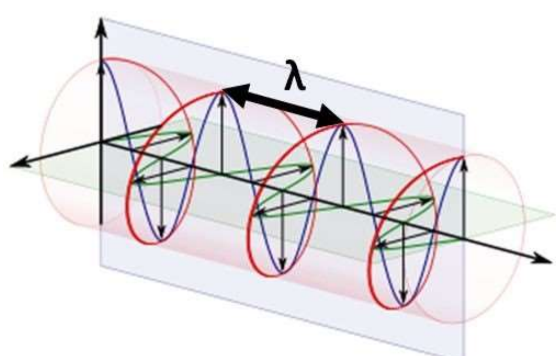
The structural similarities between the classical electromagnetic theory and QED inspires easy geometric and physical interpretations of the wavefunction. Here we need to specify what we mean with a physical interpretation, because any course in quantum mechanics will state that the interpretation of $|\psi(\boldsymbol{x}, t)|^2$ as the probability to find a particle at $\boldsymbol{x}$ and t amounts to a *physical* interpretation.[2] However, a true *physical* interpretation should explain these probabilities in terms of something real (mass or energy densities, for example), and that is where these courses leave the student bewildered. Can it be done?

It looks easy enough. A true physical interpretation will present the real and imaginary part of the elementary wavefunction $a \cdot e^{i\theta}$ as *real* field vectors driven by the same function but with a phase difference of 90 degrees:

$$a \cdot e^{i\theta} = a \cdot (\cos\theta + i \cdot \sin\theta) = a \cdot \sin(\theta + \pi/2) + i \cdot a \cdot \sin\theta$$

The visualization below – which shows a propagating circularly polarized electric field – is common but triggers many questions. How do we account for the direction of the magnetic moment (or *spin*) of a particle, for example? We analyzed such questions before – and they can be answered. Such answers are speculative, of course, but that is not the point: the question is whether a geometric interpretation makes sense at all. If we have a geometric interpretation, we have a *physical* interpretation: all that needs to be done is to also associate the real and imaginary part of the wavefunction with some physical dimension, say force per unit charge, or force per unit mass. The first interpretation looks at the wave packet as an electromagnetic oscillation. The second interpretation looks at the wave packet as a gravitational wave ($N/kg = m/s^2$).

Figure 1: A complex wave?[3]



[1] Pun intended. Earlier working titles were Schrödinger's wavefunction, or Einstein's wavefunction – to refer to their sentiment that the wavefunction must, somehow, represent something real. However, such title would have sounded very arrogant.
[2] See, for instance, the MIT OCW courses 8.04 and 8.05.
[3] Credit: https://commons.wikimedia.org/wiki/User:Dave3457. The author only added the wavelength which – in a physical interpretation of the wavefunction – can be interpreted as the *de Broglie* wavelength for a particle. For more details, see http://vixra.org/pdf/1709.0390v5.pdf.

These models are nice and intuitive, but we should confront them with the basic axioms of quantum mechanics. We will discuss some of these below.

Real or complex amplitudes?

The term amplitude is ambiguous: it may refer to the maximum amplitude of some real-valued wave or, alternatively, to a complex-valued *probability* amplitude. In the first case, we think of the *a* in the $a \cdot e^{i\theta}$ expression and, hence, it is a coefficient, a scaling factor (think of normalization) or – when building the wave packet – a *weight*. In the second, the term amplitude refers to the whole $a \cdot e^{i\theta}$ function. The question is: in any geometric and/or physical interpretation of the wavefunction we think of *a* as some real-valued number.

That may be problematic because, in quantum mechanics, we do not exclude linear operations using complex-valued coefficients. For example, when using the framework of *state vectors*, we may write something like $|X\rangle = \alpha|A\rangle + \beta|B\rangle$, and α and β would be complex numbers. We also know that, if ψ_1 and ψ_2 are solutions to the Schrödinger equation, then $\alpha\psi_1 + \beta\psi_2$ will be a solution too. Of course, we can always multiply with $1/\alpha$ and then we get $|A\rangle + \frac{\beta}{\alpha}|B\rangle$ or $\psi_1 + \frac{\beta}{\alpha}\psi_2$ to get *one* complex parameter only: the β/α ratio, which is equivalent to two *real* parameters.[4]

I am not aware of any formal proof that, by a suitable choice of the base states (or *representation* as it is referred to in quantum mechanics), we can substitute and get real-valued coefficients. However, I do note that, *in practice*, we always end up with wavefunctions with real-valued coefficients. Let me give two notable examples here: the solutions to the Schrödinger equation in a potential (the model of the hydrogen atom), and the standard representation of the wavefunction as a Fourier sum.

