Current conflicts in general relativity: Is Einstein's theory incomplete?

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A review of refutations of general relativity commonly found in today's literature is presented, with comments on the status of Einstein's theory and brief analyses of the arguments for modified gravity. Topics include dark matter and the galactic rotation curve, dark energy and cosmic acceleration, completeness and the equation of state, the speed of gravity, the singularity problem, redshift, gravitational time dilation, localized energy, and the gravitational potential. It is conjectured that the contemporary formalism of general relativity offers an incomplete description of gravitational effects, which may be the most compelling reason for seeking new theories of gravity.

I. INTRODUCTION

Researchers both inside and outside the established physics community are currently questioning the theory of General Relativity (GR) for a number of reasons. The present review article is intended to catalogue some of these objections and lend perspective on their validity. It is hoped this effort will help reduce the growing confusion that has permeated the literature at all levels, from strict peer-reviewed journals, to publications with little or no peer review, technical books, educational websites, physics forums, and unpublished communications. Also proposed here is the hypothesis that incompleteness is the most critical flaw in the current general relativistic formalism, along with the conjecture that for some physical systems, GR offers no independent information about such observables as gravitational redshift and time dilation.

A list of common reasons for refuting GR is presented below. These topics will be discussed in detail in later sections.

1) *Galactic rotation curve (dark matter)*: Many physicists and astronomers believe that general relativity fails to explain the unexpectedly rapid orbital motion of the outer regions of galaxies except through the introduction of dark matter, a supposed non-radiating transparent material that has never been directly observed astronomically, nor verified to exist in particle accelerators, despite over half a century of searching.

2) *Cosmic acceleration (dark energy):* GR does not explain the apparent increasing expansion rate of the universe without the reintroduction of Einstein's abandoned cosmological constant Λ, which must be fine-tuned in a seemingly improbable way, or the postulation of some form of phantom pressure called dark energy.

3) *Incompleteness:* Einstein's field equations are possibly incomplete in that the gravitational mass-energy density $\rho(x^\mu)$, which presumably comprises the source of the field, does not uniquely determine the metric, or equivalently, does not fully determine the geometry of spacetime, unless one selects an often *ad hoc* equation of state. Thus $\rho(x^\mu)$ does not define such observables as time dilation, redshift, and certain properties of motion, except in special cases, which points to an inconsistency in the theory.

4) *Speed of gravity*: GR predicts that gravitational effects travel at the speed of light. However many independent researchers, as well as mainstream modified gravity theorists, postulate that the effects of gravity travel at higher or lower speeds.

5) *Time dilation*: Some researchers deny that time dilation, as predicted by GR, actually exists, asserting that redshift, which is often cited as proof of time dilation, is due to other causes such as motion of the photon through a potential.

6) *Spacetime curvature*: Some theorists doubt that the curvature of spacetime is the cause of gravitational effects, or even that 4-dimensional spacetime itself has physical meaning.

7) *Energy:* GR does not offer a definition of the localized energy of the field, which some researchers consider a flaw in the theory.

8) *The singularity problem:* The GR formalism leads to coordinate singularities as well to real singularities

in the mass density. Yet the formalism is believed to break down at singularities, pointing to a contradiction.

This paper is organized as follows: In Section II, an overview of how the GR formalism is derived and applied will be presented. Sections III through X offer discussion of each of the refutations listed above. Section XI is a brief conclusion summarizing those objections to GR that may be the most valid.

II. PERSPECTIVES ON THE GENERAL RELATIVISTIC FORMALISM

General relativity, due to the subtlety and complexity of the mathematics, may rival only quantum mechanics as one of the most confusing theories ever developed. As a result, GR is sometimes improperly taught. Textbook authors and professors often rely on plausibility arguments rather than emphasizing the mathematical formalism. Plausibility arguments are however usually approximations and can be misleading. Heuristic analogies may compound the confusion and delay the tackling of Einstein's field equations, which many graduate physics students never learn to solve.

To understand GR, one must grasp that it is one and only one thing: a theory of geometry. Whether GR is correct or not is another topic. But if one wishes to apply GR, either as a practical formalism or as a tentative description, it is necessary to realize that geometry is its total content. The geometry resulting from any specific mass, energy, momentum and pressure distribution in spacetime is uniquely and exhaustively described by the line element ds, which is the 4-dimensional differential distance along a path through space and time. The line element is constructed from the product of the metric $g_{\mu\nu}$, which contains curvature information, and the differentials dx^{μ} of the coordinates, where μ normally ranges from 0 to 3, with 0 corresponding to time, and 1 to 3 to the space coordinates. The line element is usually written in squared form as $ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}$, with repeated indices indicating summation from 0 to 3.

