An Elementary Proof for infinitely many twin primes

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Abstract

Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many "advanced mathematics tools" are used to solve them, but they are still unsolved. Based on the fundamental theorem of arithmetic and Euclid's proof of endless prime numbers, we have proved there are infinitely many twin primes.

Introduction

Prime numbers¹ are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many "advanced mathematics tools" are used to solve them, but they are still unsolved.

I believe that prime numbers are "basic building blocks" of the natural numbers and they must follow some very simple basic rules and do not need "advanced mathematics tools" to solve them. One of the basic rules is the "fundamental theorem of arithmetic" and the "simplest tool" is Euclid's proof of endless prime numbers.

Fundamental theorem of arithmetic:

The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic², which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.^[1] Primes can thus be considered the "basic building blocks" of the natural numbers.

Euclid's proof³ that the set of prime numbers is endless

The proof works by showing that if we assume that there is a biggest prime number, then there is a contradiction.

We can number all the primes in ascending order, so that $P_1 = 2$, $P_2 = 3$, $P_3 = 5$ and so on. If we assume that there are just **n** primes, then the biggest prime will be labeled **P**_n. Now we can form the number Q by multiplying together all these primes and adding 1, so

$\mathbf{Q} = (\mathbf{P}_1 \times \mathbf{P}_2 \times \mathbf{P}_3 \times \mathbf{P}_4 ... \times \mathbf{P}_n) + 1$

Now we can see that if we divide Q by any of our n primes there is always a remainder of 1, so Q is not divisible by any of the primes, but we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either Q must be a prime or Q must be divisible by primes that are larger than P_n .

Our assumption that P_n is the biggest prime has led us to a contradiction, so this assumption must be false, so there is no biggest prime and the set of prime numbers is endless.

Discussions

Twin Prime Conjecture: There are infinitely many twin primes.

A twin prime is a prime number that is either 2 less or 2 more than another prime number — for example, the twin prime pairs (11 and 13; 17 and 19; 41 and 43). In other words, a twin prime is a prime that has a prime gap of two.

Twin primes become increasingly rare as one examines larger ranges, in keeping with the general tendency of gaps between adjacent primes to become larger as the numbers themselves get larger. However, it is a longstanding conjecture that there are infinitely many twin primes. Work of Yitang Zhang⁴ in 2013, as well as work by James Maynard, Terence Tao and others, has made substantial progress towards proving this conjecture, but at present it remains unsolved.

If a large number N is not divisible by 3 or any prime which is smaller or equal to N/3, it must be a prime. 1/3 of all numbers that are divisible by 7 can be divisible by 3, 1/3 of all numbers that are divisible by 11 can be divisible by 3 and 1/7 of all numbers that are divisible by 11 can be divisible by 7, 1/3 of all numbers that are divisible by 13 can be divisible by 14 can be divisible by 15 can be divisible by 15 can be divisible by 16 can be divisible by 17 can be divisible by 16 can be divisible by 17 can be divisible by 17 can be divisible by 18 can be divisible by 19 can be divisible by 19 can be divisible by 10 can be div

Let N_o represent any odd number, the chance of N_o to be a non-prime is: [(1/3) + (1/7x2/3) + (

(1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13)

+(1/19x2/3x6/7x10/11x12/13x16/17)+(1/23x2/3x6/7x10/11x12/13x16/17x18/19)

+(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) +

(1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) +

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) +

(1/61x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/5 3x58/59) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

(1/71x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +

 $(1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/5 3x58/59x60/61x66/67x70/71x72/73) + \dots] -----Formula 1$

Any odd number cannot be divisible by 2 and any odd number with 5 as its last digit is not a prime except 5.

Let \sum represent the sum of the infinite terms and $\Delta=1-\sum$, according to Euclid's proof^[2] that the set of prime numbers is endless. Δ is the chance of any odd number to be a prime. \sum may be very close to 1 when N is growing to ∞ , but always less than 1. Let $\Delta=1-\sum$, when N is growing to ∞ , Δ may be very close to 0, but always more than 0 according to Euclid's proof that the set of prime numbers is endless. If Δ is 0, then there is no prime, that is not true.