1. The correct description of the electron orbitals of the hydrogen atom is one of the main feats of quantum mechanics, and these descriptions are wavefunctions with real-valued coefficients. Of course, the wavefunction for an electron orbital will routinely include a factor like $-\frac{1}{\sqrt{2}} \cdot \sin\theta \cdot e^{i\phi}$ so, yes, there is a complex number there[5] but note how the complex factor appears: it is just a phase shift. The *envelope* for the oscillation is some real number.

2. This is also the case for the description of the wave packet in terms of a Fourier sum. We *can* use complex-valued coefficients but, *in practice*, we use real-valued coefficients. Let us also be explicit here so we are all clear on this. The description of a wave packet in space (freezing time) is given by:

$$\psi(x,0) = \int_{-\infty}^{+\infty} \Phi(k) e^{ikx} dk$$

[4] See: Prof. Dr. Barton Zwiebach, Quantum Mechanics, MIT*x* 8.01.1*x*, Chapter 1, Section 4. A complex number x + *i*y effectively consists of two parts (*x* and *y*) and can therefore reflect the (two) degrees of freedom of the physics of the situation. The example that is given is that of an elliptically polarized wave, whose shape is determined by the ratio of the axes of the ellipse (*b*/*a*) and its tilt (θ).

[5] The formula gives us the angular dependence of the amplitude for the orbital angular momentum number *l* = 1.

The $\Phi(k)$ function gives us the *weight* factors for each of the waves that make up the packet[6] and we will want to think of $\Phi(k)$ as a real-valued function, centered around some value $k_0 = \frac{p_0}{\hbar}$ and width Δk.[7]

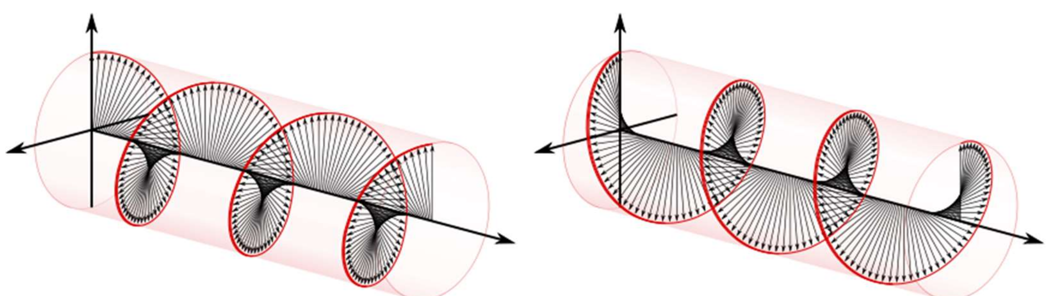
Of course, the argument above is heuristic only: it is *not* a formal proof that we can always find a suitable base to ensure real-valued coefficients. However, it should debunk the myth that the coefficients in front of wavefunctions are generally complex and that, therefore, we should not try to find a physical or geometric interpretation of the wavefunction

Theoretical spin-zero particles versus real spin-1/2 particles

It is interesting that, using suitable conventions, we can rewrite Maxwell's equations using complex numbers. Indeed, if we think of the imaginary unit as a unit vector pointing in a direction that is perpendicular to the direction of propagation of the wave, we can write the magnetic field vector as $\mathbf{B} = -i\cdot\mathbf{E}/c$.

Note the minus sign in the $\mathbf{B} = -i\cdot\mathbf{E}/c$.[8] It is there because we need to combine several conventions here. Of course, there is the classical *physical* right-hand rule for **E** and **B,** but we also need to combine the right-hand rule for the coordinate system with the convention that multiplication with the imaginary unit amounts to a *counter*clockwise rotation by 90 degrees. Hence, the minus sign is necessary for the consistency of the description. It ensures that we can associate the $a\cdot e^{i\theta}$ and $a\cdot e^{-i\theta}$ functions with left- and right-handed polarization respectively.