The computational pipeline of the general relativistic formalism for orthogonal energy-momentum tensors $T^{\mu\nu} = diag(\rho, p, p, p)$ is straightforward. One must first select a coordinate system for the spacetime region to be studied. Next, the mass-energy density ρ as a function of the coordinates x^{μ} must be specified for the region. The function $\rho(x^{\mu})$ is then substituted into the energy-momentum tensor $T^{\mu\nu}$ on the right hand side of Einstein's Field Equations (EFE):

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa T^{\mu\nu}$$

where $R^{\mu\nu}$ is the *Ricci tensor* and depends on derivatives of the metric, $g^{\mu\nu}$ is the metric to be solved for, R the scalar curvature obtained by contracting the Ricci tensor, and $\kappa = -8\pi G/c^2$ is a constant. After that, one must specify the momentum density p, or for static configurations, the pressure density, also denoted p, the latter determined by a selected equation of state that relates mass-energy density to pressure, and substitute the resulting function $p(x^i) \equiv T^{ii}$ into $T^{\mu\nu}$. The field equations are then solved to obtain the metric $g^{\mu\nu}$ and hence the line element ds. Physical observables such as redshift, time dilation, and the motion of photons and test bodies are then calculated from the line element, which is proportional to the particle Lagrangian $L = -mds/dt$ [1]. Thus, with the application of the Euler-Lagrange equation, the metric yields all test particle trajectories. (These are often calculated in a more general way using the *geodesic equation*, which can be derived by applying the Euler-Lagrange equation to the general line element.)

It is important to note that in GR, none of the physical observables are to be calculated from Newtonian quantities such as gravitational force or potential. Newtonian mechanics may provide guidelines for constructing the elements of the energy-momentum tensor, or boundary conditions on the solutions to EFE, but the concepts of force or potential play a role in plausibility arguments only. Indeed, Albert Einstein, in his original paper *Cosmological Considerations in the General Theory of Relativity* (1917) [2], used the Newtonian potential φ, along with a modified version of Laplace's equation

$$\nabla^2 \varphi - \lambda \varphi = 4\pi G \rho \,,$$

to argue the plausibility of his relativistic field equations, in which the derivatives of φ are represented by curvature $R^{\mu\nu}$ and mass density ρ by the energy momentum tensor $T^{\mu\nu}$ [3].

One reason gravitational potential so often arises in heuristic arguments is that, for many spacetime geometries, the metric has terms proportional to the classical gravitational potential *Gm/r*. These potential-like terms emerge from solving EFE, however, and are not put in by hand. More specifically, while the dependence on mass m, usually entered as an integration constant, is borrowed from Newton's law of gravity, the inverse dependence on r is not, as can be seen from Dirac's derivation of the Schwarzschild solution [4]. Moreover, no concept of potential need be assumed in the derivation of Einstein's equations. The only concept that must be assumed is that the energy, mass, momentum and pressure densities determine spacetime curvature, which in turn governs gravitational effects.

The above cautionary note is emphasized here because plausibility arguments, often based on gravitational force or potential, are frequently presented in textbooks [5,6] and on-line sources [7.8], as well as by independent researchers [9]. .For instance, Robert M. Wald in his scholarly text *General Relativity*, discusses for heuristic purposes the problem of measuring gravitational forces in the context of GR. Yet in the rigorous GR framework, such so-called forces do not exist. It would therefore be inappropriate to attempt to measure them, a fact that is not made clear. [10]. Further instances are found in James B. Hartle's textbook *Gravity, An Introduction to Einstein's General Relativity*, in which he says, "What is the difference between the rates at which signals are emitted and received at two different gravitational potentials?" [5]. Hartle continues by analyzing the effects of gravitational potential on clock rates. Yet the quantity called *gravitational potential* does not explicitly occur in the formalism of general relativity. Similarly, Steven Weinberg, in his text *Gravitation and Cosmology*, uses a plausibility argument based on gravitational force to derive the general relativistic equation of motion for a freely falling body [6]. Later however, he discusses gravitational potential more accurately in the framework of the post-Newtonian approximation, making the Newtonian nature of the quantity unambiguous [11].

Other misleading plausibility arguments are found in the clearly written critique by Miles Mathis entitled *The Speed of Gravity* [12]. Mathis states, "The strong form [of the equivalence principle] says that gravity and acceleration are the same thing. [Therefore] asking what is the speed of gravity makes no sense [because] like acceleration, gravity is not a force, it is a motion." What Mathis may be overlooking is the fact that spacetime curvature, not acceleration, constitutes the fundamental nature of gravity in GR. While it is true that test bodies accelerate in a gravitational field, and that accelerated reference frames mimic certain gravitational effects, it is also true that gravity can exist without acceleration, such as near an isolated black hole where no test bodies are present. Conversely, acceleration can exist without gravity, such as in a centrifuge rotating in free space. In view of these counterexamples, it is clear gravity is equivalent not to acceleration but to curvature. And it does after all make sense to ask at what speed changes in curvature propagate. Mathis later claims that spatial curvature does not describe linear acceleration from rest. Indeed, *spatial* curvature does not, but *spacetime* curvature does. It is the time component of the metric that is important.

In the following sections, I will offer impressions of why the eight refutations of GR noted above arise and whether they are valid objections.

III. GALACTIC ROTATION CURVE (DARK MATTER)

A large body of precise galactic redshift data tabulated over the last century has shown that the outer stars and hydrogen clouds of galaxies orbit too fast to be explained by Newtonian gravitational attraction of visible or baryonic matter alone. The pattern of orbital velocities, called the *galactic rotation curve*, remains one of the most important unsolved problems in astrophysics. The data are extensive, accurate, and independent of any specific theory, yet the solution has remained mysterious for many decades. (See full historical summary at Ref. [13].)

