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The relativistic origin of the electric charge

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Abstract Considering the electron in the hydrogen atom as classical, and bound to the proton like a planet is bound to the sun, we are led to consider that it is in free fall and therefore that we can apply the Einstein's equivalence principle, thus the special relativity can be used to study its motion. Doing so, we are able to demonstrate that the electron's charge-to-mass ratio is the subsequent relativistic frequency that appears to the observer in the laboratory. We also show that a magnetic moment, very similar to the one of the quantum mechanics, must appear, although we stay in the fields of classical and relativistic physics.

Keywords Keplerian motion · Special relativity · Electromagnetism · Electric charge

1 Introduction

The existence of the electron's charge-to-mass ratio can be proven, and its value calculated, if we consider the electron as classical and in free fall with respect to the proton. The purpose of this work is to demonstrate this, by the mean of the classical mechanics and the special relativity.

At a first sight we consider that the speed v of an electron in the hydrogen atom, assumed circular and uniform, can not be used in the dilatation factor of the special relativity [2], $\gamma = \sqrt{1 - v^2/c^2}$, because its frame of reference should not be inertial, as it is rotating. However considering the electron around the proton in a classical manner, as a planet around the sun, it should not be in an accelerated frame but in free fall. This fact changes the point of view

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because we know that in a reference frame that is in free fall, the laws of physics are the same as if there were no gravity at all, and then the laws of physics are those of special relativity [1]. This is all the purpose of the Einstein's equivalence principle. Of course we have here to consider that the force that binds the electron to the proton is equivalent to a gravitational one, and this is consistent with our clasical aproach. In this paper we propose to investigate this point of view.

We divided this work in three parts. First we recall the result of the the special relativity concerning all periodic motions, and especially the appearance of a relativistic frequency. Nothing new in this. Second we recall the basic properties of the electron (radius, frequency, speed) in the hydrogen atom, by the mean of the keplerian kinematics. Nothing new in this neither. Third, and this is the new part, we assemble the first and second part of this work by considering that the rotation speed of the electron is the one to use in the dilatation factor of the special relativity. This is where the charge-to-mass ratio appears, as well as the magnetic moment, the Bohr's magneton, both in agreement with the measured experimental values.

2 Relativistic angular frequency

A. Einstein demonstrated [2] that an observer looking at a moving clock will note a time dilatation between the frame of reference of the clock and his own frame of reference. This leads to the twin paradox of P. Langevin [3]. The time dilatation is given by the following formula :

$$\Delta t_{obs} = \gamma \Delta t \text{ with } \gamma = 1/\sqrt{1 - v^2/c^2} \tag{1}$$

In this expression Δt_{obs} is the interval of time in the observer's frame of reference, Δt is the interval of time in the moving clock's frame of reference, γ is the dilatation factor, v is the velocity of the moving clock with regards to the observer, and c is the speed of light.

This famous relationship establishes that any periodical motion will see its period affected by this time dilatation. This happens for the electron of the hydrogen atom, that orbits on a circular trajectory around the proton. In this case the observer in the laboratory will note an angular frequency shorter than the frequency measured from the electron's frame of reference. Indeed, if $\Delta t = T$ is the period of rotation, then $\omega = 2\pi/T$ is the angular frequency, and the relation (1) leads to :

$$\omega_{obs} = \omega\sqrt{1 - v^2/c^2} \tag{2}$$

Now, accepting the Planck-Einstein relationship [4], the energy of the electron in the observer's frame of reference will be :

$$E_{obs} = \hbar\omega_{obs} = \hbar\omega\sqrt{1 - v^2/c^2} \tag{3}$$

Developing this expression as a second order polynomial leads to

$$E_{obs} \approx \hbar\omega + \hbar\omega_R \text{ with } \omega_R = -\frac{1}{2}\frac{v^2}{c^2}\omega \tag{4}$$

We call ω_R relativistic angular frequency, and we are going to show that it is very closely related to the charge-to-mass ratio, at the condition that the velocity v can be the one of the electron on its orbit.