Figure 2: Left- and right-handed polarization[9]



It is, therefore, very peculiar that, in quantum mechanics, we do not have such consistency. For example, in the MIT's introductory course on quantum physics[10], it is shown that only $\psi = \exp(i\theta) = \exp[i(kx-\omega t)]$ *or* $\psi = \exp(-i\theta) = \exp[-i(kx-\omega t)] = \exp[i(\omega t-kx)]$ would be acceptable waveforms for a

[6] You will usually see a $\frac{1}{\sqrt{2\pi}}$ factor in front of the integral, and it should be there, but we left it out for clarity.
[7] The http://www.thefouriertransform.com/series/complexcoefficients.php site gives examples of Fourier transforms of common functions using complex-valued coefficients, but shows that the same results can be obtained by using real-valued coefficients.
[8] Boldface letters represent geometric vectors – the electric and magnetic field vectors **E** and **B** in this case.
[9] Credit: https://commons.wikimedia.org/wiki/User:Dave3457.
[10] See, for example, the MIT's edX Course 8.04.1*x*, Lecture Notes, Chapter 4, Section 3.

particle that is propagating in the *x*-direction – as opposed to, say, some real-valued sinusoid. We would then think some proof should follow of why one would be better than the other, or some discussion on why they might be different, but that is not the case. The professor happily concludes that "*the choice is a matter of convention and, happily, most physicists use the same convention.*"

This is very surprising – and that's an understatement. Why? We *know*, from *experience*, that theoretical or mathematical possibilities in quantum mechanics often turn out to represent real things. Think of the experimental verification of the existence of the positron (or of anti-matter in general) after Dirac had predicted its existence based on the mathematical possibility only. So why would that *not* be the case here? *Occam's Razor* tells us that we should not have any redundancy in the description. Hence, if there is a physical interpretation of the wavefunction, then we should not have to choose between the two mathematical possibilities: they would represent two different physical situations.

What could be different? There is only one candidate here: spin.

This brings us to what is – without any doubt – the most challenging objection to a physical interpretation of the wavefunction: wavefunctions of spin-1/2 particles (which is what we are thinking of here) have a weird 720° symmetry.[11] Any *real* object that we can think of has a 360-degree symmetry in space. Why? Because space is three-dimensional.

We can try to solve this contradiction in two ways. The first way is to accept the 720° symmetry and try to interpret it by accepting the measurement apparatus and the object establish some *absolute* space. The metaphor here is Dirac's belt trick. We have written about this before and, hence, we will not repeat ourselves here.[12]

The second way – much more radical – is to prove that the 720-degree symmetry would reduce to what we would expect for anything real in space – i.e. a 360-degree symmetry – when we would, effectively, use the two mentioned mathematical possibilities to distinguish between two particles that are identical but for their spin. The idea is that we would associate the $a \cdot e^{i\theta}$ and $a \cdot e^{-i\theta}$ functions with the quantum-mechanical equivalent of left- and right-handed polarization respectively. The wavefunction would then no longer describe a theoretical spin-zero particle, which should be fine – because we all know spin-zero particles don't exist: *real* particles (electrons and quarks) have spin-1/2.

Now, I do not have a mathematical proof that this would solve our problem, but I do have some serious questions on the thought experiments that is used to prove the mentioned 720° symmetry.[13] In fact, a discussion of those questions and issues is the main subject of this paper. However, before we get going on this, we should note that the $a \cdot e^{i\theta}$ and $a \cdot e^{-i\theta}$ functions are each other's complex conjugate and, therefore, reflect on a possible *physical* meaning of the complex conjugate.

[11] See, for example, Feynman's *Lectures*, Vol. III, Chapter 6.

[12] See: *Why it is hard to understand – and, therefore, explain – quantum math*, http://vixra.org/pdf/1806.0183v1.pdf (accessed on 21 October 2018).

[13] We will use the above-mentioned standard reference material here, i.e. Feynman's *Lectures*, Vol. III, Chapter 6 (*Spin One-Half*).