Astronomers and physicists are somewhat divided on the issue of the galactic rotation curve anomaly. Astronomers generally accept the hypothesis that Dark Matter (DM), which supposedly comprises the majority of galactic material, fully explains the extra orbital velocity. Their research goals, however, are largely observational, and the DM hypothesis simplifies their theoretical framework. On the other hand, a significant minority of mainstream physicists doubt that DM exists [14]. This is because, after decades of theoretical, observational and experimental research seeking any type of particle or energy that exhibits the properties of dark matter, no direct evidence for this exotic substance has been found [15]. Astronomers might disagree, pointing to phenomena such as the gravitational lensing of light from distant objects by supposed excess matter in intervening galaxies [16]. (For extensive summary with images see Ref. [17].) But these arguments are theory dependent, and the observational data are less precise and abundant. Such arguments also do not take into account the possibly significant nonlinear effects that arise from a full general relativistic treatment [18].

Most astronomers believe the DM hypothesis is entirely compatible with GR. Thus by and large they uphold general relativity as the best theory of gravity. On the other hand (although some researchers disagree, as noted below), it is commonly assumed that if DM does not exist, a modified theory of gravity is needed to explain the galactic rotation curve. Another motivation for modifying gravity is the fact that galaxies show a surprising uniformity in their would-be DM distributions, as manifest in the universal constant $a_0 = 1.2x10^{-8}\, cm/sec^2$, which accurately specifies for most spiral galaxies the radial acceleration at that distance where the excess velocity becomes dominant. This suggests that the rotation anomaly is not due to invisible matter, which should vary from galaxy to galaxy, but to an extra gravitational attraction beyond that predicted by GR. This idea has given rise to a number of modified gravity theories, including Chameleon Bigravity [15], and Modified Newtonian Dynamics (MOND) [19-21].

A few theorists argue that if DM did not exist, it would still not be necessary to modify gravity, as the rotation curve is adequately described by a full general relativistic treatment. This argument refutes the common belief that general relativistic corrections to the galactic rotation curve are insignificant due to the non-relativistic velocities and weak fields of galaxies. This belief, added to the intractable nature of the dynamical formalism, has led most researchers to dismiss the need for applying EFE to galactic orbital motion. One exception is Fred L. Cooperstock, whose calculations show that the unexpected nonlinear effects of GR may account for most of the excess orbital velocity, and that only a small amount of unseen matter is needed to make up the difference [22]. This invisible substance could be ordinary non-radiating matter, rather than the exotic variety called dark matter.

If Cooperstock's solution is correct, the galactic rotation curve would support rather than contradict GR, and the orbital motion of galaxies would no longer provide a compelling reason for modifying gravity. Furthermore, were Cooperstock's results widely acknowledged, it would render moot the search for exotic dark. A full analysis of Cooperstock's derivation, in which he solves EFE for a fluid disk using a cylindrical co-rotating coordinate system, would be required to settle the matter. Articles have appeared disputing Cooperstock's results [23,24]. But the authors fail to rigorously analyze Cooperstock's calculations, and instead criticize his simplified galactic model, or claim that he has ignored other evidence for DM such as that found in galactic cluster data. The question of whether there is a need for exotic DM or modified theories of gravity to account for galactic motion thus remains open.

IV. COSMIC ACCELERATION AND DARK ENERGY

One commonly noted problem with GR is that it does not explain the apparent increasing expansion rate of the universe without the reintroduction of Einstein's abandoned cosmological constant Λ, or without the postulation of some form of phantom substance called dark energy [25]. To offer brief background, the idea that the cosmos is expanding is based on the big bang theory, a cornerstone of the standard or ΛCDM model of cosmology. This theory is governed by the Friedman-Lemaitre-Robertson-Walker (FLRW) metric, which for spherical co-moving coordinates in flat spacetime is written:

$$ds^2 = dt^2 - a(t)(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2)$$

Using a metric of the above form, Einstein's field equations reduce to the following two simultaneous equations in terms the time-dependent scale factor a(t):

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3}$$

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi p$$

where overdots mean derivatives with respect to the time coordinate t [26]. The first of these equations is called the *Friedman equation*. Note that *a(t)*, which defines the cosmic expansion rate, is determined not just by mass density ρ, but also by pressure density p, which is fixed by an auxiliary equation of state. Using standard forms of *a(t)* which increase monotonically with time, FLRW predicts that redshift increases with distance for galaxies beyond our local cluster. This redshift is considered to arise not at the galaxies themselves, which it would if it were a Doppler effect, but in the expanding space as photons traverse the cosmos on their way to the observer.

Assuming the universe is expanding, Supernovae Type 1a redshift versus distance data, among other evidence, suggest that the cosmic expansion rate is accelerating in the present epoch [27]. Calculations based on GR however predict the expansion should decelerate. This discrepancy is often resolved in one of two related ways. The first is the Dark Energy (DE) hypothesis. According to this, some unknown energy source, possibly related to the vacuum, pushes the universe apart. The existence of DE, however, seems implausible to many researchers. This phantom energy not only has a negative sign for pressure, it supposedly makes up most of the energy in the universe [28], despite that it has never been independently observed [29]. Thus, many astrophysicists propose instead the introduction of a cosmological constant Λ, which serves the same purpose. The cosmological constant is an *ad hoc* coefficient that can be put into Einstein's field equations, and was first introduced by Einstein himself to counteract gravitational collapse in a universe he believed to be static. The constant was abandoned when the big bang theory obviated the need for cosmic repulsion, and was later reintroduced to account for cosmic acceleration. However, to match observation, Λ must be fine-tuned in a way that seems improbable [30-33]. Another problem relates to the odd coincidence that energy densities due to the cosmological constant and to matter are nearly the same in the present era [34]. Many researchers therefore reject the Λ and dark energy hypotheses.

