How scale factors determine starting time step from minimum non zero value, using superfluid universe model, assuming Hubble parameter H = 0 , with later Hubble H ~ T(temperature), after formation of causal structure to obtain initial graviton production

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Abstract

We start where we use an inflaton value due to use of a scale factor $a \sim a_{\min} t^{\gamma}$ .Also we use $\delta g_{tt} \sim a_{\min}^{2} \cdot \phi_{initial}$ as the variation of the time component of the metric tensor g_{tt} in Pre-Planckian Space-time. In doing so, what we lead up to using the Huang Superfluid universe model, which is by the modified superfluind cosmology model leading to examining $a^2 = Curvature / energy - density$ with in the Pre Planckian regime, Curvature, small but non zero, and energy density $\frac{\dot{\phi}^2}{2} + V(\phi)$. The Potential energy is given by what it would be if $a \sim a_{\min} t^{\gamma}$ leading to a relationship of $a_{initial} \propto initial - time$, where we will isolate conditions for the initial time and compare them against a root finder procedure given in another paper written by the author. Then, afterwards, assuming a modified Hubble parameter, with an initial Hubble parameter after the Causal surface with , right after a quantum bounce, determined by $H_{causal-structure-quantum-bounce} = 0$, is then $H_{initial} \sim 1/\Delta t \sim 1.66\sqrt{g_*} \cdot T^2 / m_{Planck}$. and g_* is an initial degrees of freedom value of about 110. Then, the graviton production rate is a function of time leading to a temperature T dependence, with M here is a chosen Mass scale, M of about 30 TeV, with d greater than or equal to zero, representing the Kaluza Klein dimensions assumed with the number of gravitons produced after the onset of Causal structure given by $n(T) \sim T^2 \cdot m_{Planck} \cdot (T/M)^{d+2}$. This $n(T)$ by Infinite quantum statistics is proportional to entropy

Key words Inflaton physics, Causal structure Entropy, temperature dependent initial Graviton production, Kaluza Klein dimensions

1. Referral to the Huang Superfluid Universe model

We look at [1] by Huang, as to a critical density affecting scale factor 'size of the universe' as given by

$$
\begin{aligned}
&H^2 = \frac{-k(curvature)}{a^2} + \frac{2}{3}\cdot\rho_c \\
&\& \\
&\rho_c = \frac{\dot{\phi}^2}{2} + V\left(\phi\right) \\
&\& \\
&H^2(Quantum - bounce) = 0 \\
&\Leftrightarrow a^2 = \frac{3k(curvature)}{2\rho_c} \\
&\Leftrightarrow a_{bounce} = \sqrt{\frac{3k(curvature)}{\dot{\phi}^2 + 2V\left(\phi\right)}}
\end{aligned}
\tag{1}
$$

This curvature, in the vicinity of Pre-Planckian space-time is of minimal value. Whereas Huang delineates the evolution of the scale factor as [1]

$$
\ddot{\phi}_n = -3H\dot{\phi}_n - \frac{\partial V\left(\phi_n\right)}{\partial\phi_n} \tag{2}
$$

The scalar field which Huang accesses is ϕ_n , with this being due to setting V as dependent upon the Kummel function, as written up in page 58 of [1] with, here, n going from 1 to N, in terms of scalar fields ,and

$$
\begin{aligned}
&V = \tilde{\Lambda}^4 U_b\left(z\right) \\
&\& \\
&z = 8\pi^2\phi^2 / \tilde{\Lambda}^2 \\
&\& \\
&\phi^2 = \sum_{n=1}^{N}\phi_n^2 \\
&\& \\
&\tilde{\Lambda} = Momentum - cutoff \\
&U_b\left(z\right) = c_0\tilde{\Lambda}^b\cdot\left[M(-2+b/2, N/2, z) - 1\right] \\
&M(p,q,z) = 1 + \frac{p}{q}z + \frac{p(p+1)}{q(q+1)}\frac{z^2}{2!} + ..
\end{aligned}
\tag{3}
$$

As given by [1], this potential system is from one loop Feynman diagrams as given in [2] .Our approximation is to set N as equal to 1, in the Pre Planckian regime, with the Causal structure creation zone, at the 'bubble' of space-time leading to a bifurcation of additional structure and additional space-time scalar fields, as delineated by ϕ_n. However before this happens to delineate the initial scalar field , with N=1 as within the bubble of space-time . What we are doing is to review what was put in [3] and contrast it to a (single field?) version of Eq. (3) above. In doing so we are using the Padmanbhan treatment of the linkage between scale factor, inflaton, and what was done in [3] while assuming that the Eq. (4) is for the regime of the quantum buibble, possibly of radii Plank length, and then match it to EqI3) above. i.e. probably of Planck dimensions, as having[3] .