The reality of the complex conjugate of a wavefunction

The idea of associating the complex conjugate of a wavefunction with a particle that is identical but for its (opposite) spin might be outlandish so, let us first explore a simpler idea. When we take the complex conjugate of $\psi = \exp(i\theta) = \exp[i(\mathbf{k}\cdot\boldsymbol{x}-\omega\cdot t)]$, we get $\psi^* = \exp(-i\theta) = \exp[i(-\mathbf{k}\cdot\boldsymbol{x}+\omega\cdot t)]$. Hence, $\boldsymbol{x}$ becomes $-\boldsymbol{x}$ and t becomes $-t$. Hence, we may say that the complex conjugate of a wavefunction describes whose trajectory in space and in time is being reversed.

It is not merely time symmetry: we are talking *reversibility* here. It is like playing a movie backwards. We may relate this discussion to the *Hermiticity* of (many) quantum-mechanical operators. An operator A that is operating on some state $|\psi\rangle$ will be written as[14]:

$$A|\psi\rangle$$

Now, we can then think of some (probability) amplitude that this operation produces some other state $|\varphi\rangle$, which we would write as:

$$\langle\varphi|A|\psi\rangle$$

We can now take the complex conjugate:

$$\langle\varphi|A|\psi\rangle^* = \langle\psi|A^\dagger|\varphi\rangle$$

$A^\dagger$ is, of course, the conjugate transpose of A – we write: $A^\dagger_{ij}=(A_{ji})^*$ – and we will call the operator (and the matrix) Hermitian if the conjugate transpose of this operator (or the matrix) gives us the same operator matrix, so that is if $A^\dagger = A$. Many quantum-mechanical operators are Hermitian. Because of the reversibility condition. Think of the meaning of the complex conjugate as presented above: a reversal of both the direction in time as well in space. Hence, what is the meaning of the complex conjugate of $\langle\varphi|A|\psi\rangle$?

The $\langle\varphi|A|\psi\rangle$ expression gives us the amplitude to go from some state $|\psi\rangle$ to some other state $\langle\varphi|$. Conversely, the $\langle\psi|A|\varphi\rangle = \langle\psi|A^\dagger|\varphi\rangle = = \langle\varphi|A|\psi\rangle^*$ expression tells us we were in state $|\varphi\rangle$ but now we are in the state $\langle\psi|$, and the $\langle\psi|A|\varphi\rangle$ expression gives us the amplitude for that. Hence, the Hermiticity condition amounts to a reversibility condition.

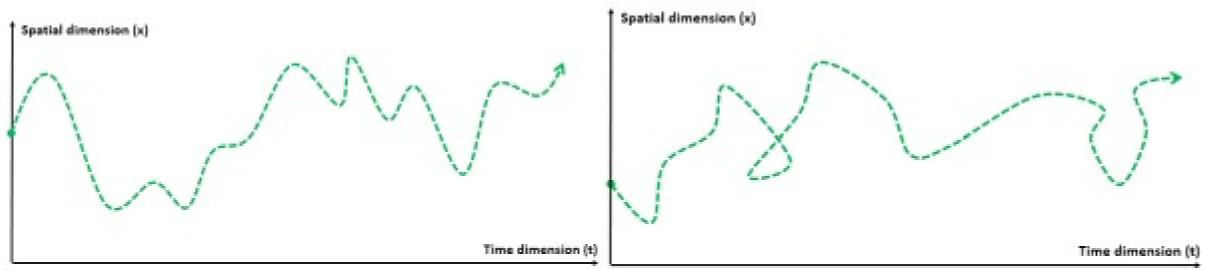
Here we need to highlight a subtle point. Time has one direction only: we cannot reverse time. We can only reverse the direction in space. We can do so by reversing the momentum of a particle. If we do so, our $\mathbf{k} = \mathbf{p}/\hbar$ becomes $-\mathbf{k} = -\mathbf{p}/\hbar$. However, the energy remains what it is and, hence, nothing happens to the $\omega\cdot t = (E/\hbar)\cdot t$ term. Hence, our wavefunction becomes $\exp[i(-\mathbf{k}\cdot\boldsymbol{x}-\omega\cdot t)]$, and we can calculate the wave velocity as negative: $v = -\omega/|\mathbf{k}| = -\omega/k$. The wave effectively travels in the opposite direction (i.e. the *negative* x-direction in one-dimensional space). Hence, we can think of opposite directions in space, but we can't reverse time. Why not?