Cosmic acceleration is arguably the phenomenon most frequently cited in peer-reviewed literature as a motivation for modified gravity [31,35-38]. Such theories are often published in mainstream journals, indicating the physics community provisionally accepts that modified gravity is relevant to current research. Among these theories are Horndeski-type scalar tensor models such as the Brans-Dicke theory [39], Born-Infeld gravity [40], Galileon theories, Gauss-Bonnet theories [41,42], f(R) theories where R is the Ricci scalar, such as the Starobinsky model [35,43,44], f(R,Q) gravity where Q is square of the Ricci tensor [45], unimodular f(R,T) gravity, where T the trace of the energy momentum tensor $T^{\mu\nu}$ [46-48], and a recently proposed local antigravity model [49]. For discussions of modified gravities, see Refs [50,51].

But is cosmic acceleration really a valid reason for modifying or rejecting the well-tested theory of GR? Arguably not. First of all, astronomical evidence for cosmic acceleration is inconclusive. Analysis of the redshift data entails fitting a set of ideal curves to a comparatively small number of data points, where the curves to be fitted are close together relative to the size of the error bars. The data itself, moreover, is accurate only insofar as Supernovae Type Ia radiate as true standard candles, a question currently being debated in peer-reviewed journals [31]. Secondly, the interpretation of the redshift data is theory dependent. Modified gravities and alternate cosmologies suggest possible scenarios in which acceleration does not exist [28,52]. R. Monjo for example proposes an inhomogeneous cosmological metric with linear rather than accelerated expansion that fits SNIa data as well as the standard model [53]. Other researchers also note that apparent cosmic acceleration arises due to the assumption of a homogeneous universe. Hua Kai-Deng and Hao Wei say, "If the cosmological principle can be relaxed, it is possible to explain the apparent cosmic acceleration ... without invoking dark energy or modified gravity. For instance, giving up the cosmic homogeneity, it is reasonable to imagine we are living in a locally underdense void." [54] What is more, cosmic acceleration only makes sense in the context of an expanding universe, whose dynamics is usually assumed to be governed by the FLRW metric, itself a cornerstone of GR. Thus any such refutation of GR assumes GR at least in part, which may seem inconsistent.

Modified gravity theories have had some success in accounting for cosmic acceleration. However, insofar as observational evidence for accelerated expansion seems inconclusive, and can possibly be accounted for by alternate theories of cosmology, the apparent increase in universal expansion rate may not provide sufficient reason to modify or replace GR.

V. INCOMPLETENESS

Einstein's field equations can be interpreted as incomplete in that mass-energy density ρ, presumably the source of gravity, does not uniquely determine all the components of the metric. For example, in the general spherical static non-vacuum case, ρ determines the r component g_{11} but not the t component g_{00}. This can be seen by examining Einstein's field equations for a static spherical non-zero mass distribution, which reduce to the simultaneous equations:

$$\kappa\rho(r) = \frac{-1}{g_{11}r^2} - \frac{1}{r^2} - \frac{g_{11}{}'}{g_{11}{}^2 r}$$

$$-\kappa\, p(r) = \frac{-1}{g_{11}r^2} - \frac{1}{r^2} - \frac{g_{00}{}'}{g_{00}g_{11}r}$$

where primes denote differentiation with respect to r. It is clear from the first equation that g_{11} is fully determined by mass-energy density *ρ(r)*. To solve for g_{00} however, an auxiliary Equation of State (EoS) relating ρ to pressure p is needed. In general applications, the EoS as a practical matter is often chosen *ad hoc*. A commonly used EoS *is* $p=w\rho$ where w is a coefficient often set to 1 or 0. The coefficient w can also be negative, as is assumed in descriptions of dark energy, although this may seem unphysical [55]. Moreover, the EoS can in general vary with space and time. Indeed, in peer-reviewed literature, models using an EoS of seeming unlimited complexity are sometimes assumed [56-58]. This leads to the awkward circumstance that in many cases the EoS yields more information about gravitational effects than do Einstein's equations themselves. In fact, almost any desired gravitational effect can be induced by tailoring the EoS, and since the EoS is derived not from gravitation theory but from the separate discipline of thermodynamics, this leads to the conjecture that EFE, and thus GR, provide no independent information at all about certain measurable gravitational effects. In particular, *Einstein's field equations provide no information about redshift and time dilation for static spherical non-zero mass distributions*. (This conjecture will be proved in a later paper.)

One contradiction arising from the requirement for an EoS is that, in the case of the static spherical *vacuum* solution, which by the Jebsen-Birkhoff theorem is uniquely the *Schwarzschild metric* [43],

$$ds^2 = (1-2m/r)dt^2 - (1-2m/r)^{-1}dr^2 \\ -r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

no equation of state is needed. The Schwarzschild metric can be derived without one, and depends only on the central mass m. At the same time, this metric, which accurately describes gravity in the vicinity of stars and planets, is the only solution to EFE that has been extensively tested in a theory-independent way. The success of the Schwarzschild metric thus implies that gravitational effects are adequately determined by mass alone. But this contradicts the formalism for the non-vacuum as described previously. Another peculiar fact is that the Schwarzschild metric has the form $g_{00} = -1/g_{11}$, as if an EoS of $p=\rho$ had been implicitly assumed. Was it? In a sense, yes, in that both ρ and p vanish for the vacuum and hence are equal. But this is a trivial application of EoS. More relevant is the fact that no EoS is applied to the mass m itself, which is put into the metric by hand as a constant of integration. It may be significant that Einstein's original static energy-momentum tensor

