Field Equations

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Abstract
Field equations occur in many physical theories. Most dynamic fields share a set of first and second order partial differential equations and differ in the kinds of artifacts that cause discontinuities. The paper restricts to first and second order partial differential equations. These equations can describe the interaction between the field and pointlike artifacts. The paper treats periodic and one-shot triggers in maximally three spatial dimensions. The paper applies quaternionic differential calculus. It uses the quaternionic nabla operator. This configuration implements the storage of dynamic geometric data as a combination of a proper timestamp and a three-dimensional spatial location in a quaternionic storage container. The storage format is Euclidean. The paper introduces warps and clamps as new types of super-tiny objects that constitute higher order objects.

Introduction

Maxwell equations apply the three-dimensional nabla operator in combination with a time derirative that applies coordinate time. The Maxwell equations derive from results of experiments. For that reason, those equations contain physical units.

In this paper, the quaternionic partial differential equations apply the quaternionic nabla. The equations do not derive from the results of experiments. Instead, the formulas apply the fact that the quaternionic nabla behaves as a quaternionic multiplying operator. The corresponding formulas do not contain physical units. This approach generates essential differences between Maxwell field equations and quaternionic partial differential equations.

The quaternionic partial differential equations do not change the data format. The format of the information that the field transmits to observers, which the field embeds is affected by the information transfer. Instead of the Euclidean storage format, which governs at the location of the observed event, the observers perceive a spacetime format, which features a Minkowski signature. The Lorentz transform describes the format conversion.

Generalized field equations

Generalized field equations hold for all basic fields. Generalized field equations fit best in a quaternionic setting.

Quaternions consist of a real number valued scalar part and a three-dimensional spatial vector that represents the imaginary part.

The multiplication rule of quaternions indicates that several independent parts constitute the product.

In this comment, we use a suffix $_r$ to indicate the scalar real part of a quaternion, and we use bold face to indicate the imaginary vector part.

$$c = c_r + \boldsymbol{c} = a\,b = (a_r + \boldsymbol{a})\,(b_r + \boldsymbol{b}) = a_r\,b_r - \langle \boldsymbol{a}, \boldsymbol{b} \rangle + a_r\,\boldsymbol{b} + \boldsymbol{a}\,b_r \pm \boldsymbol{a} \times \boldsymbol{b}$$

The ± indicates that quaternions exist in right-handed and left-handed versions.
The formula can be used to check the completeness of a set of equations that follow from the application of the product rule.

The quaternionic conjugate of a is $a^* = (a_r - \boldsymbol{a})$

From the product, rule follows the formula for the norm $|a|$ of quaternion a.

$$|a|^2 = a\, a^* = (a_r + \boldsymbol{a})\,(a_r - \boldsymbol{a}) = a_r\, a_r + \langle\, \boldsymbol{a}, \boldsymbol{a}\,\rangle$$

The quaternionic nabla ∇ acts like a multiplying operator. The (partial) differential $\nabla\psi$ represents the full first order change of field ψ.

$$\phi = \nabla\psi = \phi_r + \boldsymbol{\phi} = (\nabla_r + \boldsymbol{\nabla})\,(\psi_r + \boldsymbol{\psi}) = \nabla_r\,\psi_r - \langle \boldsymbol{\nabla}, \boldsymbol{\psi}\rangle + \nabla_r\,\boldsymbol{\psi} + \boldsymbol{\nabla}\,\psi_r \pm \boldsymbol{\nabla} \times \boldsymbol{\psi}$$

The equation is a quaternionic first order partial differential equation.

The five terms on the right side show the components that constitute the full first order change.
They represent subfields of field ϕ, and often they get special names and symbols.