We don't need to think of entropy here. Time has one direction only because – if it wouldn't – we would not be able to describe trajectories in spacetime by a well-behaved function. The diagrams below illustrate the point. The spacetime trajectory in the diagram on the right is not *kosher*, because our

[14] I should use the *hat* because the symbol without the hat is reserved for the *matrix* that does the operation and, therefore, A already assumes a representation, i.e. some chosen set of base states. However, let's skip the niceties here.

object travels back in time in not less than three sections of the graph. Spacetime trajectories need to be described by well-defined function: for every value of *t*, we should have one, and only one, value of *x*. The reverse is not true, of course: a particle can travel back to where it was. Hence, it is easy to see that our concept of time going in one direction, and in one direction only, implies that we should only allow well-behaved functions.

Figure 3: A well- and a *not*-well behaved trajectory in spacetime



It may be a self-evident point to make but it is an important one. Note that, once again, we have two *mathematical* possibilities to describe a theoretical spin-zero particle that would travel in the *negative x*-direction[15]: $\psi = \exp[i(-kx-\omega t)]$ *or* $\psi = \exp[i(kx+\omega t)]$.

Again, if we would *not* agree with the mainstream view that "the choice is a matter of convention" and that "happily, most physicists use the same convention"[16] but, instead, dare to suggest that the two mathematical possibilities represent identical particles with opposite spin (i.e. *real* spin-1/2 particles as opposed to non-existing spin-zero particles), then we get the following table.

Figure 4: Occam's Razor: mathematical possibilities versus physical realities

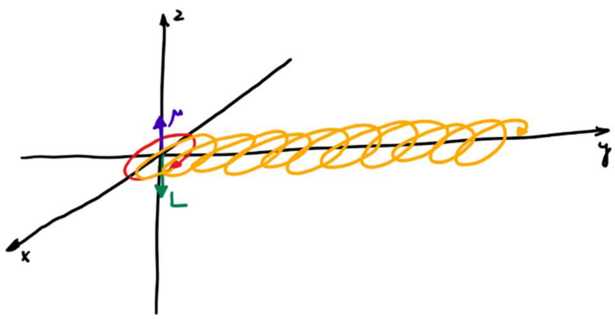
Spin and direction of travel	**Spin up ($J = +\hbar/2$)**	**Spin down ($J = -\hbar/2$)**
Positive *x*-direction	$\psi = \exp[i(kx-\omega t)]$	$\psi^* = \exp[-i(kx-\omega t)] = \exp[i(\omega t-kx)]$
Negative *x*-direction	$\chi = \exp[-i(kx+\omega t)] = \exp[i(\omega t-kx)]$	$\chi^* = \exp[i(kx+\omega t)]$

Of course, the above formulas only give us the *elementary* wavefunction. The wave *packet* will be a Fourier sum of such functions. Before we proceed, we should ask ourselves one more question: what is the physical meaning of $-\exp(i\theta)$?

Here we do need to think more carefully about the orientation of the plane of the oscillation. The illustrations of RHC and LHC waves above assume that plane is perpendicular to the direction of propagation, but there are other possibilities. In fact, a physical interpretation of the magnetic moment that we associate with the angular momentum or spin would require that plane to contain the direction of propagation, as illustrated below.

[15] We are not just switching back and forth between one- and three-dimensional wavefunctions here: think of *choosing* the reference frame such that the *x*-axis coincides with the direction of propagation of the wave.
[16] See, for example, the MIT's edX Course 8.04.1*x*, Lecture Notes, Chapter 4, Section 3.

Figure 5: Is this the *Zitterbewegung* in a Stern-Gerlach apparatus?



If this sounds outlandish to the reader, then he or she may want to think of the remarkably simple result we get when calculating the angular momentum using the Compton wavelength for the radius *a*[17]:

$$\mathrm{L} = \mathrm{I} \cdot \omega = \frac{m \cdot a^2}{2}\frac{c}{a} = \frac{mc}{2}\frac{\hbar}{mc} = \frac{\hbar}{2}$$

A minus sign in front of our exp($i\theta$) function reverses the direction of the oscillation. However, here we can use the $cos\theta = cos(-\theta)$ and $sin\theta = -sin(-\theta)$ formulas to relate $-$exp($i\theta$) to the complex conjugate. We write:

$$-\psi = -\exp(i\theta) = -(cos\theta + i\cdot sin\theta) = cos(-\theta) + i\cdot sin(-\theta) = \exp(-i\theta) = \psi^*$$

