New Rotational Doppler-Effect

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Abstract

By oblique reflection of circularly polarized photons on a rotating cylindrical mirror the frequency of the reflected photons is shifted against the ferquency of incident photons by nearly twice the rotational frequency n of the mirror: $\Delta\nu = 2\,n\,\sin\alpha$, where α is the axial angle of incidence. $\Delta\nu$ can be substantially enhanced by multiple reflections between counter-rotating coaxial mirrors.

1 Introduction

Rotational Doppler-Effects (RDE) are characterized by interaction of circularly polarized photons with rotating objects like filters, scattering surfaces, specular or diffuse reflection where the reflected photons change their frequency relative to that of the incident photons. [1] [2] [3] [4]
A specific advantage of the RDE is its continuous mode of operation, while the translational Doppler-effect usually is intermittent. Fine tuning within a fraction of one Hz up to the MHz domain would be possible.
The new RDE is free of undesired scattering or diffraction effects which may disturb the coherency of frequency shift.

2 Theoretical background

During oblique specular reflection of a photon, its spin angular momentum component parallel to the reflecting surface is inverted while its vertical components remains unchanged. Angular momentum conservation requires that the mirror must absorb the change of parallel spin angular momentum. If reflection occurs on a rotating cylinder with rotation axis parallel to a spin component of an incident photon, energy is exchanged among the photon and the rotating mirror, resulting in a frequency shift among the incident and the reflected photon, as specified below.
The analysis is substantially simplified by restriction to the axial (x-axis) component of all involved angular momentums. The axis of rotation of the mirror is located in the plane defined by the lines of propagation of incident and reflected photons

3 Setup

The setup is a cylindrical mirror or a reflecting shaft rotating around its cylindrical axis, and a circularly polarized laser-beam hitting and leaving the reflecting surface at an oblique angle α .

4 Angular momentum conservation

Take $L_0 = \hbar$ as angular momentum of an incident photon and $L_{0x} = \hbar \sin\alpha$ as the x-component of L_0. After oblique reflection on the rotating cylinder, the x-component of angular momentum is inverted to $L_{1x} = -L_{0x} = -\hbar \sin\alpha$. In result, the difference of angular momentums among the incident and the reflected photon is

$$\Delta L = L_{0x} - L_{1x} = 2\,\hbar \sin\alpha \tag{1}$$

Angular momentum conservation requires that above angular momentum increment ΔL is transferred to the rotating cylinder.

5 Energy conservation

According to Hamiltonian mechanics any transmission of angular momentum $\vec{\Delta L}$ to an object steadily rotating with $\vec{\omega}$ is combined with transmission of energy given by the scalar product

$$\Delta E = \vec{\omega}\,\vec{\Delta L} \tag{2}$$

After substitution with (1) in (2)

$$\Delta E = h\,\Delta\nu = 2\,\omega\,\hbar\,\sin\alpha = 2\,\omega\,\frac{h}{2\pi}\,\sin\alpha = 2\,n\,h\,\sin\alpha \tag{3}$$

6 Frequency shift

RDE-frequency-shift $\Delta\nu$ results from (3)

$$\Delta\nu = \Delta E/h = 2\,n\,\sin\alpha \tag{4}$$

Note that - unlike the translational Doppler-effect - $\Delta\nu$ is independent of the frequency ν of incident photons.

7 Enhanced RDE with coaxial counter-rotating cylinders

The new RDE can be substantially enhaced by a cascade of multiple reflections within a small cylindical gap between two counter-rotating coaxial cylindrical mirrors. In this case, one external and internal cylinder would be arranged such that they form a small cylindrical gap among their adjacent counter-rotating external and internal mirror faces. The beam of circularly polarized light would propagate throgh this gap in axial direction and be reflected several times in sequence. Let i be the number of reflections between the external and the internal cylinder. Then, the frequency shift would amount $i\,\Delta\nu$. Let l be the length of the cylinder, w the width of the gap and α the angle of incidence. Then the number i of reflections (amplification factor) within that gap is $i = \frac{l}{w}\cot\alpha$ and the frequency- shift

$$\Delta\nu_i = \frac{l}{w} \quad 2\,n\,\cos\alpha \tag{5}$$

Validity of (5) is restricted to $0 < \alpha < \pi/2$. The smallest possible gap w would be of an order of magnitude of the wavelength of the incident photons.

8 Synopsis

The new RDE is characterized by an utterly simple setup consisting of a rotating cylindrical mirror or shaft and a beam of circularly polarized light. The RDE frequency-shift is nearly twice the rotational frequency of the mirror. Its frequency domain covers a range between fractions of one Hz up to the MHz-domain, depending on the the rotational frequency of the cylindical mirror. Shaft rotational frequencies of $25 \times 10^6 rpm = 4,15 \times 10^5 Hz$ are already envisaged [4].
Substantial enhancement of the RDE-frequency shift would be possible with a pair of concentrically arranged counter-rotating cylindrical mirrors shaping a small cylindrical gap through which circularly polarized light must pass. The factor i of enhancement is given by the fraction of cylinder-length l and gap-width w.

9 References

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