The sum of first 20 terms = [(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17)+

(1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) +

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/59x2/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/59x2/23x28/29x30/31x36/37x40/41x42/43x46/47x52/23x28/29x30/31x36/37x40/41x42/43x46/47x52/23x28/29x20/30) + (1/59x26/27x40/41x42/43x46/47x52/23x28/29x20/20) + (1/59x26/27x40/20) + (1/59x26/20) + (1/59x26/20)

(1/61x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/5)

3x58/59x60/61) +

(1/71x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +

For the first 20 term: $\sum = 0.688872$, $\Delta = 1 - \sum = 0.311128$

The chance of N_o to be a prime is: $\Delta = 1 - [(1/3) + (1/7x^2/3) + (1/11x^2/3x^6/7) + (1/11x^2/7) + (1/11x^2/7) + (1/11x^2/7) + (1/11x^2/7) + (1/11x^2/7) + (1/11x^$

(1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/19x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11) + (1/19x2/7x10/11) + (1/19x

(1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x2/23) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x2/23) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/29x2/17x18/19x22/23) + (1/29x2/17x18/19x2) + (1/29x2) + (1/

(1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) +

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) +

(1/61x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/5 3x58/59) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

(1/71x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +

 $(1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/5 3x58/59x60/61x66/67x70/71x72/73) + \dots] -----Formula 2$

Let \$1 represents a prime with 1 as its last digit, such as 11, 31, 41, 61, 71, 101, 131, 151, 181, 191,...; \$3 represents a prime with 3 as its last digit, such as 3, 13, 23, 43, 53, 73, 83, 103, 113, 163, 193....; \$7 represents a prime with 7 as its last digit, such as, 7, 17, 37, 47, 67, 97, 107, 127, 137, 157, 167, 197...; and \$9 represents a prime with 9 as its last digit, such as 19, 29, 59, 79, 89, 109, 139, 149, 179, 199,....

Let O1 represents an odd number with 1 as its last digit, such as 11, 21, 31, 41, 51, 61, 71,...; O3 represents an odd number with 3 as its last digit, such as 3, 13, 23, 33, 43, 53, 63, 73,....; O7 represents an odd number with 7 as its last digit, such as, 7, 17, 27, 37, 47, 57, 67, 77...; and O9 represents an odd number with 9 as its last digit, such as 9, 19, 29, 39, 49, 59, 69, 79,....

Fundamental theorem of arithmetic states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.

Every odd number (O1) with 1 as its last digit is a product of unlimited terms, such as, \$1x\$1, \$3 x \$7, \$9 x \$9, \$1 x \$1 x \$1, ..., \$1 x \$3 x \$7,..., \$3 x \$3 x \$3 x \$3, ..., \$7x\$7x\$7x\$7..., but we can only consider \$1, \$7, and \$9 because they decide how large other \$1, \$3, \$7, and \$9 can be. Let the number 600 as the example. For \$1x\$1, the smallest \$1 is 11 which decides that another \$1 should not be more than 41 (11 x41=451<600, but 11x61=671>600 and 11 x 11 x 11=1331>600); for \$3x\$7, the smallest \$7 is 7 which decided \$3 should not be more than 83 (7x83=581<600, 7x3x31=651>600),...; for \$9x\$9, the smallest \$9 is 9(3x3) which decides that another \$1 should not be more than 59 (3x3x59=531<600), so the smallest \$1, \$7, and \$9 decide the possible largest \$1, \$3, \$7, and \$9 for any O1.

The chance of any odd number O1 to be a prime is: $\Delta_1 = 1 - \sum_1 = 1 - [(1/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + +(1/19x2/3x6/7x10/11x12/13x16/17) + (1/11x2/3x6/7x10/11) + (1/11x2/3x6/7x10/11x12/13x16/17) + (1/11x2/3x6/7x10/11x2/13x6/7x10/17) + (1/11x2/3x6/7x10/11x2/13x6/7x10/17) + (1/11x2/3x6/7x10/17) + (1/11x2/3x6/77) + (1/11x2/37) + (1/11x2/77) + (1/11x2/77) + (1/11x2/77) + (1/11x2/77) + (1/11x2/77) + (1/$

 $\begin{array}{l} (1/23x2/3x6/7x10/11x12/13x16/17x18/19) \\ + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) \\ + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) \\ + \end{array}$