But first, in order to give a numerical value to ω_R we shall calculate v and ω as the speed and angular frequency of the electron in its own frame of reference. This must be done by the mean of the Kepler's kinematics, as far as we consider the problem from a classical/relativistic point of view.

3 The electron in the hydrogen atom

In a classical vision of the atom, there is no reason why the electron would not respect the Kepler's laws around the nucleus. The most simple keplerian motion being circular uniform, we assume that this motion is the most fundamental for the electron in the hydrogen atom. Therefore our demonstration will be based on the kinematic properties of the keplerian motion, rather than using the famous Bohr's model [5]. The result will be nearly the same but the demonstration will be much simpler.

Let first write the fundamental equations that drives the trajectory, as given by the keplerian kinematics. Calling $\mathbf{L} = \mathbf{r} \wedge \mathbf{v}$ the kinematic momentum (the *massless angular momentum*, as R. H. Battin described it [6]), and k a (massless) constant, any keplerian circular motion shall verify the following formulas :

$$v = \omega r = L/k \ , \ r = L^2/k \text{ and } \omega = k^2/L^3 \tag{5}$$

For a gravitation problem, $k = GM$, where G is the universal gravitational constant, and M is the attracting mass. In this case L is a constant depending upon each particular case.

For the electron of the hydrogen atom we have :

$$k = \frac{e^2}{4\pi\epsilon_0 m_e} = 253.27 m^3 s^{-2} \text{ and } L = \frac{\hbar}{m_e} = 1.15710^{-4} m^2/s \tag{6}$$

where m_e is the mass of the electron, e is the electric charge and ϵ_0 is the permittivity of the vacuum.

These last values enable to calculate the radius, the velocity and the angular frequency, from the relations 5 :

$$r = 5.2910^{-11} m \ , \ \omega = 4.14310^{16} Hz \text{ and } v = 2.18910^6 m/s \tag{7}$$

The radius is the same as the one calculated By Bohr [5], but the angular frequency is twice the one of Bohr, and consequently the speed also. Note that the ratio v/c that we get here is equal to 1/137.035, which is the fine-structure constant. This result is trivial when considering the expressions 5 and 6 , with regards to the usual fine-structure constant structure given by A. Sommerfeld [7], i.e. $\alpha = e^2/(4\pi\epsilon_0\hbar c)$.

4 Assembling the relativistic and the keplerian atom

As we announced in the introduction, we are now going to see what happens if the rotation speed of the electron can be the one to use in the dilatation factor 1.

Introducing the speed and frequency values 7 into the equation 4 of the relativistic frequency, we get $\omega_R = -1.10510^{12}Hz$. This particular value is numerically the charge-to-mass ratio multiplied by 2π, so $2\pi e/m_e$. Note that we consider e as a positive number, i.e. the absolute value of the charge. To integer such a result into the classical physics, we must consider that the charge, so the coulomb, has the dimension of kilogram per second : 1C = 1kg/s. Doing so, we verify :

$$\omega_R = -\frac{1}{2}\frac{v^2}{c^2}\omega = -2\pi e/m_e \tag{8}$$

As far as we know, this relationship, although very simple, has never been proposed. Its agreement with the experiment is however excellent. It says that the charge-to-mass ratio is nothing else but a frequency, a relativistic one.

Let now show how this result leads straight forward to the forecast of the Bohr's magneton. To achieve so we must first rewrite the expression of the energy of the relation 3 :

$$E_{obs} \approx \hbar\omega - 2\pi e\hbar/m_e \tag{9}$$

The second term of this expression is equal to the Bohr's magneton [8] multiplied by 4π. This suggests that the relativistic angular frequency can be related to the magnetic moment of the electron. Let us propose a way to do so.

Imagine a force, acting on the electron, being the product of the charge by the velocity : $\mathbf{f} = -e\mathbf{v}$. Such a force will produce a work $dW = \mathbf{f}.\mathbf{v}dt$. Because the motion is uniform, v^2 is a constant, and the total work produced in a period of rotation will be :

$$W = -ev^2T = -2\pi evr = -2\pi e\hbar/m_e \tag{10}$$

W is then nothing else but the second term of the equation 9.

