Research Project Primus

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1 Theorems and Conjectures

Theorem 1.1. *A natural number $n > 2$ is a prime iff* $\prod_{k=1}^{n-1} k \equiv n-1 \pmod{\sum_{k=1}^{n-1} k}$.

Theorem 1.2. *Let $p \equiv 5 \pmod 6$ be prime then , $2p+1$ is prime iff $2p+1 \mid 3^p - 1$.*

Theorem 1.3. *Let p_n be the nth prime , then*

$$p_n = 1 + \sum_{k=1}^{2\cdot(\lfloor n\ln(n)\rfloor+1)} \left(1 - \left\lfloor \frac{1}{n} \cdot \sum_{j=2}^{k} \left\lceil \frac{3 - \sum_{i=1}^{j} \left\lfloor \frac{\lfloor \frac{j}{i} \rfloor}{\lceil \frac{j}{i} \rceil} \right\rfloor}{j} \right\rceil \right\rfloor \right)$$

Theorem 1.4. *Let $P_j(x) = 2^{-j} \cdot \left(\left(x - \sqrt{x^2-4}\right)^j + \left(x + \sqrt{x^2-4}\right)^j\right)$, where j and x are nonnegative integers . Let $N = k \cdot 2^m - 1$ such that $m > 2$, $3 \mid k$, $0 < k < 2^m$ and*

$$\begin{cases} k \equiv 1 \pmod{10} \; with \; m \equiv 2,3 \pmod 4 \\ k \equiv 3 \pmod{10} \; with \; m \equiv 0,3 \pmod 4 \\ k \equiv 7 \pmod{10} \; with \; m \equiv 1,2 \pmod 4 \\ k \equiv 9 \pmod{10} \; with \; m \equiv 0,1 \pmod 4 \end{cases}$$

Let $S_i = S_{i-1}^2 - 2$ with $S_0 = P_k(3)$, then N is prime iff $S_{m-2} \equiv 0 \pmod N$

Theorem 1.5. *Let $P_j(x) = 2^{-j} \cdot \left(\left(x - \sqrt{x^2-4}\right)^j + \left(x + \sqrt{x^2-4}\right)^j\right)$, where j and x are nonnegative integers . Let $N = k \cdot 2^m - 1$ such that $m > 2$, $3 \mid k$, $0 < k < 2^m$ and*

$$\begin{cases} k \equiv 3 \pmod{42} \; with \; m \equiv 0,2 \pmod 3 \\ k \equiv 9 \pmod{42} \; with \; m \equiv 0 \pmod 3 \\ k \equiv 15 \pmod{42} \; with \; m \equiv 1 \pmod 3 \\ k \equiv 27 \pmod{42} \; with \; m \equiv 1,2 \pmod 3 \\ k \equiv 33 \pmod{42} \; with \; m \equiv 0,1 \pmod 3 \\ k \equiv 39 \pmod{42} \; with \; m \equiv 2 \pmod 3 \end{cases}$$

Let $S_i = S_{i-1}^2 - 2$ with $S_0 = P_k(5)$, then N is prime iff $S_{m-2} \equiv 0 \pmod N$

Theorem 1.6. *Let* $P_j(x) = 2^{-j} \cdot \left(\left(x-\sqrt{x^2-4}\right)^j + \left(x+\sqrt{x^2-4}\right)^j\right)$ *, where* j *and* x *are nonnegative integers . Let* $N = k \cdot 2^m + 1$ *such that* $m > 2$ *,* $0 < k < 2^m$ *and*

$$\begin{cases} k \equiv 1 \pmod{42} \text{ with } m \equiv 2,4 \pmod{6} \\ k \equiv 5 \pmod{42} \text{ with } m \equiv 3 \pmod{6} \\ k \equiv 11 \pmod{42} \text{ with } m \equiv 3,5 \pmod{6} \\ k \equiv 13 \pmod{42} \text{ with } m \equiv 4 \pmod{6} \\ k \equiv 17 \pmod{42} \text{ with } m \equiv 5 \pmod{6} \\ k \equiv 19 \pmod{42} \text{ with } m \equiv 0 \pmod{6} \\ k \equiv 23 \pmod{42} \text{ with } m \equiv 1,3 \pmod{6} \\ k \equiv 25 \pmod{42} \text{ with } m \equiv 0,2 \pmod{6} \\ k \equiv 29 \pmod{42} \text{ with } m \equiv 1,5 \pmod{6} \\ k \equiv 31 \pmod{42} \text{ with } m \equiv 2 \pmod{6} \\ k \equiv 37 \pmod{42} \text{ with } m \equiv 0,4 \pmod{6} \\ k \equiv 41 \pmod{42} \text{ with } m \equiv 1 \pmod{6} \end{cases}$$