$$T^{\mu\nu} = diag(\rho, 0, 0, 0)\,,$$

as defined in his paper of 1917 [2], contained mass density ρ but not pressure p. This implies that Einstein interpreted the spatial components as strictly momentum, which vanishes for static configurations. Such an interpretation seems reasonable to this author in that the motions comprising pressure are random rather than unidirectional, suggesting pressure should not appear in the spatial components, but only in the mass-energy density component T^{00}. The pressure terms $T^{ii} = p$ were first suggested to Einstein in a letter from Erwin Schroedinger (1918) as a solution to the cosmological constant problem [3], and later became an established feature of GR. The history and impact of this development is a topic for future research.

As mentioned earlier, the EoS can vary with time. In the standard model of the expanding universe, for example, the EoS is assumed to change from epoch to epoch, depending on whether space is dominated by radiation, matter or the vacuum [59]. This epoch-dependent model is called the ΛCDM model, where CDM stands for Cold Dark Matter, and Λ is the cosmological constant. It is well known that if the standard EoS is assumed, the ΛCDM model accurately accounts for most astronomical observations. Thus, ΛCDM provides a useful framework for cataloguing astronomical data. However, the important point is that EFE, and hence GR, offer only partial information about how the universe evolves through time. An additional criterion for determining the cosmic scale factor *a(t)* is embodied in the EoS, and this auxiliary equation is chosen either after the fact by fitting observational data to redshift versus.distance curves, or by applying thermodynamics, a separate branch of physics [60]. The above example again shows that the requirement for an EoS to determine the metric implies general relativity may be deficient. Incompleteness thus seems the most compelling reason to modify GR.

Some authors have proposed a type of modified gravity, called *f(T)* gravity (not to be confused with torsion or teleparallel gravities sometimes also called f(T)), in which the field equations contain only functions of the trace T of the energy-momentum tensor $T^{\mu\nu}$. This obviates the need for an EoS, and may be a start toward a more complete theory of gravity.

VI. SPEED OF GRAVITY

GR is widely believed to predict that gravitational effects travel at the speed of light c. If we assume the principles of Special Relativity (SR), a formalism confirmed in arguably millions of particle accelerator experiments, c is the speed at which the effects of gravity should be expected to travel. The speed of gravity c_g cannot be greater than c, insofar as messages can in principle be sent via gravity, and if messages could travel faster than c, they could be sent into the past in certain reference frames.

There is, however, a remote chance that *non-oscillating* gravitational effects could travel at a velocity greater than c. They might for example travel at $v = c^2/u$, where u is the velocity of the source relative to the test particle. In that case, gravitational effects would be instantaneous in the rest frame of the source. Stated in terms of special relativistic spacetime diagrams, $v = c^2/u$ is the slope, in *t-r* coordinates, of the source's plane of simultaneity, where u points in the direction r. This tachyonic value of v is of interest because it matches the phase velocity of de Broglie waves as defined by the relativistic single-particle

Dirac and Klein-Gordon equations Nevertheless, it must remain true that oscillating effects such as gravitational waves, which carry energy and information, are confined to the limiting velocity c [61].

Whether a dual-velocity picture of gravitational propagation leads to contradictions is not yet known. However, the tachyonic speed of non-oscillating gravitational effects can be visualized in the following thought experiment. Imagine two stars of equal mass in circular orbits around their center of mass. First, it is known that in the framework of Newtonian celestial mechanics, which involves forces in absolute space and time, gravitational attraction must propagate instantaneously. Why? Were there any time delay, each star would feel a gravitational force pointing toward an earlier spot in the other star's orbit [62]. If visualized correctly, the reader will see that this small offset, sometimes referred to as *gravitational aberration*, exerts a slight forward force on each star, making both stars orbit faster and faster, an instability which to Newtonian order is not observed. Thus, in real physical situations, each star accelerates toward the spot where the other star is now, and the gravitational force must therefore be instantaneous. Of course, this Newtonian scenario cannot tell us the speed of gravity in GR. It is a plausibility argument only. It does however present a paradox. How can the Newtonian infinite gravitational speed be reconciled with the supposed speed c predicted by GR?

One possible answer is suggested by the following treatment of the above thought experiment. Imagine a co-rotating coordinate system with respect to which the two orbiting stars described above are at rest (neglecting the small amount of radiative orbital decay.) The two stars can now be modeled by a static double-Schwarzschild metric. Such a metric has already been derived by other authors as an exact solution to Einstein's field equations [63]. Since the metric is static in the co-rotating frame, the curvature and thus the mutual gravitational effects are also static in that frame. Defining the speed of gravity is now a matter of semantics. One might say that no effects at all are propagating in the co-rotating frame, or alternatively, that the effects of gravity propagate at infinite speed in that frame. In either case, the computed orbital motion, to Newtonian order, is the same as that of classical celestial mechanics. Again, it is important to stress that in the dual-velocity picture, these mutual gravitational effects cannot carry energy, since oscillating or energy-carrying effects must travel at c or less. (The small amount of gravitational radiation emitted from the rotating star system does of course propagate at c.)