We will remark upon utilization of the following two scalar potentials and the potential system in the following manner. In Eq.(3) we explicitly refer to a multi scalar inflaton field, which we can all as ϕ_n with values from 1 to N. But in the pre Planckian regime, we are looking at a single inflaton field version of the dynamics, which is given in Eq. (5) below.

In this case, the dyna mics of our problem will be laid out as follows

$$\phi(Before-Planckian)\xrightarrow[Causal-boundary]{}\phi_{n=1}\xrightarrow[Past-Causal-bundary]{}\phi_{n=1,\dots N}(Planckian) \qquad (4)$$

The first stage of this evolution, is given by Eq. (4) below. The Second stage has the scalar field as given in $\phi_{n=1}$ as stated for Eq. (4) below, but then mapped as the first admitted scalar field as given in Eq.(3), and then the final stage, has scalar fields which can be ranging from 1 to N in labels, which would be a physical transformation of the problem from a single field regime, to a multi scalar field regime, with similarities to super fluid helium.

In appendix A, we argue that this is similar to a particle in a quantum state, in a box, when the box is then suddenly opened up. I.e. in that quantum experiment which is in Appendix A, we have a ground state probability of P(1)=.41 that a ground state wave function would be n=1 and stay there if the length of the box were changed from L/2 to L, and we argue that we have an analogous situation here, for the linkage given for Eq.; (3), Eq.(4) and Eq. (5) given here. Having said that let us look at the Pre Planckian inflaton field, which motivates the start of our analysis

In short a single inflaton field will dominate the interior of an inflaton bubble, and then be considered as bridged to a single field version of Eq. (3) above initially. I.e. the single field inflaton, will obey the relations which were cited as given in [3] which we reproduce below as

$$\begin{aligned}
&a \approx a_{\min} t^{\gamma} \\
&\Leftrightarrow \phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln\left\{\sqrt{\frac{8\pi G V_0}{\gamma\cdot(3\gamma-1)}}\cdot t\right\} \\
&\Leftrightarrow V \approx V_0 \cdot \exp\left\{-\sqrt{\frac{16\pi G}{\gamma}}\cdot\phi(t)\right\}
\end{aligned} \qquad (5)$$

To employ this Eq (4) we are using, as was done in '[3], the following boundary condition of the bubble of Space-time as was given in [3] which we put in as being the boundary of a purported quantum bounce. This is also substantially using [4] which using the material so cited.

This would be, still,obeying [5,6,7]

$$\begin{aligned}
&g_{tt} \sim \delta g_{tt} \approx a_{\min}^2 \phi_{initial} << 1 \\
&\xrightarrow[\Pr e-Planck\to Planck]{} \delta g_{tt} \approx a_{\min}^2 \phi_{Planck} \sim 1 \\
&\Leftrightarrow \left(\frac{R_c\big|_{initial} \sim c\cdot\Delta t}{l_{Planck}}\right) \sim \vartheta(1)\Bigg|_{Planck}
\end{aligned} \qquad (6)$$

The n=N version would have ONE component of the potential largely dominated by the Eq.(4) write up, and the rest of the structure would be additional add ons according to the Kummel potential write up as given in Eq. (1) given above,

From now on, we will be examining the physics implications of finding and using Δt

2. Examining Δt from the vantage point of a minimum scale factor calculation.

To do this, we have that interpretation of Eq. (1) will lead to the following linkage of scale factor of the Universe, minimum, and the time derivative of the inflaton field, as given in Eq. (5) for the Pre Planckian regime, about the Causal structure as given in Eq. (6) above, mainly, then

$$a_{bounce} = \sqrt{\frac{3k(curvature)}{\dot{\phi}^2 + 2V(\phi)}} \sim \sqrt{\frac{12\pi G \cdot k(curvature)}{\gamma}} \cdot t \cdot \sqrt{1 + 2V_0 \cdot \gamma^2 \cdot \frac{(3\gamma - 1)}{32\pi}} \qquad (7)$$