Such a force, collinear to the velocity in a uniform circular motion, will also be at the origin of a moment : $\boldsymbol{\mu} = \mathbf{r} \wedge \mathbf{f} = -e\mathbf{L}$, where $\mathbf{L}$ is the vector kinematic momentum, which norm is given by the equation 6. It is then very trivial that the momentum is equal to twice the Bohr's magneton : $\boldsymbol{\mu} = 2\boldsymbol{\mu}_B$.

At a classical point of view, we can not say that the force $\mathbf{f}$ is an effective reality, it is rather a kind of fictive, because only the second term of the equation (9) is a reality, and it is just an energy. However it might be mathematically practical to use the idea of fictive force, remembering that it is not a true classical force following the Newton's law. In particular it is impossible to predict an orientation for such a force.

This fictive force, makes non intuitive any attempt to give a classical explanation of the electron's magnetic moment. This is a very close situation to the same concept in quantum mechanics.

5 Discussion

We proposed to see what happens if we consider the electron as classical and in free fall with regard to the proton, especially in the hydrogen atom. We then got an explanation of the charge-to-mass ratio in terms of relativistic frequency, as well as an explanation of the existence of a magnetic moment, as experimentally measured, but not expected by the classical mechanics.

Sure at a first sight we had to overcome the strict postulate of Einstein, that only inertial frames are concerned by the special relativity. However we may consider that the electron is not in an accelerated frame. Indeed if the electron is considered classical, and thus compared to a planet orbiting around the sun, it should be in free fall with respect to the proton. But we know that in a reference frame that is in free fall, the laws of physics are the same as if there were no gravity at all, and then the laws of physics are those of special relativity [2]. This is all the purpose of the Einstein's equivalence principle. Doing so we consider that the force binding the electron and the proton is equivalent to a gravitation force.

Admitting this state of the electron in the atom, we are led to calculate that a relativistic frequency must appear for an observer in the laboratory, and this frequency gives directly the charge to mass ratio of the electron. The conseque is that the charge must have the dimension of kilogram per second, kg/s. This gives the same dimension to the classical moment of a force and to the electron's magnetic moment, both having the dimension of kgm^2s^{-2}. This is clearly a bridge between the electromagnetism and the kinetics.

An other interesting point is the charge-to-mass ratio can not be split in two parts, the charge in one hand, the mass in the other. It is a frequency, not a composed parameter. Accordingly all experimental physicists know that it is impossible to measure strictly the charge alone, or strictly the mass alone, but only the charge-to-mass ratio.

We also understand why the classical objects, at a macroscopic scale, have no charge : their speed is too low with regard to the speed of light to exhibit a measurable charge-to-mass ratio. For instance this is the case for a planet orbiting around the sun.

As far as these facts are all in agreement with the experiment, we are driven to consider that the force binding the electron to the proton is equivalent to a gravitational force and therefore the charge has a relativistic origin.

References

[1] Albert Einstein, Zur Elektrodynamik bewegter Krper, An. Phys., vol. 17, 30 juin 1905, p. 891-921

[2] Markus Pssel, AEI, " The elevator, the rocket, and gravity: the equivalence principle " in: Einstein Online Vol. 01 (2005), 1009

[3] Paul Langevin, Lvolution de lespace et du temps, Scientia, [10], 1911, p. 31-54

[4] Albert Einstein, ber einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt, An. Phys., [17], no 6, 1905, p. 132148

[5] N. Bohr, On the Constitution of Atoms and Molecules, Philos. Mag. 26, 1, 1913

[6] R. H. Battin, An Introduction to the Mathematics and Methods of Astrodynamics, Revised Edition, American Institute of Aeronautics and Astronautics, Inc., Reston, 1999 (1975).

[7] A. Sommerfeld, Zur Quantumtheorie der spektrallinen, An. Phys., [51], p 51, 1916

[8] S. Procopiu, Determining the Molecular Magnetic Moment by M. Plancks Quantum Theory, Bulletin Scientifique de lAcadmie Roumaine de Sciences, Bucharest, 1913