Let $S_i = S_{i-1}^2 - 2$ *with* $S_0 = P_k(5)$ *, then* N *is prime iff* $S_{m-2} \equiv 0 \pmod{N}$ *.*

Theorem 1.7. *Let* $P_j(x) = 2^{-j} \cdot \left(\left(x-\sqrt{x^2-4}\right)^j + \left(x+\sqrt{x^2-4}\right)^j\right)$ *, where* j *and* x *are nonnegative integers . Let* $N = k \cdot 2^m + 1$ *such that* $m > 2$ *,* $0 < k < 2^m$ *and*

$$\begin{cases} k \equiv 1 \pmod{6} \text{ and } k \equiv 1,7 \pmod{10} \text{ with } m \equiv 0 \pmod{4} \\ k \equiv 5 \pmod{6} \text{ and } k \equiv 1,3 \pmod{10} \text{ with } m \equiv 1 \pmod{4} \\ k \equiv 1 \pmod{6} \text{ and } k \equiv 3,9 \pmod{10} \text{ with } m \equiv 2 \pmod{4} \\ k \equiv 5 \pmod{6} \text{ and } k \equiv 7,9 \pmod{10} \text{ with } m \equiv 3 \pmod{4} \end{cases}$$

Let $S_i = S_{i-1}^2 - 2$ *with* $S_0 = P_k(8)$ *, then* N *is prime iff* $S_{m-2} \equiv 0 \pmod{N}$ *.*

Theorem 1.8. *Let* $N = k \cdot 2^n + 1$ *with* $n > 1$ *,* k *is odd ,* $0 < k < 2^n$ *,* $3 \mid k$ *and*

$$\begin{cases} k \equiv 3 \pmod{30}, & \text{with } n \equiv 1,2 \pmod{4} \\ k \equiv 9 \pmod{30}, & \text{with } n \equiv 2,3 \pmod{4} \\ k \equiv 21 \pmod{30}, & \text{with } n \equiv 0,1 \pmod{4} \\ k \equiv 27 \pmod{30}, & \text{with } n \equiv 0,3 \pmod{4} \end{cases}$$

then N *is prime iff* $5^{\frac{N-1}{2}} \equiv -1 \pmod{N}$ *.*

Theorem 1.9. *Let* $N = k \cdot 2^n + 1$ *with* $n > 1$ *,* k *is odd ,* $0 < k < 2^n$ *,* $3 \mid k$ *and*

$$\begin{cases} k \equiv 3 \pmod{42}, & \text{with } n \equiv 2 \pmod{3} \\ k \equiv 9 \pmod{42}, & \text{with } n \equiv 0,1 \pmod{3} \\ k \equiv 15 \pmod{42}, & \text{with } n \equiv 1,2 \pmod{3} \\ k \equiv 27 \pmod{42}, & \text{with } n \equiv 1 \pmod{3} \\ k \equiv 33 \pmod{42}, & \text{with } n \equiv 0 \pmod{3} \\ k \equiv 39 \pmod{42}, & \text{with } n \equiv 0,2 \pmod{3} \end{cases}$$

then N *is prime iff* $7^{\frac{N-1}{2}} \equiv -1 \pmod{N}$ *.*

Theorem 1.10. *Let* $N = k \cdot 2^n + 1$ *with* $n > 1$ *,* k *is odd ,* $0 < k < 2^n$ *,* $3 \mid k$ *and*