This is a peculiar property that we will exploit in the next development. We should make one final note before we get into the meat of the matter. Where would this minus sign come from? We know we can always add an arbitrary phase change doesn't change the physical state: it is just like changing our zero point in time. Hence, exp($i\theta$) and exp($i\alpha$)·exp($i\theta$) = exp[$i(\theta + \alpha)$] should represent the same state. Our physical interpretation of the wavefunction does not challenge this at all. However, we should note the case of $\alpha = \pm\pi$, for which we can write:

$$\exp(\pm i\pi)\cdot\exp(i\theta) = \exp[i(\theta \pm \pi)] = -\exp(i\theta)$$

We will need this identity soon.

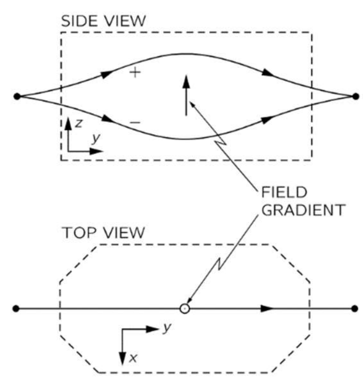
[17] The $\omega = c/a$ formula follows naturally from the same model (see: *In Search of Schrödinger's electron – and Einstein's atom*, http://vixra.org/abs/1809.0350, accessed on 21 October 2018). It is equally simple and intuitive as all the rest above, but we don't want to repeat ourselves repeatedly.

360° and 720° symmetries: what is real?

We are all familiar with the topic on hand: the angular dependence of amplitudes. To put it simply, it is about rotation matrices. The matter is best illustrated by sticking closely to Feynman's argument and so let us start with the two illustrations presenting the basic geometry of the situation on hand.

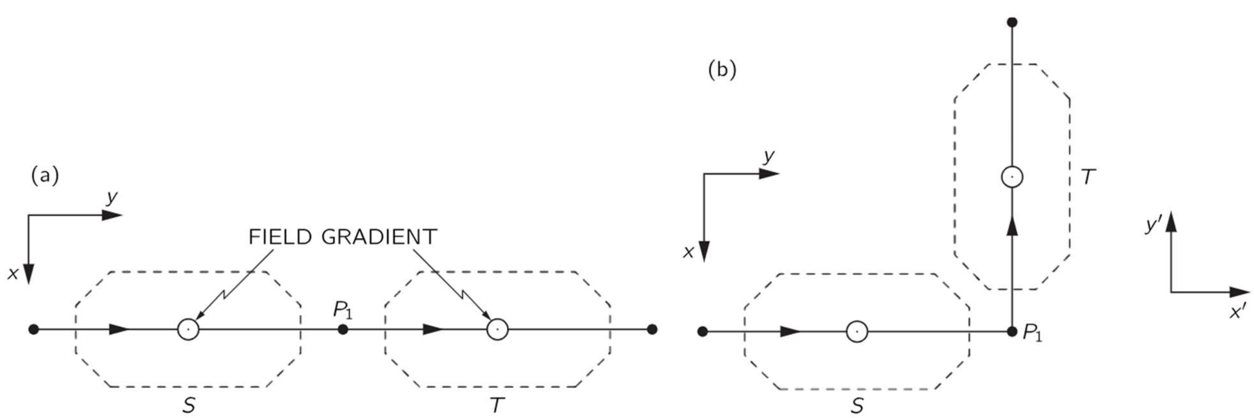
The first illustration shows the rather special Feynman-Stern-Gerlach apparatus (or the modified or *improved* Stern-Gerlach apparatus, as Feynman calls it): the apparatus splits a beam of electrons or whatever spin-1/2 particles into two and then brings them together again. We can also block one of the two channels to select spin-*up* or spin-*down* particles. The *y*-direction is the direction of propagation and the *z*-direction is the direction along which we are measuring the magnetic momentum (or, what amounts to the same, the particle's angular momentum or *spin*). The field gradient is, obviously, the direction of the inhomogeneous magnetic field that causes our spin-1/2 particles to separate according to their magnetic moment (or spin), which is either *up* or *down*. Nothing in-between.