This is for a minimum time step, t, which in our re write is, then

$$a_{bounce} \sim \Delta t \cdot \sqrt{\frac{12\pi G \cdot k(curvature)}{\gamma}} \cdot \sqrt{1 + 2V_0 \cdot \gamma^2 \cdot \frac{(3\gamma - 1)}{32\pi}} \qquad (8)$$

What we are doing is to contrast different ways of obtaining a time step Δt and then employing the tools used in [3] and [4]

Then making use of [7] while using the tools given in reference [8] with g_* is an initial degrees of freedom value of about 110 [9] , and T in Eq.(8) as a temperature, right after the formation of Causal structure, and with M here is a chosen Mass scale, M of about 30 TeV [10] we find that Eq. (9) below as given then will lead to via use of the ideas of [8] used again and again

$$H_{Early-Universe} \sim 1.66 \cdot \sqrt{g^*} \cdot \frac{T_{Early-Universe}}{M_{mass-scale}} \qquad (9)$$

Implying for a value right at the causal boundary of space time, i.e. the bounce radii of emergent

$$\Delta t \sim 1 / \left(1.66 \cdot \sqrt{g^*} \cdot \frac{T_{Early-Universe}}{M_{mass-scale}} \right) \qquad (10)$$

This will, if we utilize [7] tie in with a graviton production expression we give as, if d is the extra dimensions of assumed Kaluza – Klein space-time

$$n(T) \sim T^2 \cdot m_{Planck} \cdot (T/M)^{d+2} \qquad (11)$$

As stated before, this assumes, that Eq. (10) is by Ng. Infinite quantum statistics [11], an entropy count, with at the Causal boundary, a nonzero value, in line with [12]. And the non zero value of the scale factor is largely in tune with the ideas of quantum bounces as given in Loop quantum gravity [13] and also the non linear electrodynamic suggestions given by Camera, et. al. [14].

Having said that, we will then cite a result as given in [15] which involves a non linear equation for the Δt values used in Eq. (7) and Eq.(9) which in turn affects Eq. (10) which by infinite quantum statistics [11] implies that at a causal surface boundary, that we do not have non zero entropy.

3. Examination of the minimum time step, in Pre-Planckian Space-time as a Root of a Polynomial Equation

We initiate our work, citing [15] to the effect that we have a polynomial equation for the formation of a root finding procedure for Δt , namely if

$$\Delta t \cdot \left| \left(\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t - 1 \right) - \frac{\left(\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t - 1 \right)^2}{2} + \frac{\left(\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t - 1 \right)^3}{3} - \ldots \right| \tag{12}$$

$$\approx \left(\sqrt{\frac{\gamma}{\pi G}} \right)^{-1} \frac{48\pi\hbar}{a_{\min}^2 \cdot \Lambda}$$

From here, we then cited, in [14], using [11]a criteria as to formation of entropy, i.e. If Λ is an invariant cosmological 'constant' and if Eq. (12) holds, we can use the existence of nonzero initial entropy as the formation point of an arrow of time. given in Eq. (1) with a counting algorithm of created gravitons giving a nonzero entropy which can also be cited as similar to the Entropy given below Note that this is the boundary between the single inflaton treatment given in Eq. (5) and the more general equation

$$S_\Lambda\big|_{Arrow-of-time} = \pi \cdot \left(\frac{R_c\big|_{initial} \sim c \cdot \Delta t}{l_{Planck}} \right)^2 \neq 0 \tag{13}$$

This should be compared with Eq. (11) as a nonzero value for initial entropy at a causal surface/ boundary.

Note that the most likely result of a solution for Eq. (12) would be in the case that

$$\begin{aligned} &\left(\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot \Delta t - 1 \right) \sim \varepsilon^+ \sim tiny \\ &\Leftrightarrow \Delta t \sim Planck - time \end{aligned} \tag{14}$$

What Eq. (13) gives us then is an estimate as to a truncated value of time step which is tied into the arrow of time consideration as to the later part of this document. This is also linked to the causal barrier idea also alluded to in this document.

All this leads to a conclusion which is to the inter connectivity of initial conditions and nonzero entropy.

4. Conclusion. Inter connection between minimum scale factor, Δt , and Eq. (10). Much more to explore

That there may be a linkage between a minimum scale factor, a minimum time step and initial graviton production is nothing other than stunning. Also, this can be linked to possible falsification of a prior suggestion brought up in [15] which we cite below. Can we also, in all of this, examine if there is an invariant cosmological constant, or if it varies with an initial electromagnetic field, as is suggested next.