$$\begin{cases} k \equiv 3 \pmod{66}, & \textit{with } n \equiv 1,2,6,8,9 \pmod{10} \\ k \equiv 9 \pmod{66}, & \textit{with } n \equiv 0,1,3,4,8 \pmod{10} \\ k \equiv 15 \pmod{66}, & \textit{with } n \equiv 2,4,5,7,8 \pmod{10} \\ k \equiv 21 \pmod{66}, & \textit{with } n \equiv 1,2,4,5,9 \pmod{10} \\ k \equiv 27 \pmod{66}, & \textit{with } n \equiv 0,2,3,5,6 \pmod{10} \\ k \equiv 39 \pmod{66}, & \textit{with } n \equiv 0,1,5,7,8 \pmod{10} \\ k \equiv 45 \pmod{66}, & \textit{with } n \equiv 0,4,6,7,9 \pmod{10} \\ k \equiv 51 \pmod{66}, & \textit{with } n \equiv 0,2,3,7,9 \pmod{10} \\ k \equiv 57 \pmod{66}, & \textit{with } n \equiv 3,5,6,8,9 \pmod{10} \\ k \equiv 63 \pmod{66}, & \textit{with } n \equiv 1,3,4,6,7 \pmod{10} \end{cases}$$

then N *is prime iff* $11^{\frac{N-1}{2}} \equiv -1 \pmod{N}$ *.*

Theorem 1.11. *A positive integer* n *is prime iff* $\varphi(n)! \equiv -1 \pmod{n}$

Theorem 1.12. *For* $m \geq 1$ *number* n *greater than one is prime iff*

$$(n^m - 1)! \equiv (n-1)^{\left\lceil \frac{(-1)^{m+1}}{2} \right\rceil} \cdot n^{\frac{n^m - mn + m - 1}{n-1}} \pmod{n^{\frac{n^m - mn + m + n - 2}{n-1}}}$$

Theorem 1.13. *Sequence* S_i *is defined as* $S_i = \begin{cases} 8 & \textit{if } i = 0; \\ (S_{i-1}^2 - 2)^2 - 2 & \textit{otherwise}. \end{cases}$ *then ,* $F_n = 2^{2^n} + 1, (n \geq 2)$ *is a prime if and only if* F_n *divides* $S_{2^{n-1}-1}$ *.*

Theorem 1.14. *Let* $p \equiv 1 \pmod{6}$ *be prime and let* $5 \nmid 4p + 1$ *, then* $4p + 1$ *is prime iff* $4p + 1 \mid 2^{2p} + 1$ *.*

Theorem 1.15. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2 - 4}\right)^m + \left(x + \sqrt{x^2 - 4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $F_n(b) = b^{2^n} + 1$ *such that* $n \geq 2$ *and* b *is even number . Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_b(6)$*, thus If* $F_n(b)$ *is prime, then* $S_{2^n - 1} \equiv 2 \pmod{F_n(b)}$*.*

Theorem 1.16. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2 - 4}\right)^m + \left(x + \sqrt{x^2 - 4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $E_n(b) = \frac{b^{2^n} + 1}{2}$ *such that* $n > 1$*,* b *is odd number greater than one . Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_b(6)$*, thus If* $E_n(b)$ *is prime, then* $S_{2^n - 1} \equiv 6 \pmod{E_n(b)}$*.*

Theorem 1.17. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2 - 4}\right)^m + \left(x + \sqrt{x^2 - 4}\right)^m \right)$ *.*

Let $N_p(b) = \frac{b^p + 1}{b + 1}$ *, where* p *is an odd prime and* b *is an odd natural number greater than one .*

CASE(1). $b \equiv 1, 9 \pmod{12}$ *, or* $b \equiv 3, 7 \pmod{12}$ *and* $p \equiv 1 \pmod{4}$ *, or* $b \equiv 5 \pmod{12}$ *and* $p \equiv 1, 7 \pmod{12}$ *, or* $b \equiv 11 \pmod{12}$ *and* $p \equiv 1, 11 \pmod{12}$ *.*

CASE(2). $b \equiv 3, 7 \pmod{12}$ *and* $p \equiv 3 \pmod{4}$ *, or* $b \equiv 5 \pmod{12}$ *and* $p \equiv 5, 11 \pmod{12}$ *, or* $b \equiv 11 \pmod{12}$ *and* $p \equiv 5, 7 \pmod{12}$ *.*

Let $S_i = P_b(S_{i-1})$ *with* $S_0 = P_b(4)$ *. Suppose* $N_p(b)$ *is prime , then :*

- $S_{p-1} \equiv P_b(4) \pmod{N_p(b)}$ *if Case(1) holds ;*
- $S_{p-1} \equiv P_{b+2}(4) \pmod{N_p(b)}$ *if Case(2) holds ;*

Theorem 1.18. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$.

