Prime Function Complete Proof

Analysis, Infinite Equation, Base 2^n Distributive Property

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Abstract

Function and method for solving the distribution of prime numbers accurately using the combination of step functions, polynomial functions, inverse functions and continuous functions. Equation $\lim_{n\to\infty} p(n) = \{3 + 2(n + x_p) | x_p = x_3 + x_5 + x_7 + x_{11} + \cdots x_p\}$ is true for all integer where $n > 0$ for the distribution and generation of exact values of prime numbers without exception. This formula is efficient by means of modern supercomputers for the task of adding new expression for x_p.

1 Introduction

Prime numbers perhaps are the most interesting group of numbers in real number system due to its unpredictable nature, unusual distribution and complexity. They follow an asymptotic path for positive integers.

2 Definition

Let c be any integer

Let m be any integer ≥ 4

Let n be any integer > 0

Let p be any prime number to be tested for all n

Let n_p as non-prime number

2.1 Range for testing prime p

Integer m shall limit to test within the integer c, whereas c has a range between $2 \leq c \leq \lfloor\sqrt{m}\rfloor$. Prime identities shall equal to $\frac{p}{c} = \left\lfloor\frac{p}{c}\right\rfloor$.

2.2 Properties

All even integer m are n_p.

Any integer m divisible by p are n_p where $p \leq \lfloor \sqrt{m} \rfloor$.

Integer 5 is the first real prime number because of the constraint of $p \leq \lfloor \sqrt{m} \rfloor$, where $\lfloor \sqrt{m} \rfloor \neq \sqrt{m} \wedge \lfloor \sqrt{m} \rfloor > 1$.

3 Function

Prime Function p defined as:

$p(n) = \{3 + 2(n + x_p) | x_p = x_3 + x_5 + x_7 + x_{11} + \cdots x_p\}$ for $n > 0$. This is the most accurate and most strenuous prime producing function as it contains infinite number of terms.

3.1 Function $\boldsymbol{p}$ producing all odd integer:

$p_o = 3 + 2n$

3.2 Function $\boldsymbol{p}$ where $\boldsymbol{p(mod\ \ 3) \neq 0}$:

$p_3 = 3 + 2(n + x_3)$

3.3 Function $\boldsymbol{p}$ where $\boldsymbol{p(mod\ \ 5) \neq 0}$:

$$p_5 = \left\{3 + 2(n + x_3 + x_5) \,\middle|\, a_5 = (n-2)(mod\ \ 8) + 2 \,\middle|\, x_5 = \left\lfloor \frac{a_5 - 1}{7} \right\rfloor + 2\left\lfloor \frac{n-2}{8} \right\rfloor + \left\lfloor \frac{1}{n} \right\rfloor \right\}$$

3.4 Function $\boldsymbol{p}$ where $\boldsymbol{p(mod\ \ 7) \neq 0}$:

$p_7 = \{3 + 2(n + x_3 + x_5 + x_7) | a_5 | x_5 | a_7 | x_7\}$

$\therefore a_5 = (n - 2 + x_7)(mod\ \ 8) + 2$

$\therefore x_5 = \left\lfloor \frac{a_5-1}{7} \right\rfloor + 2\left\lfloor \frac{n-2+x_7}{8} \right\rfloor + \left\lfloor \frac{1}{n+x_7} \right\rfloor$

$\therefore a_5 = (n - 14 + x_7)(mod\ \ 48) + 14$

$$\therefore x_7 = \left\lfloor \frac{a_7-1}{13} \right\rfloor + \left\lfloor \frac{a_7-1}{19} \right\rfloor + \left\lfloor \frac{a_7-1}{22} \right\rfloor - \left\lfloor \frac{a_7-1}{26} \right\rfloor + \left\lfloor \frac{a_7-1}{28} \right\rfloor + \left\lfloor \frac{a_7-1}{31} \right\rfloor + \left\lfloor \frac{a_7-1}{37} \right\rfloor - \left\lfloor \frac{a_7-1}{38} \right\rfloor - \left\lfloor \frac{a_7-1}{39} \right\rfloor - \left\lfloor \frac{a_7-1}{44} \right\rfloor + \left\lfloor \frac{a_7-1}{48} \right\rfloor + \left\lfloor \frac{a_7-1}{50} \right\rfloor - \left\lfloor \frac{a_7-1}{56} \right\rfloor - \left\lfloor \frac{a_7-1}{57} \right\rfloor + 8\left\lfloor \frac{n-14}{48} \right\rfloor + \left\lfloor \frac{(n+2)}{2n} \right\rfloor$$

Where $p(n) = \{n | n \geq 1\}$; step function $p(n)$ is prime for all integers $n = 1, 2, 3, \ldots, 13$. The primes for $n = 1, 2, 3, \ldots, 13$ are $5, 7, 11, 13, \ldots, 47$. Function $p(n)$ has a property of $p(n) \neq \{3n|p \wedge 5n|p \wedge p|p \ldots |n > 1\}$.

