ON THE DICHOTOMY OF CAUSALITY AND MEASUREMENT

G.N.N.MARTIN

ABSTRACT. There is a fundamental incompatibility between the logic of causality and that of measurement: causality is defined at a point, whereas measurement is defined over a volume. This problem is illustrated by Schrödinger's wave equation for an electron, where the wave equation describes the evolution of the electron by describing the evolution of the wave at each point, but measurement is made on the electron as a whole.

1. Causality versus measurement

I hope it is self evident that measurement is defined over a volume. There are indeed quantities such as density that we associate with a point, but the measurement of such quantities do involve measurement over a volume: density is the mass of a volume divided by the magnitude of the volume.

It is a little less obvious that causality, at least, the causality that we understand, is associated with a point. By causality, I mean those calculations that enable us to predict the future, or deduce the past: these calculations typically work at a point. An example may help.

Consider the problem of predicting how two billiard balls will react when they collide. We can simplify the problem by assuming the balls can be represented by points at their centre of mass. We assume the balls are rigid, and when they collide, every part of a ball changes velocity at the same time.

In a less approximate analysis we recognise that the billiard balls must distort, and the distortion creates internal forces that must travel through the ball. One side of the ball starts to react before the other, and reacts somewhat differently. We only get an accurate description of how the balls interact if we can come up with a formula that tells us how any given point behaves as the stresses and strains change at that point. Such a formula expresses *local action*.

It is impractical to use local action to describe the behaviour of a billiard ball, but we use local action to predict the behaviour of waves in the sea or of sound waves. Maxwell used local action to predict the existence of electromagnetic waves. With waves, the difficulty is in deciding quite what it is you are measuring. A billiard ball is a clearly defined object, but a sea wave or a sound wave is not.

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The fact that in some circumstances electromagnetic waves behave like particles and electrons behave like waves suggests that we might use local action to describe their behaviour.

2. Schrödinger's wave equation

Schrödinger's wave equation is the bridge between the causality defined by action at a point, and the action over a volume required by measurement.

2.1. The particulate wave. Schrödinger's equation solves (approximately ¹) the wave equation $\frac{\partial^2 \psi}{\partial t^2} - \nabla^2 \psi = -m^2 \psi$, where ψ is a complex number and the value of m is calibrated by de Broglie's relationship. For a given set of boundary conditions the equation has a number of solutions known as eigensolutions, each of which can be scaled so that the integral over the entire wave of the squared magnitude of ψ (i.e. of $\psi\psi^*$) is unity. I will refer to these appropriately scaled eigensolutions as *particulate waves*.

Each of these particulate waves has an associated mass. If we calculate the possible particulate waves for a hydrogen atom we find the difference between their masses corresponds to the energy of the lines in the hydrogen spectrum.

2.2. The electron as a particle. You may think that the problem of understanding how the electron can behave as a particle arises because we have modelled the electron as a wave, but that is not so. When we call something a particle, we do not attempt to say how or why it behaves as a particle: the behaviour is a given. The characteristic of a particle is that any attempt to measure its state measures the state of the entire particle.

A particulate wave is the unit of observable change. The electrons around an atom are a waveform obeying the appropriate equation. The electrons interact with the nucleus, and both interact with the surroundings, and the energy that we can reasonably ascribe to the electrons is not restricted to the sum of the energies of particulate waves. An atom can acquire (or lose) energy by interactions at a point, but the change is undetectable. A detectable change acts over a volume, taking one (or more) complete electrons from one (or more) particulate waveform to another. We have no idea how it does this, but we can deduce that it is so, since the only observables have energies consistent with that. I will refer to this detectable change as a *particle event*.

2.3. (Spooky) action at a distance: a particle event appears to require a change to take place 'at the same time' over a volume. I say 'appears' because we do not know how it happens nor exactly what 'at the same time' means. 'Spooky action at a distance' is an extreme example of such an event.

It is possible to entangle a pair of electrons, such that when a particle event disentangles them they will have opposite spin, even if the electrons are very far apart when they

¹It is not relevant whether it is the wave equation or the Schrödinger equation or neither which is exactly correct. The point is that both calculate the action at a point.

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are disentangled. This interdependence illustrates 'spooky action at a distance' and for some time people tried to show that the apparent interdependence was a product of their common history. Eventually JSBell showed (effectively) that the predictions of quantum theory definitely implied action at a distance. Given the continuing experimental verification of quantum theory, spooky action at a distance has become incontrovertible.

Any measurement or observation acts over a volume, implying action at a distance. The action seems less spooky because the distances are usually small.

3. Implications

It is hard to draw any firm conclusions, because there is so much that is unexplained both in the nature of the waves and in the nature of a particle event. But it is hard to resist speculating.

3.1. Entangled particles. Where an atom has several electrons bound to it, do the electrons exist as distinct particles, or is the particle only an artefact of a non-local event? Perhaps the electrons are just a cloud of waves from which a particulate wave is carved when a particle event occurs. If so, entangled particles never exist as particles, they are a bundle of waves from which various particulate waves can be carved. Any supposedly entangled particles are seemingly disentangled by an attempt to verify the entanglement, but that supposition reverses cause and effect. It is not the desire to ascertain the state of the particle that causes it to be disentangled. A particulate wave consisting of a single particle is by its nature a disentangled particle.

This would explain the paradox that Schrödinger illustrated with his 'entangled cat'.

3.2. Further possible local action. If particle events cause observables, then perhaps the corollary is true: where an interaction produces no observables, then the interaction can be described by local action. The force acting on an electron in an electric field is usually conceived as quantised, mediated by photons, but in that case all the electrons in an atom would be maintained as particulate waves, or in the more usual parlance, disentangled. And again, when two entangled electrons fly in different directions, each can be guided by an electromagnetic field, yet the two electrons remain entangled.

3.3. The nature of waves. This suggests to me that although the particle event is the smallest measurable change, the quantum is not the smallest thing we can model. Waves may be more fundamental than quanta.

The nature of the waves raises similar questions. If the waves are not vibrations within a particle, like the vibrations of the material of colliding billiard balls, what is the medium of the vibration? What is being distorted? The obvious albeit wildly speculative possibility is that the medium is space time itself, and particles are just artefacts of particle events. That would solve one of the two mysteries of particles: it does not explain how a particle can behave as a particle, but it would explain how the 'particles'

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behave as waves. And it has the further attraction of possibly unifying gravitational waves with fundamental particles.

4. CONCLUSION

The particle is a consequence of the nature of measurement, since measurement necessarily measures a non-zero volume. The wave is a consequence of the causality that is associated with local action. Perhaps waves are more fundamental than quanta and there may be some waves which are not quantised.

5. Appendix: detecting waves.

I have asserted that only the particulate events are observable, but the reader might object that we can certainly observe the effect of gravity, or of an electric field, both of which I have argued might interact at a point rather than by exchange of quanta. We can certainly observe the effect of a waveform if we understand the waveform well enough to predict how it varies in space and time. However, we observe the effect by observing the motion of a body, and the observables result from particulate events of the body.