Let $M_p(a) = \frac{a^p-1}{a-1}$ *, where* p *is an odd prime and* a *is an odd natural number greater than one*

CASE(1). $a \equiv 3, 11 \pmod{12}$ *, or* $a \equiv 5, 9 \pmod{12}$ *and* $p \equiv 1 \pmod{4}$ *, or* $a \equiv 7 \pmod{12}$ *and* $p \equiv 1, 7 \pmod{12}$ *, or* $a \equiv 1 \pmod{12}$ *and* $p \equiv 1, 11 \pmod{12}$ *.*

CASE(2). $a \equiv 5, 9 \pmod{12}$ *and* $p \equiv 3 \pmod{4}$ *, or* $a \equiv 7 \pmod{12}$ *and* $p \equiv 5, 11 \pmod{12}$ *, or* $a \equiv 1 \pmod{12}$ *and* $p \equiv 5, 7 \pmod{12}$ *.*

Let $S_i = P_a(S_{i-1})$ *with* $S_0 = P_a(4)$ *. Suppose* $M_p(a)$ *is prime , then :*

- $S_{p-1} \equiv P_a(4) \pmod{M_p(a)}$ *if Case(1) holds ;*
- $S_{p-1} \equiv P_{a-2}(4) \pmod{M_p(a)}$ *if Case(2) holds ;*

Conjecture 1.1. *Let* $b_n = b_{n-2} + lcm(n-1, b_{n-2})$ *with* $b_1 = 2$ *,* $b_2 = 2$ *and* $n > 2$ *. Let* $a_n = b_{n+2}/b_n - 1$ *, then*

1. Every term of this sequence a_i *is either prime or* 1 *. 2. Every odd prime number is member of this sequence . 3. Every new prime in sequence is a next prime from the largest prime already listed .*

Conjecture 1.2. *Let* $b_n = b_{n-1} + lcm(\lfloor \sqrt{n^3} \rfloor, b_{n-1})$ *with* $b_1 = 2$ *and* $n > 1$ *. Let* $a_n = b_{n+1}/b_n - 1$ *, then*

1. Every term of this sequence a_i *is either prime or* 1 *. 2. Every odd prime of the form* $\lfloor \sqrt{n^3} \rfloor$ *is member of this sequence . 3. Every new prime of the form* $\lfloor \sqrt{n^3} \rfloor$ *in sequence is a next prime from the largest prime already listed .*

Conjecture 1.3. *Let* $b_n = b_{n-1} + lcm(\lfloor \sqrt{2} \cdot n \rfloor, b_{n-1})$ *with* $b_1 = 2$ *and* $n > 1$ *. Let* $a_n = b_{n+1}/b_n - 1$ *, then*

1. Every term of this sequence a_i *is either prime or* 1 *. 2. Every prime of the form* $\lfloor \sqrt{2} \cdot n \rfloor$ *is member of this sequence . 3. Every new prime of the form* $\lfloor \sqrt{2} \cdot n \rfloor$ *in sequence is a next prime from the largest prime already listed .*

Conjecture 1.4. *Let* $b_n = b_{n-1} + lcm(\lfloor \sqrt{3} \cdot n \rfloor, b_{n-1})$ *with* $b_1 = 3$ *and* $n > 1$ *. Let* $a_n = b_{n+1}/b_n - 1$ *, then*

1. Every term of this sequence a_i *is either prime or* 1 *. 2. Every prime of the form* $\lfloor \sqrt{3} \cdot n \rfloor$ *is member of this sequence . 3. Every new prime of the form* $\lfloor \sqrt{3} \cdot n \rfloor$ *in sequence is a next prime from the largest prime already listed .*

Conjecture 1.5. *Let* b *and* n *be a natural numbers ,* $b \geq 2$ *,* $n > 2$ *and* $n \neq 9$ *. Then* n *is prime if and only if* $\sum\limits_{k=1}^{n-1} \left(b^k - 1\right)^{n-1} \equiv n \pmod{\frac{b^n-1}{b-1}}$