4 Initial Analysis $p(mod\ \ 3) \neq 0$

In order to work on prime number function accurately, we need two things. First, by determining their distributive property where $\lfloor \sqrt{p} \rfloor - \sqrt{p} = 0$. Second by expanding its domain n with new expression in terms of n through step function.

5 Infinite Equation and Prime Range Reducer

All prime numbers $p(n)$ follow pattern at all prime value. We can reduce the range by establishing or adding a new constraint for each prime. Let say the function

$\therefore p_5 = \left\{3 + 2(n + x_3 + x_5) \,\middle|\, a_5 = (n-2)(mod \;\; 8) + 2 \,\middle|\, x_5 = \left\lfloor \frac{a_5 - 1}{7} \right\rfloor + 2\left\lfloor \frac{n-2}{8} \right\rfloor + \left\lfloor \frac{1}{n} \right\rfloor \right\}$ is true for all, where $p(mod \;\; 3) \neq 0 \;\wedge\; p(mod \;\; 5) \neq 0$. We can establish a new prime function by changing its constraint $n = n + x_n$ for the equation of

$\therefore p_7 = \left\{3 + 2(n + x_3 + x_5 + x_7) \,\middle|\, a_5 = (n - 2 + x_7)(mod \;\; 8) + 2 \,\middle|\, x_5 = \left\lfloor \frac{a_5 - 1}{7} \right\rfloor + 2\left\lfloor \frac{n-2+x_7}{8} \right\rfloor + \left\lfloor \frac{1}{n + x_7} \right\rfloor \right\}$ where x_7 is an additional constraint for eliminating all numbers of $7|p$ and thus the new equation is $p(mod \;\; 7) \neq 0$.

6 General Equation

$$\lim_{n \to \infty} p(n) = \{3 + 2(n + x_p) | x_p = x_3 + x_5 + x_7 + x_{11} + \cdots x_p\}$$

7 Proof that $p(n)$ followed by logarithmic path pattern for reducing the range

Computed up to $23|p$ due to limited processing power.

$p(n)$	Pattern	Factor	2^n
$p(mod \;\; n) \neq 0$	repeats every		n
5	2	2^1	1
7	8	2^3	3
11	48	$2^4(3)$	5.58496250072116
13	480	$2^5(3)(5)$	8.90689059560852
17	5760	$2^7(3^2)(5)$	12.4918530963297
19	92160	$2^{11}(3^2)(5)$	16.4918530963297
23	1658880	$2^{12}(3^4)(5)$	20.661778097772
29	>90000000	??????	?????24-25

Table 1

8 Proof

8.1 Original Equation p_o:

n	P	p_o	T
	prime numbers	odd integers	TRUE FALSE
		$3+2n$	
1	5	5	TRUE
2	7	7	TRUE
3	11	9	FALSE
4	13	11	FALSE
5	17	13	FALSE

$p_0 = 2n + 3$ is true for $n \leq 2$

8.2 Equation p_3:

n	P	p_o	$3\|p$	p_3	T
	prime numbers	odd integers	position	$3\|p$	TRUE FALSE
	$3+2n$	$n=n$	$\Rightarrow$	$n = n + x_3$	
				$x_3 = \left\lfloor \frac{n-1}{2} \right\rfloor$	
1	5	5		5	TRUE
2	7	7		7	TRUE
3	11	9	1	11	TRUE
4	13	11		13	TRUE
5	17	13		17	TRUE
6	19	15	4	19	TRUE
7	23	17		23	TRUE
8	29	19		25	FALSE
9	31	21	7	29	FALSE
10	37	23		31	FALSE
11	41	25		35	FALSE
12	43	27	10	37	FALSE

$p_3 = \left\{3 + 2n \,\middle|\, n = n + x_3 \,\middle|\, x_3 = \left\lfloor \frac{n-1}{2} \right\rfloor\right\}$ is true for $n \leq 7$; eliminating all $3|p$.