The question of gravitational aberration has been a source of confusion in the literature. Some authors claim that the absence of gravitational aberration for orbiting bodies would constitute proof of an instantaneous gravitational interaction. Others, such as S. Carlip, argue that in a formal general relativistic treatment, aberration terms almost perfectly cancel even though c_g is assumed to be c, and therefore the lack of aberration does not imply $c_g \;\square\; c$ [64]. It is unclear, however, whether Carlip's professed formal treatment, which employs a novel light-cone coordinate description of a mass-changing object called a *photon rocket* [65], is based on rigorous principles.

There remains in Carlip's calculation a small higher-order residual gravitational aberration. Curiously, mathematical physicist Michal Krizek proposes that such an aberration is actually observed, and is the partial cause, along with tidal forces, of the increase in mean distance between the Earth and the Moon [66].

Can the speed of gravity c_g be less than c? Some peer-reviewed theories of modified gravity, including quantized massive graviton theories, predict that it can (for extensive discussion see Ref. [67]). If true, the speed of gravity would not be the same in every reference frame. It might for example travel at a speed relative the source, much like Ritz's old ballistic theory of light [68]. But to many theorists this seems implausible, especially in view of recent observations. Specifically, the reported near-simultaneous LIGO gravitational wave detection GW170817 and gamma ray burst GRB 170817a, received with a time lag of only 1.7 seconds from an event thought to be some 130 million light years away, seem to indicate gravity waves and electromagnetic waves travel at the same speed [69]. More precisely, $c_g = c$ to an accuracy of 10^{-15} [70,71]. The small time lag is believed to be due to size of the source. Many astrophysicists have therefore concluded that these near-simultaneous GW and GRB detections disprove modified gravity theories in which $c_g \neq c$ [72-74], or that such theories must be strongly constrained [71,75]. For example, Crisostomi and Koyama say, [76] "The almost simultaneous detection of gravitational waves and gamma-ray bursts from the merging of a neutron stars binary system unequivocally fixed the speed of gravity c_{GW} to be the same as the speed of light c." However, that this conclusion should be called *unequivocal* may be premature. Engineers and scientists familiar with large-scale government-funded research, especially involving extensive computer analysis, sometimes find that the results are prone to error. Even if disparities rarely occurred, doubts might still be raised. Indeed, independent theorist and critic Miles Mathis doubts there is any truth at all to the professed LIGO gravitational wave detections, and while Mathis's technical arguments have apparently not been peer-reviewed, his allegations of disregard for the scientific method on the part of the LIGO team may be justified [77]. It therefore seems reasonable that the raw data from the LIGO observations, as well as the experimental apparatus and its underlying assumptions, be analyzed by independent parties before conflicting theories are abandoned. To the knowledge of this author, an independent analysis has not been conducted. (See however James Creswell of the Niels Bohr Institute and associates, who perform an extensive analysis of LIGO detector noise and conclude that the gravity wave signals are questionable, stating, "A clear distinction between signal and noise therefore remains to be established in order to determine the contribution of gravitational waves to the detected signals." [78]) Note that as recently as two decades ago, independent verification was the hallmark of physics. This standard should not be compromised. Meanwhile, it is still too early to call an end to all research into different speeds of gravity.

VII. GRAVITATIONAL TIME DILATION

Some theorists deny that time dilation, as predicted by GR, actually exists, claiming that redshift, which is often treated as equivalent to time dilation, is due to other causes such as photon motion through a gravitational potential. First, there seems to be confusion in the literature about the relation between time dilation and redshift, which will be discussed below. So the immediate question is, are there ways to measure time dilation without relying on redshift? One method is via the Shapiro time delay, which is the time delay of light as it traverses the field of the Sun [79]. This delay has been measured to a high degree of accuracy. The simplest explanation is that the delay is due in part to time dilation along the path of the photon as it passes close to the gravitational source, and in part to relativistic path length increase. Alternatively, the time delay might be attributed to a slowing of the speed of light as seen from infinity. But time dilation and the slowing of the speed of light are formally equivalent. They are two different descriptions of a single property of the metric, namely that $g_{00} < 1$. In any case, the Shapiro time delay does indeed verify time dilation independently of redshift.

Blurring of the distinction between gravitational time dilation and gravitational redshift is so prevalent, many authors use the terms almost interchangeably, even though they might be different phenomena. For example, there is no way in principle to directly measure cosmic time dilation, which may not even exist given that $g_{00} = 1$ in the FLRW metric, although cosmic redshift is certainly observed. The confusion is further compounded by the fact that some authors contend that time dilation *causes* redshift, or that *gravitational potential* causes redshift. That such claims lead to contradictions has been demonstrated by Vasily Yanchilin [9]. In his paper entitled *The Experiment with a Laser to Refute General Relativity*, he points out that general relativists, in textbooks and peer-reviewed journals alike, contradict themselves by purporting on the one hand that gravitational redshift, for example in a Schwarzschild field, is caused by energy loss as photons climb through the gravitational potential, and on the other hand, by time dilation at the emitter. If both were true, Yanchilin explains, we would see twice the redshift we do. So it must be one or the other. This seems patently logical, and Yanchilin proposes an earth-based experiment to distinguish between the two purported causes. However there is a subtle point that Yanchilin and others may have missed. The notions that redshift is caused by energy loss in transit or by time dilation at the source are both plausibility arguments, put forth to help students visualize why redshift occurs in a gravitational field [80]. These arguments are misleading. Indeed, they may have misled Yanchilin into designing an experiment that will fail to prove what he seeks to prove, as will be discussed below.