Conjecture 1.6. *Let* a *,* b *and* n *be a natural numbers ,* $b > a > 1$ *,* $n > 2$ *and* $n \notin \{4, 9, 25\}$ *. Then* n *is prime iff* $\prod\limits_{k=1}^{n-1} \left(b^k - a\right) \equiv \frac{a^n-1}{a-1} \pmod{\frac{b^n-1}{b-1}}$

Conjecture 1.7. *Let* a *,* b *and* n *be a natural numbers ,* $b > a > 0$ *,* $n > 2$ *and* $n \notin \{4, 9, 25\}$ *. Then* n *is prime iff* $\prod\limits_{k=1}^{n-1} \left(b^k + a\right) \equiv \frac{a^n+1}{a+1} \pmod{\frac{b^n-1}{b-1}}$

Conjecture 1.8. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$*, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - 1$ *such that* $k > 0$ *,* $3 \nmid k$ *,* $k<2^n$ *,* $b > 0$ *,* b *is even number,* $3 \nmid b$ *and* $n > 2$ *. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{kb/2}(P_{b/2}(4))$ *, then* N *is prime iff* $S_{n-2} \equiv 0 \pmod{N}$ *.*

Conjecture 1.9. *Let* $P_j(x) = 2^{-j} \cdot \left(\left(x - \sqrt{x^2-4}\right)^j + \left(x + \sqrt{x^2-4}\right)^j \right)$*, where* j *and* x *are nonnegative integers. Let* $N = k \cdot 2^m + 1$ *with* k *odd ,* $0<k<2^m$ *and* $m > 2$ *. Let* F_n *be the nth Fibonacci number and let* $S_i = S_{i-1}^2 - 2$ *with* $S_0 = P_k(F_n)$ *, then* N *is prime iff there exists* F_n *for which* $S_{m-2} \equiv 0 \pmod{N}$ *.*

Conjecture 1.10. *Let* $P_j(x) = 2^{-j} \cdot \left(\left(x - \sqrt{x^2-4}\right)^j + \left(x + \sqrt{x^2-4}\right)^j \right)$*, where* j *and* x *are nonnegative integers. Let* $F_m(b) = b^{2^m} + 1$ *with* b *even ,* $b > 0$ *and* $m \geq 2$ *. Let* F_n *be the nth Fibonacci number and let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(P_{b/2}(F_n))$ *, then* $F_m(b)$ *is prime iff there exists* F_n *for which* $S_{m-2} \equiv 0 \pmod{F_m(b)}$ *.*

Conjecture 1.11. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n - b - 1$ *such that* $n > 2$*,* $b \equiv 0, 6 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv P_{(b+2)/2}(6) \pmod{N}$*.*

Conjecture 1.12. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n - b - 1$ *such that* $n > 2$*,* $b \equiv 2, 4 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv -P_{b/2}(6) \pmod{N}$*.*

Conjecture 1.13. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n + b + 1$ *such that* $n > 2$*,* $b \equiv 0, 6 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv P_{b/2}(6) \pmod{N}$*.*

Conjecture 1.14. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n + b + 1$ *such that* $n > 2$*,* $b \equiv 2, 4 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv -P_{(b+2)/2}(6) \pmod{N}$*.*

Conjecture 1.15. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n - b + 1$ *such that* $n > 3$*,* $b \equiv 0, 2 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv P_{b/2}(6) \pmod{N}$*.*

Conjecture 1.16. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n - b + 1$ *such that* $n > 3$*,* $b \equiv 4, 6 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv -P_{(b-2)/2}(6) \pmod{N}$*.*

Conjecture 1.17. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n + b - 1$ *such that* $n > 3$*,* $b \equiv 0, 2 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv P_{(b-2)/2}(6) \pmod{N}$*.*

Conjecture 1.18. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n + b - 1$ *such that* $n > 3$*,* $b \equiv 4, 6 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv -P_{b/2}(6) \pmod{N}$*.*

Conjecture 1.19. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$

Let $N = k \cdot 3^n - 2$ *such that* $n > 3$ *,* $k \equiv 1,3 \pmod 8$ *and* $k > 0$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{3k}(6)$ *, thus If* N *is prime then* $S_{n-1} \equiv P_3(6) \pmod N$