One way to look at it would be to suggest that as done by H. Kadlecova [16] in the 12 Marcel Grossman meeting that the typical energy stress tensor, using, instead, Gyratons, with an electro-magnetic energy density addition to effective Electromagnetic cosmological value as given by

$$\rho_{E\&M-contribution} \sim 8\pi G \cdot \left(E^2 + B^2 \right) \tag{15}$$

I.e. that there be, due to effective E and M fields a boost from an initially low vacuum energy to a higher ones, as given by Kadlecova [16.17]

$$\Lambda_{+} = \Lambda + \rho_{E\&M-contribution} \tag{16}$$

If true, this may affect Eq. (12) as given in the text. Ere also should keep in mind the issues brought up by Abbot et.al. and Corda, as far as foundational gravity as cited in [18,19,20] as well. I.e. parsing correctly would entail understanding the foundations of experimental gravity.

Finally, and not least, this construction of a single field inflaton field, as given up to the Causal structure boundary is, if it is done correctly, probably linked to one of the many post causal inflaton fields, as referenced in [1], and Eq. (1) of this presentation. The transition from one to possibly many inflaton fields, and a super fluid model of the universe be a way, as the author visualizes, of initiating turbulence at the start of the formation of a causal structure, with an analogy to superfluid induced turbulence as alluded to in [1]. A topic the author will explore later. And also if we can observe the following generated GW, as given with defined Frequency

$$frequency\big|_{initial} \sim 1/\left(R_c\big|_{initial} \sim c \cdot \Delta t \right) \xrightarrow[c \equiv 1]{} 1/\Delta t \tag{17}$$

One of the open questions this also leads to , is, if [21], in terms of the Cyclic conformal cosmology of Penrose is admissible, with this construction or is ruled out.

What we are also considering is, although not explicitly stated, a similar mechanism as is given in the Higgs formation of mass as is written up in pages 480 to page 483 of [22] , and also a way to a possible linkage to [23] in terms of gravitons, and Higgs theory. In particular,

quote:

Higgs mechanism at the graviton level as a consequence of the Vainshtein mechanism ,

end of quote,

from [23] may be developed in a future update of this document. Another alternative, to consider, in this temperature dependent regime, is also given by [24].

One final consideration. In [24] Oda has a rendering of the Cosmological Constant as given by the paragraph right after equation (42) of [24]

Quote

where the cosmological constant takes the form Λ = (2−(5 /4)times D) times m^2, which is negative for D > 1. We conjecture that in this class of potentials, the cosmological constant might be always negative since the 't Hooft model belongs to this class.

End of quote

The radical suggestion the author has, that in the Pre Planckian regime, in the regime right next, or included within the bubble, that the effective spatial dimension, D, would be 1, i.e. a dramatic reduction of effective 'dimensionality' with the effect that in the Pre-Planckian space-time, that one has, due to this, an effective POSITIVE cosmological constant. I.e. that the Oda conjecture applied literally should be with respect to the nucleated bubble of Pre Planckian space-time.

The author welcomes disagreements with this conjecture, and also wishes constructive engagement as to this point from interested readers.

We also wish to point to a recent paper, by Canate, Jime, and Salgado [25] as to the question if Geometric hair, in black hole theory is supported . by analytical and geometric models. The authors refer to several modified gravity models which impact the expansion of the universe. Minding that the Corda suggestion [19] as to how early universe models as to Tensor-Scalar models influence what is known about early universe experimental gravity data sets which could be expected, the additional benefit of our analysis, may be in helping to delineate what modified gravity models are admissible as far as the early universe, which in turn will directly impact the characterization of if or not black holes, indeed have geometic hair. If we go in addition to this, a review of [26] , where the author did a thought experiment as to what a causal discontinuity did as to the available fluctuations, and [27] on an inquiry as to if extra dimensions are necessary at all, and [28] as to how certain black hole results may be replicated, as far as the question of entanglement entropy in the early universe, we find that the model so given above, may have some very unexpected inter relationships with black hole physics, but also with the early universe at the same time.