(1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/59x2/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/59x2/23x28/29x30/31x36/37x40/41x42/43x46/47x52/23x28/29x30/31x36/37x40/41x42/43x46/47x52/23x28/29x20/30) + (1/59x26/27x40/41x42/43x46/47x52/23x28/29x20/20) + (1/59x26/27x40/20) + (1/59x26/20) +

(1/61x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59) +

(1/71x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +

 $(1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) + \ldots] -----Formula 3.$

For N=600, the number of primes with 1 as its last digit=600/10 - 600/10 [(1/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/19x2/3x6/7x10/11x12/13x16/17) +

(1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/21x2) + (1/21x2)

(1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) +

(1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

 $\begin{array}{l}(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + \\(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + \end{array}$

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +

(1/83x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73x78/79)]=60-60[0.333333 + 0.051948 + 0.039960 + 0.023753 + 0.018590 + 0.014102 + 0.012738 + 0.009069 + 0.008436 + 0.006543 + 0.005766 + 0.004377 + 0.003597]=60-60x0.532212=28. There are 25 primes with 1 as its last digit, if we count 1, the difference between real number and the calculated number is only 2. The distribution of primes is not uniform and 600 is not a big number, so the difference is reasonable, when the number N becomes lager, the difference will be ≤ 1 .

Every odd number with 3 as its last digit is a product of unlimited terms, such as, 3x, 7x, 3x, 3x, 1x, 7x, 3x, 1x, 1x, 7x, 3x, 1x, 1x,

The chance of any odd number O3 to be a prime is: $\Delta_3=1-\sum_3=1-[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +$

 $(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + \dots] ------Formula 4$

For N=600, the number of primes with 3 as its last digit=600/10 - 600/10 [(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/17x2/3x6/7x0/11x12/11x12/13) + (1/17x2/3x6/7x0/11x12/11x12/13) + (1/17x2/3x6/7x0/11x12/11x12/13) + (1/17x2/3x6/7x0/11x12/13) + (1/17x2/3x6/7x0/11x12/13) + (1/17x2/3x6/7x0/11x12/13) + (1/17x2/3x6/7x0/11x12/13) + (1/17x2/3) + (1/17x2/3) + (1/17x2/3) + (1/17x2/3) + (1/17x2/3)) + (1/17x2/3) + (1/17x2/3) + (1/17x2/3)) + (1/17x2/3)) + (1/17x2/3)) + (1/17x2/3)) + (1/17x2/3)) + (1/17x2/3)) + (1/17x2/

(1/23x2/3x6/7x10/11x12/13x16/17x18/19)

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) = 60-60[0.333333 + 0.095238 + 0.039960 + 0.028207 + 0.018590 + 0.010328 + 0.008436 + 0.007538 + 0.006543 + 0.004910 + 0.004377] = 60-60x0.55746=26.6.There are 26 primes with 3 as its last digit, the difference between real number and the calculated number is $0.6 \le 1$.

Every odd number with 7 as its last digit is a product of unlimited terms, such as, 7x, 3x, 3x,

The chance of any odd number O7 to be a prime is: $\Delta_7=1-\sum_7=1-[(1/3) + (1/7x^2/3) + (1/7x^2/3)]$

(1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +

 $(1/23x2/3x6/7x10/11x12/13x16/17x18/19) \ +$

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

 $(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + \dots] ------Formula 4$

For N=600, the number of primes with 7 as its last digit=600/10 - 600/10 [(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +

(1/23x2/3x6/7x10/11x12/13x16/17x18/19)

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

 $\begin{array}{l} (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) = 60-60[0.333333 + 0.095238 + 0.039960 + 0.028207 + 0.018590 + 0.010328 + 0.008436 + 0.007538 + 0.006543] = 60-60x0.548173 = 27.1. There are 28 primes with 7 as its last digit, the difference between real number and the calculated number is <math display="inline">0.9 \leq 1$.

Every odd number (O9) with 9 as its last digit is a product of unlimited terms, such as, 1x, 7, 3x, 3x, 1x, 1x, 9, ..., 1x, 7x, 7, ..., 3x, 3x,

for 7x, the smallest \$7 is 7 which decided another \$7 should not be more than 67 (7x67=469<600, but 7x97=679>600),...; for 3x, the smallest \$3 is 3 which decides that \$9 should not be more than 193 (3x193=579<600), so the smallest \$9, \$7, and \$3 decide the possible largest \$1, \$3, \$7, and \$9 for any O9.