Figure 6: Feynman's modified (or *improved*) Stern-Gerlach apparatus



The objective is to find rotation matrices: we want to know how the wavefunction changes if we rotate it along the *z*-axis (the analysis for rotations along the other axes comes later). So that is what's shown below. On the left-hand side, our particles go through two apparatuses who are perfectly aligned (the rotation angle is zero). In the right-hand side, we have a rotation angle of 90 degrees (π/2).

Figure 7: Successive modified Stern-Gerlach apparatuses



The amplitudes for the *up* and *down* state – as our particle enters the second apparatus – may or may not be the same. We know we can no longer define them in terms of the base states that came with the first apparatus (*S*): being *up* or *down* with respect to *S* is not the same thing as being *up* or *down* with respect to *T*. We can write, more generally, something like this:

$$C'_j = \sum_i R_{ji}^{TS} C_i$$

Of course, we know the *probabilities* to be *up* or *down* are going to be the same, so we should probably *not* write something like $C'_{up} = C_{up}$ and $C'_{down} = C_{down}$ but writing something like $|C'_{up}|=|C_{up}|$ and $|C'_{down}|=|C_{down}|$ is plausible. So, the amplitudes differ by a phase factor only. Feynman writes:

$$C'_{up} = e^{i\lambda}C_{up} \text{ and } C'_{down} = e^{i\mu}C_{down}$$

Again, using the rule that we can always shift the phase of the amplitudes with some arbitrary number, we find that μ must be equal to –λ, so the equations become:

$$C'_{up} = e^{i\lambda}C_{up} \text{ and } C'_{down} = e^{-i\lambda}C_{down}$$

In the special case where the rotation angle is zero (so that's the left-hand diagram), we have that λ = 0. Same representation, same amplitudes. Simple. But, of course, we want to see what λ and – λ are going to be when the rotation angle – which we'll denote by ϕ - is *not* equal to zero. Feynman starts by making a reasonable assumption: λ and ϕ are probably proportional, so let's try to see where we get by writing:

$$\lambda = m\phi$$

Of course, when we rotate the thing by 360 degrees (ϕ = 2π), we are back where we were, so we write:

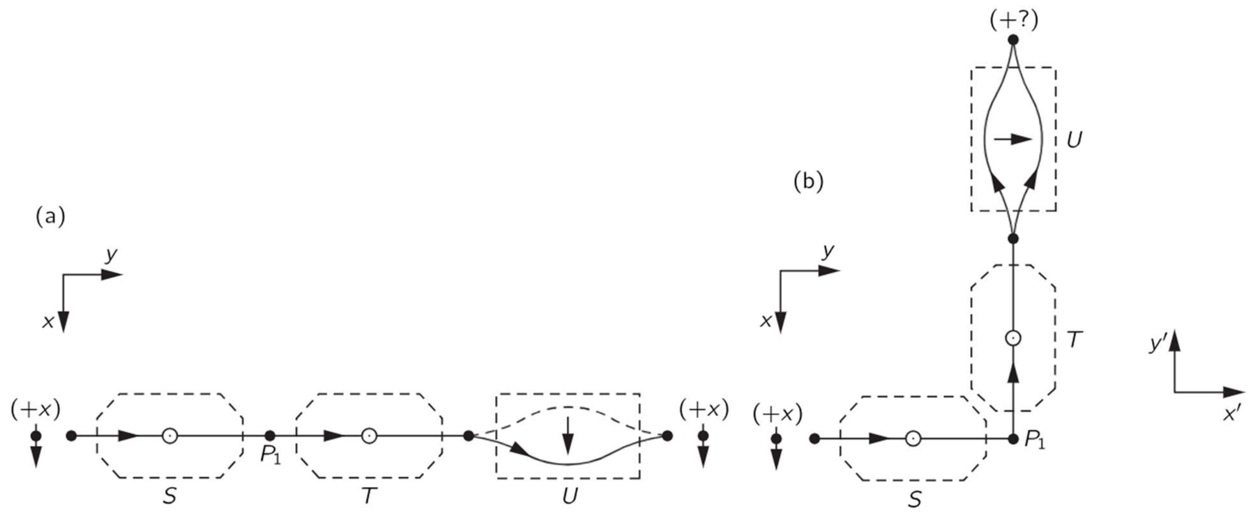
$$C'_{up} = e^{i\lambda}C_{up} = e^{im\phi}C_{up} = e^{im2\pi}C_{up} = C_{up}$$

$$C'_{down} = e^{-i\lambda}C_{down} = e^{-im\phi}C_{down} = e^{-im2\pi}C_{down} = C_{down}$$

For these two equalities to hold, *m* must be 1, right? So, we do have a 360-degree symmetry rather than this weird 720-degree symmetry, right?