Conjecture 1.20. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$

Let $N = k \cdot 3^n - 2$ *such that* $n > 3$ *,* $k \equiv 5,7 \pmod 8$ *and* $k > 0$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{3k}(6)$ *, thus If* N *is prime then* $S_{n-1} \equiv P_1(6) \pmod N$

Conjecture 1.21. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot 3^n + 2$ *such that* $n > 2$ *,* $k \equiv 1,3 \pmod 8$ *and* $k > 0$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{3k}(6)$ *, thus If* N *is prime then* $S_{n-1} \equiv P_3(6) \pmod N$

Conjecture 1.22. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot 3^n + 2$ *such that* $n > 2$ *,* $k \equiv 5,7 \pmod 8$ *and* $k > 0$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{3k}(6)$ *, thus If* N *is prime then* $S_{n-1} \equiv P_1(6) \pmod N$

Conjecture 1.23. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - c$ *such that* $b \equiv 0 \pmod 2), n > bc, k > 0, c > 0$ *and* $c \equiv 1,7 \pmod 8$ *Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6))$*, thus If* N *is prime then* $S_{n-1} \equiv P_{(b/2)\cdot\lceil c/2 \rceil}(6) \pmod N$

Conjecture 1.24. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - c$ *such that* $b \equiv 0,4,8 \pmod{12}, n > bc, k > 0, c > 0$ *and* $c \equiv 3,5 \pmod 8$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6))$*, thus If* N *is prime then* $S_{n-1} \equiv P_{(b/2)\cdot\lfloor c/2 \rfloor}(6) \pmod N$

Conjecture 1.25. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - c$ *such that* $b \equiv 2,6,10 \pmod{12}, n > bc, k > 0, c > 0$ *and* $c \equiv 3,5 \pmod 8$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6))$*, thus If* N *is prime then* $S_{n-1} \equiv -P_{(b/2)\cdot\lfloor c/2 \rfloor}(6) \pmod N$

Conjecture 1.26. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n + c$ *such that* $b \equiv 0 \pmod 2), n > bc, k > 0, c > 0$ *and* $c \equiv 1,7 \pmod 8$ *Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6))$*, thus If* N *is prime then* $S_{n-1} \equiv P_{(b/2)\cdot\lfloor c/2 \rfloor}(6) \pmod N$

Conjecture 1.27. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n + c$ *such that* $b \equiv 0,4,8 \pmod{12}, n > bc, k > 0, c > 0$ *and* $c \equiv 3,5 \pmod 8$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6))$*, thus If* N *is prime then* $S_{n-1} \equiv P_{(b/2)\cdot\lceil c/2 \rceil}(6) \pmod N$

Conjecture 1.28. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n + c$ *such that* $b \equiv 2,6,10 \pmod{12}, n > bc, k > 0, c > 0$ *and* $c \equiv 3,5 \pmod 8$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6))$*, thus If* N *is prime then* $S_{n-1} \equiv -P_{(b/2)\cdot\lceil c/2 \rceil}(6) \pmod N$

Conjecture 1.29. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = 2 \cdot 3^n - 1$ *such that* $n > 1$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_3(a)$ *, where* $a = \begin{cases} 6, & \text{if } n \equiv 0 \pmod{2} \\ 8, & \text{if } n \equiv 1 \pmod{2} \end{cases}$ *thus ,* N *is prime iff* $S_{n-1} \equiv a \pmod{N}$

Conjecture 1.30. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = 8 \cdot 3^n - 1$ *such that* $n > 1$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{12}(4)$ *thus ,* N *is prime iff* $S_{n-1} \equiv 4 \pmod{N}$

Conjecture 1.31. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot 6^n - 1$ *such that* $n > 2$ *,* $k > 0$ *,* $k \equiv 2, 5 \pmod{7}$ *and* $k < 6^n$ *Let* $S_i = P_6(S_{i-1})$ *with* $S_0 = P_{3k}(P_3(5))$ *, thus* N *is prime iff* $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 1.32. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot 6^n - 1$ *such that* $n > 2$ *,* $k > 0$ *,* $k \equiv 3, 4 \pmod{5}$ *and* $k < 6^n$ *Let* $S_i = P_6(S_{i-1})$ *with* $S_0 = P_{3k}(P_3(3))$ *, thus* N *is prime iff* $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 1.33. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - 1$ *such that* $n > 2$ *,* $k < 2^n$ *and*