The chance of any odd number O9 to be a prime is: $\Delta_9=1-\sum_9=1-[(1/3) + (1/7x^2/3) + (1/7x^2/3)]$

(1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/19x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11) + (1/19x2/7x10/11) + (1/19x2/7x0/11) + (1/19x2/7x0/11) + (1/19x2/7x0/11) + (1/19x2/7x0/11) + (1/19x2/7x0/11) + (1/19x2/7x0/11) + (1/19x

 $(1/23x2/3x6/7x10/11x12/13x16/17x18/19) \ + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) \ + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x2/23) \ + (1/29x2/3x6/7x10/11x12/13x16/17x18/19) \ + (1/29x2/3x6/7x10/11x12/11$

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/59x2/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/59x2/23x28/29x30/31x36/37x40/41x42/43x46/47x52/23x28/29x30/31x36/37x40/41x42/43x46/47x52/23x28/29x20/30) + (1/59x26/27x40/41x42/43x46/47x52/53) + (1/59x26/27x40/41x42/43x46/47x52/53) + (1/59x26/27x40/47x60/27x60/27x60/47x60/47x60/47x

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +

 $(1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) + \dots] ------Formula 5.$

For N=600, the number of primes with 9 as its last digit=600/10 - 600/10[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/2)x(10) + (

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +

(1/83x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73) +

(1/103x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97x100/101) +

(1/113x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97x100/101x106/107) +

(1/163x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97x100/101x106/107x112/113x126/127x130/131x136/137x150/151x156/157) +

 $(1/193x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97x100/101x106/107x112/113x126/127x130/131x136/137x150/151x156/157x162/163x166/167x172/173x180/181x190/191)] = 60-60[0.333333 + 0.095238 + 0.039960 + 0.028207 + 0.023753 + 0.018590 + 0.014102 + 0.010328 + 0.008436 + 0.007538 + 0.006543 + 0.004910 + 0.004377 + 0.004222 + 0.003294 + 0.002974 + 0.001972 + 0.001618] = 60-60x0.609395=23.4, there are 25 primes with 9 as its last digit, the difference is 2.6, when N become larger, the difference will be <math>\leq 1$.

Above results show there are almost equal number of \$1, \$3, \$7, and \$9, ~1/4 of all primes are \$1, ~1/4 of all primes are \$3, ~1/4 of all primes are \$7, and ~1/4 of all primes are \$9. Euclid proved primes are endless, an infinite number x1/4 is still infinite, so \$1, \$3, \$7, and \$9 are endless. We can also prove \$1, \$3, \$7, and \$9 are endless with Euclid's proof: We can number all the primes in ascending order (exclude 2 and 5), so that $P_{11} = 11$, $P_{12} = 31$, $P_{13} = 41$, $P_{21} = 3$, $P_{22} = 13$, $P_{23} = 23$, $P_{31} = 7$, $P_{32} = 27$, $P_{33} = 47$, $P_{41} = 19$, $P_{42} = 29$, $P_{43} = 59$, and so on. If we assume that there are just **n** primes with 1 as their last digit, **n** primes with 3 as their last digit, **n** primes with 7 as their last digit, and **n** primes with 9 as their last digit, then the biggest primes with 1, 3, 7 or 9 as its last digit will be labeled **P**_{1n}, **P**_{2n}, **P**_{3n}, and **P**_{4n}. Now we can form the number Q by multiplying together all these primes and adding 10(the difference between primes with 1, 3, 7, or 9 as its last digit is at least 10), so

$$\mathbf{Q} = (\mathbf{P}_{11} \times \mathbf{P}_{21} \times \mathbf{P}_{31} \times \mathbf{P}_{41} \dots \times \mathbf{P}_{1n} \times \mathbf{P}_{2n} \times \mathbf{P}_{3n} \times \mathbf{P}_{4n}) + 10$$

Now we can see that if we divide Q by any of our n primes there is always a remainder of 10, so Q is not divisible by any of the primes, but we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either Q must be a prime or Q must be divisible by primes that are larger than P_{1n} , P_{2n} , P_{3n} , or P_{4n} .