A rigorous analysis of the behavior of light as it climbs through a gravitational field shows that, while photon energy $E=h\nu$, where ν is the proper frequency measured along the photon's path, is indeed lost during transit, and time, as viewed from infinity, is dilated at the emitter, these are two different descriptions of a single property of the metric, which in static cases is simply that $g_{00} < 1$. These phenomena do not *cause* redshift; spacetime curvature does. In fact, spacetime curvature causes all three phenomena: time dilation at the emitter, photon energy loss in transit, and redshift at the detector. And all three have the same value, obtained from $g_{\mu\nu}$.

The ultimate arbiter is the metric. When redshift is calculated from $g_{\mu\nu}$, the result is unambiguous. There is one value of redshift, and it is not doubled. So if Yanchilin successfully conducts his experiment, in which light is to be emitted both upwards and downwards from a central height in a tall building, and the results tabulated by a frequency counter at that same central height, he will measure the correct GR redshift. However, believing the two plausibility arguments are mutually exclusive, he may misinterpret his results as a confirmation of photon energy loss, and hence as a repudiation of time dilation. Intending to disprove GR, he may find that many physicists will only claim he has proven it. Yet Yanchilin has simply carried to its logical conclusion a set of common misconceptions. I would venture that the fault lies in today's education system, in which plausibility arguments are emphasized while mathematical formalism is neglected.

VIII. SPACETIME AND CURVATURE

Some researchers doubt that the curvature of spacetime, as embodied in the metric, is the origin of gravitational effects, or even that 4-dimensional Minkowski spacetime is a valid physical concept. In the latter case, they are refuting special relativity (See for example Ref [81]). A number of authors are currently investigating new physics beyond SR, and peer-reviewed articles state there is a consensus among physicists that the spacetime structure of SR will have to be modified in order to quantize gravity [82]. There is also renewed interest in Lorentz-violating theories such as Horava gravity, whose low energy limit is dynamically equivalent to the Einstein-aether theory [83,84]. Yet in a classical (non-quantum) context, a formalism describing time, space and linear motion more concise and accurate than SR has, to the knowledge of this author, never been derived. Occam's razor alone says this validates SR.

It is true of course that time and space have very different properties. One such property is the signature in the line element, as can be seen from the 2D spherical Minkowski line element $ds^2 = dt^2 - dr^2$. The sign of the temporal term is opposite that of the radial term, implying that if t is a dimension, it is in some sense an imaginary one. Another such property is the arrow of time. Space, in contrast, has no arrow. These disparities may make space and time hard to conceptualize as a homogeneous entity. Some critics thus reject spacetime altogether, and attempt to explain the constancy of the speed of light, which forms the mathematical basis of SR, by attributing the shortening of rulers and slowing of clocks to electromagnetic or mechanical processes [81]. However, since every moving clock and object slows and shortens, it might as well be said that time dilates and length contracts, as there is no way in principle to distinguish time and length from clocks and objects. In any event, refutations of SR are rarely mentioned in modern peer-reviewed journals except in the context of quantization. This does not mean, of course, that spacetime could not eventually be replaced by a simpler or more accurate construct, conceived perhaps as a product of brilliant intuition.

That gravity arises due to the curvature of spacetime is more frequently doubted. Some researchers accept Minkowski spacetime, yet reject the idea that pseudo-Riemannian geometry, which is defined by a (possibly) curved line element in which one term is of opposite sign, determines the properties of space, time and motion in a gravitational field. Among such theories are teleparallel gravity (TEGR) [85] or torsion-f(T) gravity [86,87].

The notion that gravity is caused by curved spacetime springs from the principle of equivalence. This principle may be paraphrased by saying that all point-like test particles, regardless of their mass or composition, follow the same trajectory in a gravitational field. So to doubt that gravity is geometry is to doubt the principle of equivalence. Yet the principle of equivalence has been demonstrated to a high degree of accuracy. In response to this fact, physicists who refute geometric gravity have proposed a hierarchy of equivalence principles, from strong to weak [88,89], claiming that only the weaker versions have been proven. This allows small deviations from pseudo-Riemannian geometry, which may be needed, for example, in attempts to quantize gravity.

As an aside, it can be argued that if gravity is geometry, then it cannot in principle be quantized. Geometric gravity does not involve any forces that might be mediated by gravitons. All apparent forces are pseudo forces. Thus, centrifugal force is as real or unreal as centripetal force. Both occur when an object deviates from a geodesic. (An example is found in the apparent forces at the near and far walls of an orbiting space station.) So if one wishes to quantize the attractive gravitational force, one should also quantize centrifugal force, which seems absurd. It is perhaps relevant that after almost a century of effort, no attempt to quantize gravity has been fully successful [84]. On the other hand, quantization efforts are justified insofar as GR does not tell us how spacetime curvature propagates outward from a massive body, only that it does so at the speed of light. To address this omission, it may be necessary to extend GR to include gravitons or some other mediating mechanism.

Whether gravity is or is not geometry is a separate question from whether GR is valid. GR of course requires that gravity be geometry. But there is an unlimited set of geometric gravity theories, often called *metric theories*, that differ from GR. These theories involve curved metrics, possibly in higher dimensions, but the metrics are not necessarily solutions to

Einstein's field equations. Examples include modified gravity theories such as *f(R)* gravities, in which the field equations contain higher order terms in the scalar curvature R [35,43,44], or *f(R,T)* theories, where *T* is the trace of the energy-momentum tensor [46-48]. The variations are endless.

