How many points are there in a line segment? – A new answer from discrete space viewpoint

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ABSTRACT

While it is known that Euclid's five axioms include a proposition that a line consists at least of two points, modern geometry avoid consistently any discussion on the precise definition of point, line, etc. It is our aim to clarify one of notorious question in Euclidean geometry: how many points are there in a line segment? – from discrete space viewpoint. In retrospect, it may offer an alternative of quantum gravity, i.e. by exploring discrete gravitational theories.

1. Introduction

So many students from all ages have asked this question: how many points are there in a line segment? And a good math teacher will answer politely: in the circumference of a circle there are infinite number of points[1]. Similarly one can also ask: how many lines are there in a rectangular? The answer again is known: there are infinite number of lines in given rectangular.

But a careful student will ask again: but what is the definition of point and line? Teacher will answer again: a point is a circle with zero diameter, and line is composed of infinite points.

If our beloved student persists, he/she will continue to ask: but teacher, if a circle has zero diameter, then an infinite number of zeroes will not make a finite line, isn't it?

At this time, there is fair chance that the teacher feel upset and say: “shut up and calculate.”

That is what usually happens in most primary school mathematics classroom, and the situation is not getting better in undergraduate classroom. Only in graduate math class, then the students are allowed to ask numerous questions, such as foundations of mathematics etc.

Here we will offer a simpler solution of the above posed question from a discrete space viewpoint, with very wide implications, including distinction between quantization and discretization.

2. Solution: the space consists of circles with finite diameter

The obvious paradox that we set in the introduction section can be simplified as follows:

$$0+0+0+\ldots\text{ad infinitum} = 0$$

Therefore the basic postulate that a line segment consists of circles with zero diameter is contradictory by itself.

Our proposed solution is to assume that the space consists of circles with small but finite diameter (z), therefore if a line segment consists of circles like that, we have:

$$z+z+z+\ldots\text{ad infinitum} = \text{finite line}$$

One implication of this proposition is that we should better consider the geometry of space not as continuum, but as a discrete space. And we must remember that discretization of space is much more fundamental than quantization.

Moreover, we can consider the following:

a. It can be shown that similar indeterminacy problem plagues the very definition of differential calculus, as no one knows that actual size of dx. See H.J.M. Bos [2]:

> **2.15.** *I turn now to a difficulty which necessarily arises in any attempt to set up an infinitesimal calculus which takes the differential as fundamental concept, namely the indeterminacy of differentials.*
>
> *The first differential dx of the variable x is infinitely small with respect to x, and it has the same dimension as x. These are the only conditions it has to satisfy, and they do not determine a unique dx, for if dx satisfies the conditions then clearly so do $2dx$ and $\frac{1}{2}dx$ and in general all $a\,dx$ for finite numbers a. That is, all quantities that have the same dimension and the same order of infinity as dx might serve as dx.*
>
> *Moreover, there are elements not from this class which satisfy the conditions for dx; for instance dx^2/a and $\sqrt{a\,dx}$, for finite positive a of the same dimension as x. dx^2/a is infinitely small with respect to dx and $\sqrt{a\,dx}$ is infinitely large with respect to dx, so that there is even not a privileged class of infinite smallness from which dx has to be chosen; there is no "first" class of infinite smallness adjacent to finiteness. Thus first-order differentials involve a fundamental indeterminacy.*

b. Boyer has shown that Planck blackbody radiation can be derived from discrete charge assumption (without partition as assumed by Planck). See [3].

c. Lee Smolin has described three approaches to quantum gravity in his book[4]. But considering our proposition above, it seems that the notion of quantum gravity may be not necessary. Instead, we should consider discrete gravity theories.

d. Gary W. Gibbons and George F.R. Ellis have considered a discrete Newtonian cosmology. That is a good start [5].

e. Gerard 't Hooft has proposed a discrete deterministic interpretation of QM.[6] But it seems the use of both discrete and quantum language are superfluous. We need to let go the quantum terminology with its own excess baggage.

f. At astronomical scale, Conrad Ranzan has proposed a cellular universe, which is essectially a Newtonian Steady-State model but with a discrete cellular space model.[7] In our view, such an approach needs to be explored and investigated further. See also our recent paper, where we suggest an ultradiscrete KdV as model of cosmology [8]. See also Lindquist-Wheeler model [9][10].

3. Concluding Remarks

An old question and paradox in Euclidean geometry may be resolved consistently, once we accept and assume a discrete space instead of continuum model which is full of indeterminacies.

Many implications and further developments can be expected both in particle physics realm and also in cosmology theorizing. More observation and experiments are recommended to verify whether the space is discrete or continuous.

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