Our assumption that P_{1n} , P_{2n} , P_{3n} , or P_{4n} is the biggest prime with 1, 3, 7, or 9 as its last digit has led us to a contradiction, so this assumption must be false, so there is no biggest prime with 1, 3, 7, or 9 as its last digit and the set of prime numbers with 1, 3, 7, or 9 as its last digit is endless.

The chance for O3 following \$1, to be a non-prime is: $\sum_{3} = [(1/3) + (1/7x^{2}/3) + (1/7x^{2$

(1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +

 $(1/23x2/3x6/7x10/11x12/13x16/17x18/19) \ +$

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

 $(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + \dots]$

 $3x58/59x60/61x66/67x70/71) + \dots$] ------Formula 4

Let assume the pair, $\$1_x$, $\$3_x$ is the largest twin pair, however we know both \$1 and \$3 are endless, there are $\$1_{x+1}$, $\$1_{x+2}$, $\$1_{x+3}$,... $\$1_{x+n}$, and $\$3_{x+1}$, $\$3_{x+2}$, $\$3_{x+3}$,... $\$3_{x+n}$. We need only find if there is at least 1 of $\$3_{x+n}$ follow 1 of $\$1_{x+n}$. Any \$1 can be divisible by only 1 and itself, so $\$1_{x+n}$ plus 2 (also plus 4, 6, 8, 10, 12, 14, 16, or 18) cannot be divisible by $\$1_{x+n}$ because the smallest \$1 is 11 and the next \$1 is 31.

The chance for O3 following a prime, 1_x , to be a prime is: $\Delta_{x3}=1-\sum_{x3}=1-\{[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/17x2/3x6/7x10/11x12/11x12/13) + (1/17x2/3x6/7x10/11x12/11x1$

(1/23x2/3x6/7x10/11x12/13x16/17x18/19) +

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/53x2/3x28/29x30/31x36/37x40/41x42/43x46/47) + (1/5)x2(1/5)x2(1/5)x(1/5)) + (1/5)x

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

 $(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + \dots] - 1/\$1_{x+1} \}$

The O3 next to 1_{x+1} is only 2 (also is true for 4, 6, 8, 10, 12, 14, 16, or 18 difference) different from 1_{x+1} , so the term is a single term 1/ For limited n primes, 1_{x+n} , we will have:

 $\Delta_{x3} = 1 - \sum_{x3} = n - n \{ [(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/23x2/3x6/7x10/11x12/17x12/17x12/17x18/19) + (1/23x2/17x10/11x12/17x18$

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/53x2/3x6/37x40/41x42/43x46/47) + (1/53x2/3x6/37x40/41x42/43x6/37x40/47) + (1/53x2/3x6/37x40/47) + (1/53x2/3x6/37x60/47) + (1/53x2/3x6/37x60/47) + (1/53x2/3x6/37x60/47) + (1/53x2/3x6/37x60/47) + (1/53x2/3x6/37) + (1/53x2/37) + (

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/5 3x58/59x60/61) + $(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + \dots] - [1/\$1_{x+1} + 1/\$1_{x+2} + 1/\$1_{x+3}, \dots + 1/\$1_{x+n}]$

If the number n of primes 1_{x+1} , 1_{x+2} , 1_{x+3} ,..., 1_{x+n} , $\geq 1_{x+1}$, then: $n(1/1_{x+1}+1/1_{x+2}+1/1_{x+3},...,1/1_{x+3}) = (1+1/1_{x+1}/1_{x+2}+1/1_{x+3},...,1/1_{x+3},...,1/1_{x+1}/1_{x+1}) = (1+1/1_{x+1}/1_{x+2}+1/1_{x+3},...,1/1_{x+3},...,1/1_{x+1}/1_{x+3}) = (1+1/1_{x+1}/1_{x+2}+1/1_{x+3},...,1/1_{x+1}/1_{x+3}) = (1+1/1_{x+1}/1_{x+2}+1/1_{x+3},...,1/1_{x+1}/1_{x+3}) = (1+1/1_{x+1}/1_{x+2}+1/1_{x+3},...,1/1_{x+1}/1_{x+3}) = (1+1/1_{x+1}/1_{x+2}+1/1_{x+3},...,1/1_{x+1}/1_{x+3}) = (1+1/1_{x+1}/1_{x+2}+1/1_{x+1}/1_{x+3},...,1/1_{x+1}/1_{x+3}) = (1+1/1_{x+1}/1_{x+2}+1/1_{x+1}/1_{x+3},...,1/1_{x+1}/1_{x+3}) = (1+1/1_{x+1}/1_{x+2}+1/1_{x+1}/1_{x+3},...,1/1_{x+1}/1_{x+3}) = (1+1/1_{x+1}/1_{x+2}+1/1_{x+3}) = (1+1/1_{x+1}/1_{x+3}) = (1+1/1_{x+1}/1_{x+$