Meanwhile, unless the equivalence principle can be disproved, there is no reason to reject curved spacetime as a description of how objects behave under the influence of gravity. Even if the metric is considered to be only a shorthand notation for gravitational effects, this does not change the fact that by Occam's razor, curved spacetime provides the simplest and most accurate formalism for gravity known today.

IX. ENERGY AND THE GR FORMALISM

That GR does not offer a clear definition of the localized energy of the field is considered by some to be a defect in the theory. P.A.M. Dirac, in his concise textbook General Theory of Relativity [90], summarizes the situation as follows, "It is not possible to obtain an expression for the energy of the gravitational field satisfying both the conditions: (i) when added to other forms of energy the total energy is conserved, and (ii) the energy within a definite region at a certain time is independent of the coordinate system. Thus in general, gravitational energy cannot be localized." Authors in peer-reviewed journals occasionally raise objections to the lack of local conserved energy, and suggest possible conserved quantities other than energy [91].

The absence in GR of a definite field energy meeting the requirements given by Dirac does not imply that Einstein's theory is incomplete or should be modified. Conservation of energy is a classical law by virtue of the concept of potential energy, an arguably contrived quantity which is proportional to the potential. Yet potential, as explained before, is not intrinsic to GR. Therefore, GR should not be expected to comply with conservation of energy.

X. THE SINGULARITY PROBLEM

The formalism of GR predicts real physical singularities, such as those at $t=0$ in the FLRW metric (the time of the big bang) or $r=0$ in the Schwarzschild metric, as well as coordinate singularities such as that at $r=2m$, the horizon of a black hole. Yet the mathematical formalism is believed to break down at singularities [92]. Is this a contradiction in the theory? Some mainstream physicists contend that it is, citing for example a problem known as *geodesic incompleteness*, by which a photon traveling on a geodesic would cease to exist at a singularity [93,94]. Thus, there are ongoing efforts modify GR so that singularities do not arise [71].

Many researchers claim that a correct theory of quantized gravity will remove all singularities. These endeavors toward quantization are well documented in mainstream journals [95,96]. Yet the so-called *singularity problem* may not constitute a valid reason for rejecting or modifying GR. It could be true of course that singularities are unphysical. For example, it can be shown from the Schwarzschild metric that a black hole would take forever to form by gravitational attraction alone [97]. Therefore, unless black holes are primordial or created by other forces, they do not exist in a universe governed by GR. (Some astrophysicists ignore this result. As Naoki Tsukamoto says, in the introduction to an article on black hole shadows, "Recently, LIGO detected three gravitational wave events from binary black hole systems. The events showed stellar-mass black holes really exist in our universe." [98]) Bouncing cosmological models have also been proposed that avoid the singularity at the big bang [48,92,99]. In any case, singularities do not seem to pose a problem from a mathematical standpoint. Coordinate singularities can be transformed away, while so-called real singularities can be handled as mathematical limits.

XI. CONCLUSION

Of the many reasons theorists refute general relativity, there are two that stand out as possibly the most compelling: 1) the galactic rotation curve anomaly, and 2) incompleteness, or the need for an equation of state. Finding a modified gravity theory that accounts for the galactic rotation curve has proven surprisingly difficult. One problem is that GR describes solar system observations to a high degree of accuracy, yet a naive scaling of the galactic rotation curve to fit the orbits of outer planets gives erroneous results. Thus, any modified gravity theory must employ some screening mechanism whereby GR holds at smaller scales, but not on the scale of galaxies or the cosmos. Many such mechanisms exist, but so far no modified gravity theory has gained acceptance as a replacement for GR. This problem is widely discussed in Physical Review D [73,100]. (For a summary of screening mechanisms see Ref. [36].)

More significantly, GR's requirement for an equation of state seems proof of the incompleteness of the theory, though to the knowledge of this author, such a deficiency is never acknowledged in the literature. Physicists invariably select an EoS as a matter of course. The EoS is usually chosen either *ad* *hoc*, or based on thermodynamic arguments. The EoS can be as complicated as desired, and in principle tailored to produce almost any physical result. For example, in the static spherical non-vacuum case, the mass density ρ(r) determines only the g_{11} component of the metric. The g_{00} component, which describes observables such as time dilation and redshift, depends on the EoS, and if the EoS is suitably varied, can in practice be anything conceivable. Thus, these time-coordinate observables do not in general depend on the mass distribution. This fact contradicts the common interpretation of the Schwarzschild metric, according to which such observables depend on mass alone. It is seldom if ever mentioned that the Schwarzschild metric, the only metric to have been observationally tested in a theory-independent way, does not require an EoS and therefore seems at odds with the rest of the theory.

Criticisms of general relativity abound, yet no suitable replacement has been proposed. It might be possible to derive a theory of gravity based on a field equation that does not require an EoS, for example in which the energy-momentum tensor $T^{\mu\nu}$ is replaced by a function f(T) of the scalar *T*, the trace of $T^{\mu\nu}$. But such a theory is unlikely to explain the galactic rotation curve. Many of the questions raised in this article therefore remain open.

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[Many references at Physical Review D are publicly available on arXiv. If links are not provided below, they may be found by searching for author and title at Google Scholar.]

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