$\boldsymbol{\nabla}\,\psi_r$ is the gradient of ψ_r
$\langle \boldsymbol{\nabla}, \boldsymbol{\psi}\rangle$ is the divergence of $\boldsymbol{\psi}$.
$\boldsymbol{\nabla} \times \boldsymbol{\psi}$ is the curl of $\boldsymbol{\psi}$

$\phi_r = \nabla_r\,\psi_r - \langle \boldsymbol{\nabla}, \boldsymbol{\psi}\rangle$ (This is not part of Maxwell equations!)
$\boldsymbol{\phi} = \nabla_r\,\boldsymbol{\psi} + \boldsymbol{\nabla}\,\psi_r \pm \boldsymbol{\nabla} \times \boldsymbol{\psi}$
$\boldsymbol{E} = -\nabla_r\,\boldsymbol{\psi} - \boldsymbol{\nabla}\,\psi_r$
$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{\psi}$

From the above formulas follows that the Maxwell equations do not form a complete set.

Physicists use gauge equations to make Maxwell equations more complete.

$$\chi = \nabla^* \phi = \nabla^*\,\nabla\psi = (\nabla_r - \boldsymbol{\nabla})(\nabla_r + \boldsymbol{\nabla})\,(\psi_r + \boldsymbol{\psi}) = (\nabla_r\,\nabla_r + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla}\rangle)\,\psi$$

and

$$\zeta = (\nabla_r\,\nabla_r - \langle \boldsymbol{\nabla}, \boldsymbol{\nabla}\rangle)\,\psi$$

are quaternionic second order partial differential equations.

The first equation splits into two first order partial differential equations.

The last second order partial differential equation cannot split into two quaternionic first order partial differential equations. This equation offers waves as parts of its set of solution. For that reason, it is also called a wave equation.

$$\nabla_r\,\nabla_r\,\psi = \langle \boldsymbol{\nabla}, \boldsymbol{\nabla}\rangle\,\psi = \omega\,\psi \Longrightarrow \mathrm{f} = \exp(2\pi\mathbb{i}\omega x\tau)$$

In odd numbers of participating dimensions, both second order partial differential equations offer shock fronts as part of its set of solutions.

$$\mathrm{f}(c\tau + x\mathbb{i}) + g\,\mathrm{f}(c\tau - x\mathbb{i})\ ;\ \text{one-dimensional fronts}$$

$$\mathrm{f}(c\tau + r\mathbb{i})/r + g\,\mathrm{f}(c\tau - r\mathbb{i})/r\ ;\ \text{spherical fronts}$$

After integration over a sufficient period the spherical shock front results in the Green’s function of the field under spherical conditions.

$\mathfrak{Q} = (\nabla_r\,\nabla_r - \langle \boldsymbol{\nabla}, \boldsymbol{\nabla}\rangle)$ is equivalent to d'Alembert's operator.
$\boxdot = \nabla^*\,\nabla = \nabla\,\nabla^* = (\nabla_r\,\nabla_r + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla}\rangle$ describes the variance of the subject

Gauge equations must extend Maxwell equations to derive the second order partial wave equation.

Maxwell equations use coordinate time, where quaternionic differential equations use proper time. Regarding quaternions, the norm of the quaternion plays the role of coordinate time. These time values apply not in their absolute versions. Thus, only time intervals apply.

Hilbert spaces can only cope with number systems that are division rings. In a division ring, all non-zero members own a unique inverse. Only three suitable division rings exist. These are the real numbers, the complex numbers, and the quaternions. Thus dynamic geometric data that are characterized by a Minkowski signature must first be dismantled into real numbers before they can serve in a Hilbert space. Quaternions can serve without dismantling.
Quantum physicists use Hilbert spaces for the modelling of their theory. Quaternionic quantum mechanics appears to represent a natural choice.