Finally in a reminder as to purported bridges between the pre Planckian bubble, as would be for the physics, of linkage between Eq. (3) and Eq. (4) the author wishes to reiterate the following points

Eq,.(4) in Pre Planckian physics up to a causal barrier, would be for a single field inflaton. The author is stating that the INITIAL inflaton field, if the causal structure structure is linked to the forming of Eq. (3) by assuming that the Eq.(4) construction would go to N=1 of Eq. (3). This would be equivalent , with the other inflaton fields, N= 2 to N= N, being filled out at the same time the physics of [26] was fulfilled.

The details of this would be in some respects also similar to a 2^{nd} order phase transition. [29] which is a point which will require additional modeling,. That is the transition from N=1 initial scalar field potential, to many scalar field potentials. We state unequivocally though that the details would have some overlap with the ideas outlined in [29] as to the quark gluon plasma and electroweak, but would not have the convenient simple phase diagrams as outlined in [29] / Amd then using Eq. (4) and Eq. (A2) of Appendix A, below, we argue we then will have a probability of the suddenly liberated from just n = 1 ground state, of what we were looking at the causal barrier to be, that instead we will have a probability of P(1)~ .41, as given by approximation in Appendix A, that the single field inflaton would be held to, in main value, with a 59- 60 per cent probability that other inflaton states would be evolved to, as implied by [30]. The exact particulars of this would be in refinement of an argument as qualitatively alluded to in [29] below, with major refiments.

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APPENDIX A, summary of material from [30] as to quantum mechanical probability for particle to stay in ground state. For a box, with a wave functional as described below.

Assume a normalized quantum mechanical wave functional, ψ , as given by

$$\psi = \sqrt{\frac{2}{L}}; if \quad 0 \le x \le L/2$$

$$\psi = 0; \quad if \quad L/2 \le x \le L \qquad \text{(A1)}$$

If so then, the probability that one has a wave functional value with n=1 in the situation defined by Eq. (A1) is given as

$$\psi(x)=\sum_{n=1}^{\infty}A(n)\sqrt{\frac{2}{L}}\sin\left[\frac{n\pi x}{L}\right]$$
$$A(n)=\int_{0}^{L/2}\sqrt{\frac{2}{L}}\sin\left[\frac{n\pi x}{L}\right]\sqrt{\frac{2}{L}}dx=\frac{4}{n\pi}\sin^2\left[\frac{n\pi}{4}\right]$$
$$P(n)=\left(\frac{4}{n\pi}\sin^2\left[\frac{n\pi}{4}\right]\right)^2 \qquad \text{(A2)}$$
$$\therefore P(1)=\frac{4}{\pi^2}\cong .41$$

References

[1] Huang, K. " A Superfluid Universe", World Scientific, Singapore, Republic of Singapore, 2017

[2] JHalpern, K. and Hiang, K. , Phys Review 53, 3252, (1996)

[3] Padmanabhan, T., "Understanding Our Universe; Current Status, and Open Issues", pp 175-204, of, "100 Years of Relativity, Space-Time, Structure: Einstein and Beyond", World Scientific, P.T.E. LTD, Singapore, Republic of Singapore, 2005. http://arxiv.org/abs/gr-qc/0503107

[4] Beckwith, A. (2017) Constraints, in Pre-Planckian Space-Time via Padmabhan's $\Lambda_{initial}\cdot H^{-2}_{initial}\approx o(1)$ Approximation Leading to Initial Inflaton Constraints and Its Relation to Early Universe Graviton Production. *Journal of High Energy Physics, Gravitation and Cosmology*, **3**, 322-327. doi: 10.4236/jhepgc.2017.32027.

[5] Beckwith, A. (2016) Gedanken Experiment for Refining the Unruh Metric Tensor Uncertainty Principle via Schwarzschild Geometry and Planckian Space-Time with Initial Nonzero Entropy and Applying the Riemannian-Penrose Inequality and Initial Kinetic Energy for a Lower Bound to Graviton Mass (Massive Gravity). *Journal of High Energy Physics, Gravitation and Cosmology*, **2**, 106-124. doi: 10.4236/jhepgc.2016.21012

[6] Giovanni , M. (2008) A Primer on the Physics of the Cosmic Microwave Background. World Press Scientific, Hackensack. New Jersey, USA http://dx.doi.org/10.1142/6730

[7] **Dowker,F. "Causal sets and the deep structure of space-time", https://arxiv.org/abs/gr-qc/0508109**

[8] Sarkar, U., "Particle and Astro particle Physics", Series in High Energy Physics and Gravitation, Taylor & Francis, New York, 2008