$3|p$ is repeated every 3 interval of $n = \{3, 6, 9, 12, \ldots \infty\}$ for $p_o = \{9, 15, 21, 27, \ldots \infty\}$.

x_3 denotes for this interval.

8.3 Equation $\boldsymbol{p_5}$:

n	P	p_3	$5\|p$	p_5	T
	prime numbers	3|p	position	$5\|p$	TRUE FALSE
	$3+2n$	$n=n+x_3$	$\Rightarrow$	$n=n+x_3+x_5$	
		$x_3=\left\lfloor\frac{n-1}{2}\right\rfloor$	$\Rightarrow$	$x_3=\left\lfloor\frac{n-1+x_5}{2}\right\rfloor$	
				$a_5=(n-2)(mod\ \ 8)+2$	
				$x_5=\left\lfloor\frac{a_5-1}{7}\right\rfloor+2\left\lfloor\frac{n-2}{8}\right\rfloor+\left\lfloor\frac{1}{n}\right\rfloor$	
1	5	5		5	TRUE
2	7	7		7	TRUE
3	11	11		11	TRUE
4	13	13		13	TRUE
5	17	17		17	TRUE
6	19	19		19	TRUE
7	23	23		23	TRUE
8	29	25	1	29	TRUE
11	41	35	4	41	TRUE
13	47	41		47	TRUE
14	53	43		49	FALSE
18	71	55	11	67	FALSE
21	83	65	14	77	FALSE
28	113	85	21	103	FALSE
31	137	95	24	113	FALSE

$p_5=3+2n\,\left|n=n+x_3+x_5\right|x_3=\left\lfloor\frac{n-1+x_5}{2}\right\rfloor\left|x_5=\frac{a_5-1}{7}+2\left\lfloor\frac{n-2}{8}\right\rfloor+\left\lfloor\frac{1}{n}\right\rfloor\right|a_5=(n-2)(mod\ \ 8)+2$ is true for $n\leq 13$; eliminating all $5|p$.

$5|p$ is repeated every $3\wedge 7$ interval of $n=\{\ 8,11,18,21\ldots\infty\ \}$ for $p_o=\{\ 25,35,55,65\ldots\infty\ \}$. See Table 1.

x_5 denotes for this interval.

8.4 Equation $\boldsymbol{p_7}$:

n	P	p_5	$7\|p$	p_7	T
	prime numbers	$5\|p$	position	$7\|p$	TRUE FALSE
	$3+2n$	$n = n + x_3 + x_5$	$\Rightarrow$	$n = n + x_3 + x_5 + x_7$	
		$x_3 = \left\lfloor \frac{n-1+x_5}{2} \right\rfloor$	$\Rightarrow$	$x_3 = \left\lfloor \frac{n-1+x_5+x_7}{2} \right\rfloor$	
		$a_5 = (n-2)(mod \ \ 8) + 2$	$\Rightarrow$	$a_5 = (n-2+x_7)(mod \ \ 8) + 2$	
		$x_5 = \left\lfloor \frac{a_5-1}{7} \right\rfloor + 2\left\lfloor \frac{n-2}{8} \right\rfloor + \left\lfloor \frac{1}{n} \right\rfloor$	$\Rightarrow$	$x_5 = \left\lfloor \frac{a_5-1}{7} \right\rfloor + 2\left\lfloor \frac{n-2+x_7}{8} \right\rfloor + \left\lfloor \frac{1}{n+x_7} \right\rfloor$	
				$a_7 = (n-14)(mod \ \ 48) + 14$	
				$x_7(see \ \ bottom)$	
$x_7 = \left\lfloor \frac{a_7-1}{13} \right\rfloor + \left\lfloor \frac{a_7-1}{19} \right\rfloor + \left\lfloor \frac{a_7-1}{22} \right\rfloor - \left\lfloor \frac{a_7-1}{26} \right\rfloor + \left\lfloor \frac{a_7-1}{28} \right\rfloor + \left\lfloor \frac{a_7-1}{31} \right\rfloor + \left\lfloor \frac{a_7-1}{37} \right\rfloor - \left\lfloor \frac{a_7-1}{38} \right\rfloor - \left\lfloor \frac{a_7-1}{39} \right\rfloor - \left\lfloor \frac{a_7-1}{44} \right\rfloor + \left\lfloor \frac{a_7-1}{48} \right\rfloor + \left\lfloor \frac{a_7-1}{50} \right\rfloor - \left\lfloor \frac{a_7-1}{56} \right\rfloor - \left\lfloor \frac{a_7-1}{57} \right\rfloor + 8\left\lfloor \frac{n-14}{48} \right\rfloor + \left\lfloor \frac{(n+2)}{2n} \right\rfloor$					
1	5	5		5	TRUE
14	53	49	1	53	TRUE
21	83	77	8	83	TRUE
25	103	91	12	103	TRUE
28	113	103		113	TRUE
32	139	119	19	137	FALSE
36	163	133	23	151	FALSE
43	197	161	30	181	FALSE
55	269	203	42	233	FALSE
58	281	217	45	247	FALSE
70	359	259	57	299	FALSE
77	401	287	64	331	FALSE
81	431	301	68	349	FALSE
88	463	329	75	379	FALSE
92	491	343	79	397	FALSE
99	547	371	86	431	FALSE
111	617	413	98	479	FALSE
114	641	427	101	491	FALSE
126	719	469	113	547	FALSE