The Poisson equation

$$\Phi = \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \psi = G \circ \varphi$$

describes how the field reacts with its Green's function G on a distribution φ of point-like triggers.

$$G(\boldsymbol{q}-\boldsymbol{c})=1/|\boldsymbol{q}-\boldsymbol{c}|$$

$$\langle \boldsymbol{\nabla} \times \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle = 0$$

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{\psi}) = \boldsymbol{\nabla} \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle - \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \boldsymbol{\psi}$$

$$(\boldsymbol{\nabla}\boldsymbol{\nabla}) \psi = (\boldsymbol{\nabla} \times \boldsymbol{\nabla}) \psi - \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \psi = (\boldsymbol{\nabla} \times \boldsymbol{\nabla}) \boldsymbol{\psi} - \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \psi = \boldsymbol{\nabla} \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle - 2 \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \psi + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \psi_r$$

The term $(\boldsymbol{\nabla} \times \boldsymbol{\nabla}) \psi$ indicates the curvature of field ψ.

The term $\langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \psi$ indicates the stress of the field ψ.

$$(\boldsymbol{\nabla} \times \boldsymbol{\nabla}) \psi + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \psi = \boldsymbol{\nabla} \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle - \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \psi_r$$

With respect to a local part of a closed boundary that is oriented perpendicular to vector $\boldsymbol{n}$ the partial differentials relate as

$$\boldsymbol{\nabla} \psi = \boldsymbol{\nabla} (\psi_r + \boldsymbol{\psi}) = \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle + \boldsymbol{\nabla} \psi_r \pm \boldsymbol{\nabla} \times \boldsymbol{\psi} \Leftrightarrow \boldsymbol{n} \psi = \boldsymbol{n} (\psi_r + \boldsymbol{\psi}) = \langle \boldsymbol{n}, \boldsymbol{\psi} \rangle + \boldsymbol{n} \psi_r \pm \boldsymbol{n} \times \boldsymbol{\psi}$$

This is exploited in the generalized Stokes theorem

$$\iiint \boldsymbol{\nabla} \psi \, dV = \oiint \boldsymbol{n} \psi \, dS$$

This result turns the differential continuity equation into an integral balance equation.
It also elucidates what the terms in the continuity equation mean.

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{\psi}) = \boldsymbol{\nabla} \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle - \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \boldsymbol{\psi} \Leftrightarrow \boldsymbol{n} \times (\boldsymbol{n} \times \boldsymbol{\psi}) = \boldsymbol{n} \langle \boldsymbol{n}, \boldsymbol{\psi} \rangle - \langle \boldsymbol{n}, \boldsymbol{n} \rangle \boldsymbol{\psi}$$

$$\iiint \boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{\psi}) \, dV = \oiint \langle \boldsymbol{n}, \boldsymbol{\psi} \rangle \boldsymbol{n} \, dS - \oiint \boldsymbol{\psi} \, dS$$

Waves, warps, and clamps

Waves require a periodic harmonic actuator. For odd numbers of participating dimensions of a one-shot actuator, the equations deliver shock fronts.

The author suggests ***warp*** as the name for the solutions of the homogeneous second order partial differential equations, which represent one-dimensional shock fronts. One-dimensional triggers cause warps. During travel, warps keep their shape and their amplitude.

Strings of equidistant warps feature a frequency. If each warp carries a standard bit of energy and if all strings feature the same spatial length, then the strings obey the Einstein-Planck relation $E=h\nu$. The strings can bridge huge distances without losing their integrity. Therefore, these strings can represent ***photons***.

The author suggests ***clamp*** as the name for the solutions of the homogeneous second order partial differential equations, which represent three-dimensional spherical shock fronts. Three-dimensional isotropic triggers cause clamps. During travel, the shape of the front stays constant, but the amplitude of the front diminishes as 1/r with distance r from the trigger location. The clamp integrates into the Green's function of the field.

Because the clamps deform their embedding field, will clamps carry a standard bit of mass. However, the deformation of the carrier quickly fades away. Dense swarms of clamps that are recurrently regenerated can produce a persistent deformation of the embedding field.

Elementary particles, hop around in a stochastic hopping path that results in a dense and coherent swarm of clamps. In this way, elementary particles get their mass, which is proportional to the number of elements of the swarm.
The warps and clamps form super-tiny objects that constitute photons and elementary particles.

References

https://www.docs.com/hans-van-leunen