$$\begin{cases} k \equiv 3 \pmod{30} \textit{ with } b \equiv 2 \pmod{10} \textit{ and } n \equiv 0, 3 \pmod{4} \\ k \equiv 3 \pmod{30} \textit{ with } b \equiv 4 \pmod{10} \textit{ and } n \equiv 0, 2 \pmod{4} \\ k \equiv 3 \pmod{30} \textit{ with } b \equiv 6 \pmod{10} \textit{ and } n \equiv 0, 1, 2, 3 \pmod{4} \\ k \equiv 3 \pmod{30} \textit{ with } b \equiv 8 \pmod{10} \textit{ and } n \equiv 0, 1 \pmod{4} \end{cases}$$

Let $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(18))$ *, then* N *is prime iff* $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 1.34. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - 1$ *such that* $n > 2$ *,* $k < 2^n$ *and*

$$\begin{cases} k \equiv 9 \pmod{30} \textit{ with } b \equiv 2 \pmod{10} \textit{ and } n \equiv 0, 1 \pmod{4} \\ k \equiv 9 \pmod{30} \textit{ with } b \equiv 4 \pmod{10} \textit{ and } n \equiv 0, 2 \pmod{4} \\ k \equiv 9 \pmod{30} \textit{ with } b \equiv 6 \pmod{10} \textit{ and } n \equiv 0, 1, 2, 3 \pmod{4} \\ k \equiv 9 \pmod{30} \textit{ with } b \equiv 8 \pmod{10} \textit{ and } n \equiv 0, 3 \pmod{4} \end{cases}$$

Let $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(18))$ *, then* N *is prime iff* $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 1.35. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - 1$ *such that* $n > 2$ *,* $k < 2^n$ *and*

$$\begin{cases} k \equiv 21 \pmod{30} \textit{ with } b \equiv 2 \pmod{10} \textit{ and } n \equiv 2, 3 \pmod{4} \\ k \equiv 21 \pmod{30} \textit{ with } b \equiv 4 \pmod{10} \textit{ and } n \equiv 1, 3 \pmod{4} \\ k \equiv 21 \pmod{30} \textit{ with } b \equiv 8 \pmod{10} \textit{ and } n \equiv 1, 2 \pmod{4} \end{cases}$$

Let $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(3))$ *, then* N *is prime iff* $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 1.36. *Let* F_p *be the pth Fibonacci number .If* p *is prime , not* 5 *, and* $M \geq 2$ *then* $M^{F_p} \equiv M^{(p-1)^{(1-(\frac{p}{5}))/2}} \pmod{\frac{M^p-1}{M-1}}$

Conjecture 1.37. *Let b and n be a natural numbers , $b \geq 2$, $n > 1$ and $n \notin \{4, 8, 9\}$. Then n is prime if and only if* $\sum\limits_{k=1}^{n} \left(b^k + 1\right)^{n-1} \equiv n \pmod{\dfrac{b^n - 1}{b - 1}}$

Conjecture 1.38. *If q is the smallest prime greater than $\prod\limits_{i=1}^{n} C_i + 1$, where $\prod\limits_{i=1}^{n} C_i$ is the product of the first n composite numbers , then $q - \prod\limits_{i=1}^{n} C_i$ is prime .*

Conjecture 1.39. *If q is the greatest prime less than $\prod\limits_{i=1}^{n} C_i - 1$, where $\prod\limits_{i=1}^{n} C_i$ is the product of the first n composite numbers , then $\prod\limits_{i=1}^{n} C_i - q$ is prime .*

Conjecture 1.40. *Let n be an odd number and $n > 1$. Let $T_n(x)$ be Chebyshev polynomial of the first kind and let $P_n(x)$ be Legendre polynomial , then n is a prime number if and only if the following congruences hold simultaneously • $T_n(3) \equiv 3 \pmod{n}$ • $P_n(3) \equiv 3 \pmod{n}$*

Conjecture 1.41. *Let n be a natural number greater than two . Let r be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$. Let $T_n(x)$ be Chebyshev polynomial of the first kind , then n is a prime number if and only if $T_n(x) \equiv x^n \pmod{x^r - 1, n}$.*