$p_7 = \left\{3 + 2n \;\middle|\; n = n + x_3 + x_5 + x_7 \left| x_3 = \left\lfloor \frac{n-1+x_5+x_7}{2} \right\rfloor \right| \left| x_5 = \left\lfloor \frac{a_5-1}{7} \right\rfloor + 2\left\lfloor \frac{n-2+x_7}{8} \right\rfloor + \left\lfloor \frac{1}{n+x_7} \right\rfloor \right| a_5 = (n-2+x_7)(mod\;\; 8) + 2 \;\; |a_7| x_7 \right\}$ is true for $n \leq 28$ eliminating all $7|p$.
$7|p$ is repeated every $7 \wedge 4 \wedge 7 \wedge 4 \wedge 7 \wedge 12 \wedge 3 \wedge 12$. See Table 1.
x_7 denotes for this interval.

9 Software Programs

Programs use for calculating the number of interval repetition.

9.1 Visual Studio C#:

```
public static long equation(long x)
{
      //Accuracy up to p=47
       long a, a1, a2;
       double b;
       long y;

       a1 = ((x - 2) % 8) / 6;
       a2 = (x - 2) / 8;
       a2 = 2 * a2;
       a = a1 + a2;

       b = (double)(x + a - 1) / 2;

       y = Math.Abs((int)(4 * b) + 5 + 2 * (int)b);

       return y;
}


private void PrimeCompute()
{
       long primeRem, primeDiv;
       long[] primesToBeRemoved = new long[100];
       long iNumPrimeRem, iRowIncreaser = 0;
       long primeTrial;
       long iPattern = 0, iNumPattern = 0;
       long iPrime;
       long maxTry;
       long[] iDiffArray = new long[50000000];
       long[] iPatternArray = new long[50000000];
       long[] pat = new long[25];

       primeDiv = 29;
       maxTry = 90000000;

       //Calculation
       long i;
       iNumPrimeRem = 1;

       do
       {
              iNumPrimeRem++;
```

```
        if (iNumPrimeRem > 1)
        {
                i = Methods.equation(iNumPrimeRem);
                primesToBeRemoved[iNumPrimeRem - 1] = Methods.equation(iNumPrimeRem);
        }
}
while (Methods.equation(iNumPrimeRem + 1) < primeDiv);

primeRem = Methods.equation(iNumPrimeRem);
iNumPrimeRem = iNumPrimeRem - 1;
iPrime = iNumPrimeRem + 2;

do
{
label1:

        primeTrial = Methods.equation(iPrime + iRowIncreaser);

        for (i = 1; i <= iNumPrimeRem; i++)
        {
                if (primeTrial % primesToBeRemoved[i] == 0)
                {
                        iRowIncreaser++;
                        goto label1;
                }

                if (i == iNumPrimeRem)
                {
                        iPrime++;
                        iPattern++;
                        if (primeTrial != primeDiv & primeTrial % primeDiv == 0)
                        {
                                if (iDiffArray[1] == 0)
                                {
                                        iPattern = 1;
                                        iNumPattern = 1;
                                }
                                else
                                { iNumPattern++; }
                                iDiffArray[iNumPattern] = iPattern;
                        }
                }
        }
}
while (iNumPattern < maxTry);

//Difference
for (i = 1; i <= iNumPattern - 1; i++)
{ iPatternArray[i] = iDiffArray[i + 1] - iDiffArray[i]; }

//Pattern
int n;
long j = 0;

n = pat.Count() - 1;

for (i = 1; i <= n; i++)
{ pat[i] = iPatternArray[i]; }
```

```
        i = 0;

        do
        {
            i++;
            j++;

            if (pat[i] != iPatternArray[j + n])
            { i = 0; }
        } while (i < n);

        MessageBox.Show(j.ToString());
        Application.Exit();
    }
```