The chance for O9 following a non-prime, \$7, to be a non-prime is: $\sum_{9} = [(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +$

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +

 $(1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) + \dots]$

The chance for O9 following a prime, \$7, to be a prime is: $\Delta_9=1-\sum_9=1-[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +$

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +

 $(1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) + \dots]$

Let assume the pair, $\$7_x$, $\$9_x$ is the largest twin pair, however we know both \$7 and \$9 are endless, there are $\$7_{x+1}$, $\$7_{x+2}$, $\$7_{x+3}$,... $\$7_{x+n}$, and $\$9_{x+1}$, $\$9_{x+2}$, $\$9_{x+3}$,... $\$9_{x+n}$. We need only find if there is at least 1 of $\$9_{x+n}$ follow 1 of $\$7_{x+n}$. Any \$7 can be divisible by only 1 and itself, so $\$7_{x+n}$ plus 2 (also plus 4, 6, 8, 10, 12, 16, or 18) cannot be divisible by $\$7_{x+n}$ because the smallest \$7 is 7 and the next \$7 is 17.

The chance for O9 following a prime, 7_x , to be a prime is: $\Delta_{x9}=1-\sum_{x9}=1-\{[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/29x2/3x28/29x30/31) + (1/29x2/3x28/29x30/31) + (1/29x2/3x28/29x30/31) + (1/29x2/3x28/29x30/31) + (1/29x2/23x28/29x30/31) + (1/29x2/23x28/29x30/3) + (1/29x2/23x28/29x30/2) + (1/29x2/23x28/29x28/$

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +

 $\begin{array}{l} (1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/5 \\ 3x58/59x60/61x66/67x70/71x72/73) + \ldots]-1/\$1_{x+1} \end{array}$

The O9 next to $$7_{x+1}$ is only 2 (also is true for 4, 6, 8, 10, 12, 16, or 18 difference) different from $$7_{x+1}$, so the term is a single term $1/$7_{x+1}$. For limited n primes, $$7_{x+n}$, we will have:

 $\Delta_{x9} = 1 - \sum_{x9} = n - n \{ [(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) \}$

+(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) +

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/59x2/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/59x2/23x28/29x30/31x36/37x40/41x42/43x46/47x52/23x28/29x30/31x36/37x40/41x42/43x46/47x52/23x28/29x20/30) + (1/59x26/27x40/47x60/47x

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +

 $\begin{array}{l} (1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/5\\ 3x58/59x60/61x66/67x70/71x72/73) + \ldots] - [1/\$7_{x+1} + 1/\$7_{x+2} + 1/\$7_{x+3}, \ldots + 1/\$7_{x+n}] \rbrace$

If the number n of primes 7_{x+1} , 7_{x+2} , 7_{x+3} ,..., $7_{x+n} \ge 7_{x+1}$, then: $n(1/7_{x+1}+1/87_{x+2}+1/87_{x+3}+1/87_{x+3}+1/87_{x+3}) = (1+87_{x+1}/87_{x+2}+87_{x+1}/87_{x+3},...,87_{x+1}/87_{x+n}) = (1+87_{x+1}/87_{x+2}+87_{x+1}/87_{x+3},...,87_{x+1}/87_{x+1}) = (1+87_{x+1}/87_{x+2}+87_{x+1}/87_{x+3},...,87_{x+1}/87_{x+1}) = (1+87_{x+1}/87_{x+2}+87_{x+1}/87_{x+1}) = (1+87_{x+1}/87_{x+2}+87_{x+1}/87_{x+1}) = (1+87_{x+1}/87_{x+2}+87_{x+1}/87_{x+1}) = (1+87_{x+1}/87_{x+1}) = ($