[9] **Kolb, E. and Turner, M.,"The Early Universe", Westview Frontiers in physics series, 1991, volume 69**

[10] Rubakov, V. " Large and Infinite Extra Dimensions", https://arxiv.org/abs/hep-ph/0104152 . Phys.Usp.44:871-893,2001; Usp.Fiz.Nauk 171:913-938,2001

[11]Jack Ng, Y. (2008) Space-Time Foam: From Entropy and Holography to Infinite Statistics and Nonlocality. Entropy, 10, 441-461. http://dx.doi.org/10.3390/e10040441

[12] Keifer, C. " Can the Arrow of Time be Understood from Quantum Cosmology?", pp 191-203, of "The Arrows of Time, A Debate in Cosmology", with Mersini-Houghton, L, and Vaas, R, as editors, in Springer Verlag, Fundamental Theories in Physics, 2012; volume 172, Heidelberg, Federal Republic of Germany

[13] Rovelli, C. and Vidotto, F, "Covariant Loop Quantum Gravity", Cambridge University Press, Cambridge, United Kingdom, 2015

[14] Camara, C.S., de Garcia Maia, M.R., Carvalho, J.C. and Lima, J.A.S. (2004) Nonsingular FRW Cosmology and Non Linear Dynamics. Arxiv astro-ph/0402311 Version 1, Feb 12, 2004

[15] **Beckwith, A., "How a Minimum Time Step Based in Pre Planckian Space-Time if Friedman Equation H Set Equal to Zero Leads to the Arrow of Time", http://vixra.org/abs/1702.0290**

[16] Kadlecova H. , " Gravitational Field of Gyratons Propagating on Backgrounds Formed by Direct Product space-time's", pp 1899-1901; in the 12 Marcel Grossman Meetings of General Relativity Proceedings edited by T. Damour, and R. Jantzen, 2012, World Press Scientific, Singapore, Republic of Singapore (Vol 3)

[17] Kadlecova, H., Zelinikov, A., Krtous P., and Podolsky, J., Phys Rev D, 80, 024004 (2009)

[18] B. P. Abbott et al., "Observation of Gravitational Waves from a Binary Black Hole Merger", PRL 116, 061102 (2016); https://physics.aps.org/featured-article-pdf/10.1103/PhysRevLett.116.061102

[19] Corda, C. (2009) Interferometric Detection of Gravitational Waves: The Definitive Test for General Relativity. International Journal of Modern Physics D, 18, 2275-2282. ArXiv: 0905.2502 [gr-qc]

[20] B. P. Abbot et.al. "GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence" Published in **Phys**. **Rev**. **Lett**. **116**, 241103 (**2016**)

[21] Penrose, R "Cycles of Time", Alfred Knopf, 2011, New York, New York

[22] Thompeson, M. " Modern Particle Physics" Cambridge University Press, Cambridge, UK, 2008

[23] Arraut I, "The Higgs mechanism at the graviton level: The Vainshtein mechanism in time-domains", https://arxiv.org/abs/1505.06215v3

[24] Oda, I, "Remarks on Higgs Mechanism for Gravitons", Phys.Lett.B690:322-327,2010, https://arxiv.org/abs/1004.3078

[25] Canate, P. Jaime, L, and Salgado, M. "Spherically Symmetric black holes in f® gravity: Is geometric scalar hair supported?. Classical and Quantum Grav(33), 155055, (42pp)

[26] Beckwith, A. (2016) Open Question: Could a Causal Discontinuity Explain Fluctuations in the CMBR Radiation Spectrum? *Journal of High Energy Physics, Gravitation and Cosmology*, **2**, 186-208. Doi: 10.4236/jhepgc.2016.22018

[27] Clarkson, C. and Seahra, S." A gravitational wave window on extra dimensions" *Class. Quant. Grave.* 24, F33 (2007)

[28] Mueller, R. And Lousto, C ," Entanglement entropy in curved spacetimes with event horizons" Phys.Rev. D52 (1995) 4512-4517, https://arxiv.org/abs/gr-qc/9504049

[29] STICLET, D. , "Phase Transitions in the Early Universe Electroweak and QCD Phase Transitions", http://fyzika.uniza.sk/~melo/DoruSticlet.pdf

[30] Shankar, R. “Fundamentals of Physics, II, Electromagnetics, Optimes and Quantum Mechanics” Yale University Press, New Haven, Conn. 2016