9.2 Visual Basic

```
Function equation(x As Long) As Double

Dim a As Long, b As Double

a = Int(((x - 2) Mod 8) / 6) + 2 * Int((x - 2) / 8)

b = (x + a - 1) / 2

equation = Abs(4 * b + 5 + 2 * Int(b))

End Function


Sub primeEquationAndRemovalProg()

Dim primeRem As Long, primeDiv As Long

Dim primesToBeRemoved(1 To 10) As Long

Dim iNumPrimeRem As Long, i As Long, iRowIncreaser As Long

Dim primeTrial As Long

Dim iPattern As Long, iNumPattern As Long

Dim iPrime As Long

Dim maxTry As Long

Dim iDiffArray(1 To 20000000) As Long, iPatternArray(1 To 20000000) As Long

Dim pat(1 To 25) As Integer


primeRem = 23

primeDiv = 29

maxTry = 90000000
```

```
iNumPrimeRem = 1
Do Until equation(iNumPrimeRem) = primeRem
iNumPrimeRem = iNumPrimeRem + 1
    If iNumPrimeRem > 1 Then
    primesToBeRemoved(iNumPrimeRem - 1) = equation(iNumPrimeRem)
    End If
Loop

iNumPrimeRem = iNumPrimeRem - 1
iPrime = iNumPrimeRem + 2

Do Until iNumPattern = maxTry
label1:
primeTrial = equation(iPrime + iRowIncreaser)

    For i = 1 To iNumPrimeRem
    If primeTrial Mod primesToBeRemoved(i) = 0 Then
    iRowIncreaser = iRowIncreaser + 1
    GoTo label1
    End If

    If i = iNumPrimeRem Then
    iPrime = iPrime + 1
    iPattern = iPattern + 1

        If primeTrial <> primeDiv And primeTrial Mod primeDiv = 0 Then
            If iDiffArray(1) = 0 Then
            iPattern = 1
            iNumPattern = 1
            Else
            iNumPattern = iNumPattern + 1
```

```
                End If
            iDiffArray(iNumPattern) = iPattern
            End If
        End If
        Next
Loop

'Difference
For i = 1 To (iNumPattern - 1)
iPatternArray(i) = iDiffArray(i + 1) - iDiffArray(i)
Next

'Pattern
Dim n As Integer, j As Long

n = UBound(pat) - LBound(pat) + 1

For i = 1 To n
pat(i) = iPatternArray(i)
Next

i = 0

Do Until i = n

i = i + 1
j = j + 1

If pat(i) <> iPatternArray(j + n) Then
i = 0
End If
```

```
Loop

MsgBox (j)

End Sub
```

10 Importance and Practical Application

10.1 For the development of quantum computers.

10.2 For attacking the 150-year-old Riemann Hypothesis in the field of pure mathematics.

11 Conclusion

This prime producing function is the most accurate function as it produces prime numbers accurately without exception. As the accuracy increases, also with the number of terms. If we calculate $p(mod \quad 11) \neq 0$, we can reach the first 100th terms; $p(mod \quad 13) \neq 0$ for the 1000th terms; $p(mod \quad 23) \neq 0$ for the first 1 millionth; $p(mod \quad 31) \neq 0$ for 1 billionth; so on and so forth. Due to unpredictable nature of primes, it is hereby concluded that the equation of prime has limitless number of terms as it is being proven. I will state a conjecture; it will be named **"Prime Function Conjecture"**, it is state that there's no such prime producing function exist with limited number of terms.

References

1. https://en.wikipedia.org/wiki/Prime_number_theorem#Statement_of_the_theorem
2. https://en.wikipedia.org/wiki/Formula_for_primes
3. https://en.wikipedia.org/wiki/Prime_number