The chance for O1 following \$9, to be a non-prime is: $\sum_{1} = [(1/3) + (1/7x^2/3) + (1/7x^2/3)]$

(1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +

(1/23x2/3x6/7x10/11x12/13x16/17x18/19) +

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

 $(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + \dots]$

The chance for O1 following \$9, to be a prime is: $\Delta_1=1-\sum_1=1-[(1/3) + (1/7x^2/3) + (1/7x^2/3)]$

(1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +

(1/23x2/3x6/7x10/11x12/13x16/17x18/19) +

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

 $(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + \dots]$

Let assume the pair, $\$9_x$, $\$1_x$ is the largest twin pair, however we know both \$9 and \$1 are endless, there are $\$9_{x+1}$, $\$9_{x+2}$, $\$9_{x+3}$,... $\$9_{x+n}$, and $\$1_{x+1}$, $\$1_{x+2}$, $\$1_{x+3}$,... $\$1_{x+n}$. We need only find if there is at least 1 of $\$1_{x+n}$ follow 1 of $\$9_{x+n}$. Any \$9 can be divisible by only 1 and itself, so $\$9_{x+n}$ plus 2 (also plus 4, 6, 8, 10, 12, 14, 16, or 18) cannot be divisible by $\$9_{x+n}$ because the smallest \$9 is 19 and the next \$9 is 29.

The chance for O1 following a prime, $\$9_x$, to be a prime is: $\Delta_{x1}=1-\sum_{x1}=1-\{[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/4x2/3x6/7x40/41x42/43) + (1/4x2/3x6/7x40/41x40/44$

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

 $(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + \dots] - 1/\$1_{x+1} \}$

The O1 next to 9_{x+1} is only 2 (also is true for 4, 6, 8, 10, 12, 14, 16, or 18 difference) different from 9_{x+1} , so the term is a single term $1/\$9_{x+1}$. For limited n primes, 9_{x+n} , we will have:

$$\begin{split} &\Delta_{x1} = 1 - \sum_{x1} = n - n \{ [(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/23x2/17x10/11x12/17x18/19) + (1/23x2/17x18/19) + (1/23x2/17x18/19) + (1/23x2/17x18/19) + (1/23x2/17x18/19) + (1/23x2/17x18/19) + (1/23x2/17x18/17x18/19) + (1/23x2/17x18/17x18$$

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +

(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +

(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

 $(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + \dots] - [1/\$9_{x+1} + 1/\$9_{x+2} + 1/\$9_{x+3}, \dots + 1/\$9_{x+n}] \}$

If the number n of primes 9_{x+1} , 9_{x+2} , 9_{x+3} ,..., $9_{x+n} \ge 9_{x+1}$, then: $n(1/9_{x+1}+1/9_{x+2}+1/9_{x+3},...,1/9_{x+3}) = (9_{x+1}/9_{x+1}) = (9_{x+1}/9_{x+1}+9_{x+1}/9_{x+2}+9_{x+1}/9_{x+3},...,9_{x+1}/9_{x+3}) = (1+9_{x+1}/9_{x+2}+9_{x+1}/9_{x+3},...,9_{x+1}/9_{x+3}) = (1+9_{x+1}/9_{x+2}+9_{x+1}/9_{x+3}) = (1+9_{x+1}/9_{x+3},...,9_{x+1}/9_{x+3}) = (1+9_{x+1}/9_{x+2}+9_{x+1}/9_{x+3}) = (1+9_{x+1}/9_{x+3}) = (1+9_$

It is easy to prove (\$3, \$7, p+4 prime), (\$7, \$1, p+4 prime), (\$1, \$7, sexy prime), (\$7, \$3), (\$3, \$9), (p, p+8), (p, p+10), (p, p+12), (p, p+14), (p, p+16), and (p, p+18) are endless.

References:

- 1. Dudley, Underwood (1978), Elementary number theory (2nd ed.), W. H. Freeman and Co., Section 2, Theorem 2.
- 2. Dudley, Underwood (1978), Elementary number theory (2nd ed.), W. H. Freeman and Co., Section 2, Lemma 5.
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