Proof of the Fourth Landau's Problem

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1 Definition of the Fourth Landau's Problem

Definition: Is the set of primes of the form $(n^2 + 1)$ infinite?

2 Algorithm for Proof of the Fourth Landau's Problem

The proof of the **Fourth Landau's Problem** is a consequence of the **Proof of the Legendre's Conjecture**.
Let the odd number be y.
Half of the numbers $(n^2 + 1)$, following odd $n^2 = y^2$ are even and correspondingly composite numbers.
The other half $(n^2 + 1)$ follows an even number of $n^2 = (y + 1)^2$.
Then:

$$n^2 + 1 = (y + 1)^2 + 1 = y^2 + 2y + 2 = y(y + 2) + 2. \tag{1}$$

Let's represent (1) with respect to the odd number y_k following the given y:

$$n^2 + 1 = y_k(y_k - 2) + 2. \tag{2}$$

But the number represented by the expression (2) is the first in one of the two sets of **Proof of the Legendre's Conjecture**:

$$\{y_n \mid y_k(y_k - 2) < y_n < y_k^2,\ y_k \geq 3\}. \tag{3}$$

According to the **Proof of the Legendre's Conjecture**, the frequency of the appearance of composite numbers in the entire given segment:

$$Z_{y_{comp}}(\{y_{comp}\}) = 33{,}3\dots3\%(\{3y \mid y \geq 3,\ 3y = y_n\}) +$$
$$+ \sum_{m=3} Z_{y_{om}} \left(\{y_{om}y_m \mid y_m \geq y_{om},\ y_{om}y_m = y_n,\ y_{om} < N_{y_n},\ \frac{y_m}{3} \notin \mathbb{N}, \dots, \frac{y_m}{y_{o(m-1)}} \notin \mathbb{N}\} \right) < 100\%, \tag{4}$$

where: the number of digits represented by $(\dots)$ in the first term, $\to \infty$;

m is the number of a member of a sequence of odd primes;

y_{comp} is a composite odd number in a given segment of a sequence of odd numbers $\{y\}$;

$Z_{y_{comp}}$ is the frequency of the appearance of composite numbers (in %) in a given segment of the sequence $\{y\}$;

N_{y_n} is the number of terms in the set (3);

$N_{y_n} = y_k - 1$;

y_m is a sequence of odd numbers with the conditions given in the formula.

Therefore, although with increasing y_k the probability of the appearance of a composite number in the first term of the set (3) increases, it never reaches 100%.

That is, the set of primes of the form $(n^2 + 1)$ is infinite.

Publications: http://samlib.ru/editors/b/bezymjannyj_a/w5.shtml