Conjecture 1.42. *Let n be a natural number greater than two and $n \neq 5$. Let $T_n(x)$ be Chebyshev polynomial of the first kind . If there exists an integer a , $1 < a < n$, such that $T_{n-1}(a) \equiv 1 \pmod{n}$ and for every prime factor q of $n - 1$, $T_{(n-1)/q}(a) \not\equiv 1 \pmod{n}$ then n is prime . If no such number a exists then n is composite .*

Conjecture 1.43. *Let $P_a(x) = 2^{-a} \cdot \left(\left(x - \sqrt{x^2 - 4}\right)^a + \left(x + \sqrt{x^2 - 4}\right)^a\right)$. Let $N = k \cdot b^m \pm 1$ with b an even positive integer , $0<k<b^m$ and $m > 2$. Let F_n be the nth Fibonacci number and let $S_i = P_b(S_{i-1})$ with $S_0 = P_{kb/2}(P_{b/2}(F_n))$, then N is prime iff there exists F_n for which $S_{m-2} \equiv 0 \pmod{N}$.*

Conjecture 1.44. *Let n be a natural number greater than one . Let r be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$. Let $L_n(x)$ be Lucas polynomial , then n is a prime number if and only if $L_n(x) \equiv x^n \pmod{x^r - 1, n}$.*

Conjecture 1.45. *Let b and n be a natural numbers , $b \geq 2$, then $\frac{b^n-1}{b-1} \cdot \frac{b^{\sigma(n)}-1}{b-1} \equiv b + 1 \pmod{\frac{b^{\varphi(n)}-1}{b-1}}$ for all primes and no composite with the exception of 4 and 6 .*

Conjecture 1.46. *Let b and n be a natural numbers , $b \geq 2$, then $\frac{b^{\varphi(n)}-1}{b-1}(b^{\tau(n)} - 1) + b \equiv b^{n-1} \pmod{\frac{b^n-1}{b-1}}$ for all primes and no composite with the exception of 4 .*

Conjecture 1.47. *Let p be prime number greater than three and let $T_n(x)$ be Chebyshev polynomial of the first kind , then $T_{p-1}(2) \equiv 1 \pmod{p}$ if and only if $p \equiv 1, 11 \pmod{12}$.*

Conjecture 1.48. *Let p be prime number greater than two and let $T_n(x)$ be Chebyshev polynomial of the first kind , then $T_{p-1}(3) \equiv 1 \pmod{p}$ if and only if $p \equiv 1, 7 \pmod{8}$.*

Conjecture 1.49. *Let p be prime number greater than three and let $T_n(x)$ be Chebyshev polynomial of the first kind , then $T_{p-1}(5) \equiv 1 \pmod{p}$ if and only if $p \equiv 1, 5, 19, 23 \pmod{24}$* .

Conjecture 1.50. *Let n be an odd natural number greater than one , let k be a natural number such that $k \leq n$, then n is prime if and only if :* $\displaystyle\sum_{i=0}^{k-1} i^{n-1} + \sum_{j=0}^{n-k} j^{n-1} \equiv -1 \pmod{n}$

Conjecture 1.51. *Let n be a natural number greater than one and let $T_n(x)$ be Chebyshev polynomial of the first kind , then n is prime if and only if :*$\displaystyle\sum_{k=0}^{n-1} 2T_{n-1}\left(\frac{k}{2}\right) \equiv -1 \pmod{n}$.

Conjecture 1.52. *Let n be a natural number greater than one and let $L_n(x)$ be Lucas polynomial , then n is prime if and only if :*$\displaystyle\sum_{k=0}^{n-1} L_{n-1}(k) \equiv -1 \pmod{n}$.

Conjecture 1.53. *Let p be an odd prime number , let $R_p(3) = \frac{3^p-1}{2}$ and let $S_i = S_{i-1}^3 + 3S_{i-1}$ with $S_0 = 36$, then $R_p(3)$ is prime number iff $S_{p-1} \equiv 36 \pmod{R_p(3)}$* .

Conjecture 1.54. *Let p be an odd prime number greater than three , let $R_p(-3) = \frac{3^p+1}{4}$ and let $S_i = S_{i-1}^3 + 3S_{i-1}$ with $S_0 = 36$, then $R_p(-3)$ is prime number iff $S_{p-1} \equiv 36 \pmod{R_p(-3)}$* .