Well… No. Not according to Feynman. He constructs a terribly complicated – and, in my view, potentially flawed – argument designed to sort of prove that the symmetry must be a 720-degree symmetry or, what amounts to the same, to prove that *m* = ½. The argument is based on a thought experiment that imagines a third apparatus *U*, as shown below.

Figure 8: Feynman's series of apparatus



The argument goes as follows: we have some filter in front of the *S* apparatus that produces a pure +*x* state. In other words, our particle (think of an electron) is *up* but, importantly, it's up along the *x*-direction. This orientation has nothing to do with the *S* and *T* representations, because these apparatuses measure spin along the *z*-direction. However, the *U* apparatus does measure spin along the *x*-direction and, hence, Feynman expects the particle to sail through but use one channel only, as depicted above. The result is that we still have a particle coming out with its spin *up* in the *x*-direction (+*x*).

What happens in the second set-up? We have the same electrons – with *up* spin along the *x*-direction – going through and coming out of apparatus *S*, but then they take a turn, so its wavefunction (that's what an amplitude is) must change. And then the particle goes through *T* and *U*, which analyze spin along the *y*-direction with respect to *S*. So far, so good. So, what can we say about the state of our electron when it comes out of *U* in the set-up on the right-hand side.

Well… Let us assume that the argument above is correct and that *m* is equal to 1. Let us now also consider a set-up for which the *T* and *U* apparatuses are rotated over a 180-degree angle (π). Hence, we sort of fold *T* onto *S*, so to speak. So, our rotation makes the particle go back in the direction where it came from – through the *T* and *U* apparatus. Now, if *m* = 1, then we get:

$$C'_{up} = e^{i\lambda}C_{up} = e^{i\pi}C_{up} = -C_{up}$$

$$C'_{down} = e^{-i\lambda}C_{down} = e^{-i\pi}C_{down} = -C_{down}$$

According to Feynman, this result cannot be possible. Let us quote him here:

> "This result ($C'_{up} = -C_{up}$ and $C'_{down} = -C_{down}$) is just the original state all over again. Both amplitudes are just multiplied by −1 which gives back the original physical system. (It is again a case of a common phase change.) This means that if the angle between *T* and *S* in (b) is increased to 180°, the system (with respect to *T*) would be indistinguishable from the zero-degree situation, and the particles would again go through the (+) state of the *U* apparatus. At 180°, though, the (+) state of the *U* apparatus is the (−*x*) state of the original S apparatus. So a

(+*x*) state would become a (−*x*) state. But we have done nothing to change the original state; the answer is wrong. We cannot have m = 1."[18]

This is where our physical interpretation – which, rather than making an arbitrary choice, maps all *mathematical* possibilities to all possible *physical* situations – differs from the mainstream interpretation. The $C'_{up} = -C_{up}$ and $C'_{down} = -C_{down}$ do represent two different realities – two different physical *states*, that is. Putting a minus sign in front of the wavefunction amounts to taking its complex conjugate. Hence, it effectively *does* reverse the spin direction.

Of course, the attentive reader will immediately cry wolf. We *do* have a common phase change here, don't we? Therefore, Feynman must be right and the $C'_{up} = -C_{up}$ and $C'_{down} = -C_{down}$ amplitudes *must* represent the same states. The answer is: no. There is no *common* phase change here. The phase change is +π for the *up* state and −π for the *down* state.

Q.E.D. Quantum electrodynamics. Quod eram demonstrandum.

Jean Louis Van Belle, 22 October 2018

References

This paper discusses general principles in physics only. Hence, references can be limited to references to general textbooks and courses and physics textbooks only. The two key references here are the MIT introductory course on quantum physics and Feynman's *Lectures* – both of which can be consulted *online*. As the author is most familiar with Feynman's *Lectures*, most general references are to those. In such cases, the reference refers to the volume, the chapter and the section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

While all of the illustrations in this paper are open source (or – in one case – created by the author), references and credits have been added.

[18] Feynman *Lectures*, Vol. III, Chapter 6, Section 3.