Methods for Calculating the Pressure Field in the Tube Flow

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Abstract

In this paper we outline methods for calculating the pressure field inside flow conduits in the one-dimensional flow models where the pressure is dependent on the axial coordinate only. The investigation is general with regard to the tube mechanical properties (rigid or distensible), and with regard to the cross sectional variation along the tube length (constant or variable). The investigation is also general with respect to the fluid rheology as being Newtonian or non-Newtonian.

Keywords: fluid mechanics; axial pressure field; tube flow; rigid tube; distensible tube; Newtonian fluid; non-Newtonian fluid; uniform conduit; non-uniform conduit.

1 Introduction

In the flow of Newtonian and non-Newtonian fluids through conduits of different geometries and various wall mechanical properties the relation that is usually in demand is the volumetric flow rate as a function of the pressure at the two boundaries with no need for a detailed knowledge about the pressure field inside the conduit, because in the case of rigid tubes the flow rate generally depends on the difference between the pressure at the inlet and outlet boundaries, while for the distensible tubes the flow rate is dependent on the actual values of the pressure at the two boundaries [1]. Moreover, in the common cases of purely-viscous laminar axi-symmetric flow in rigid conduits with a constant radius along their length the pressure field can be obtained by simple linear interpolation and symmetric arguments with no need for elaborate calculations. Therefore, in most cases the pressure field inside the conduit is either not of primary interest to the investigator or it can be obtained with a minimal effort.

However, in some other circumstances the pressure as a function of the luminal space coordinates may be needed and it cannot be obtained from simple arguments

due to the complexity of flow situation arising for instance from a history dependent non-Newtonian fluid or tube wall distensibility or complicated tube geometry. For example, in assessing the durability and resistance of a pipeline in the risk assessment analysis or in the case of predicting the yield point of a yield-stress fluid in a distensible tube [2] detailed knowledge of more complex pressure fields is required. In such circumstances, elaborate analytical and numerical techniques are needed to obtain the pressure field.

Various methods have been used in the past to calculate the pressure field in the flow conduits. Many of these methods are based on the use of analytical techniques and apply to special cases of flow, fluid and mechanical and geometrical characteristics of conduit. Numerical discretization techniques such as finite element and finite difference methods have also been used for finding the pressure fields although in most circumstances the pressure field may be regarded as a byproduct of the primary computational objectives.

The literature of fluid dynamics contains numerous studies dedicated to the investigation and analysis of pressure fields inside flow conduits and storage vessels. For example, Miekisz [3] investigated the temporal and spatial dependency of the pressure inside an elastic tube in the context of hemodynamic viscous flow using a linear telegraph equation with the employment of analytical mathematical techniques. Oka [4] studied the pressure field in a tapered tube and developed a formula for calculating the pressure gradient as a function of the axial coordinate for the viscous flow of non-Newtonian fluids using power law, Bingham and Casson models. In a series of studies, Degasperi and coworkers [5–8] investigated the pressure field distribution in tubes with different geometries under transient and steady state conditions in the context of injection and outgassing of vacuum devices using analytical and numerical techniques. Naili and Thiriet [9] studied the pressure field in straight rigid tubes with a uniform cross section in the axial direction under Newtonian incompressible flow conditions using five different cross sectional shapes. There are many other studies (e.g. [10–16]) which are not dedicated to computing the pressure although the methods and results can be used to obtain the pressure field. There are also numerous studies related to the pressure wave propagation in distensible tubes and blood vessels (e.g. [17–24]); a subject which is not included in the scope of the present paper.

In the one-dimensional flow models, where the flow is assumed to have axial dependency only in the z direction of the cylindrical coordinate system, the pressure is well defined at each axial point with no dependency on the azimuthal angle or

the radial distance. For rigid tubes with constant radius along their axial length the pressure is generally a linear function of the axial coordinate, i.e. constant pressure gradient, and hence the axial pressure can be trivially evaluated from the given boundary conditions through linear interpolation. However, for rigid tubes with varying cross section along their axial direction and for distensible tubes in general the pressure is not a linear function of the axial coordinate and hence the axial pressure cannot be computed from the given boundary conditions using a simple linear interpolation formula. A similar situation may also arise in the flow of history dependent (i.e. viscoelastic and thixotropic/rheopectic) non-Newtonian fluids in rigid tubes with a constant radius due to the possible change of rheology and flow profile along the tube length and hence the axial pressure field may not be a linear function of the axial coordinate.

In the present investigation we outline some methods that have been used or can be used in the calculation of the axial pressure field in the one-dimensional flow models. Although, the investigation is general with regard to the type of fluid as Newtonian or non-Newtonian, and the type of conduit i.e. with constant or variable cross section and rigid or distensible mechanical wall characteristics, it does not extend to the case of history dependent fluids due to the complexity of these problems and the lack of viable and sufficiently simple flow models that can be used to analyze such flows. For the generality of discussion and results, we assume throughout this investigation a laminar, steady state, incompressible, slip-free, one-dimensional, fully developed flow with negligible body and inertial forces and minor entrance and exit effects although some of these assumptions and conditions can be relaxed in some circumstances.

2   Methods

In this section we outline some of the methods that can be used for finding the axial pressure field inside a tube containing a fluid and subject to a pressure gradient. As indicated already, the tube is more general than being circular cylindrical with rigid walls and constant cross section along its length under Newtonian flow conditions. However, there is no single method that is sufficiently general to apply to all these variations of flow, fluid and conduit in all circumstances as will be clarified in the forthcoming subsections.

2.1 Solving Navier-Stokes Equations

Using Navier-Stokes flow equations, which are based on the mass and linear momentum conservation principles, is the most common way for describing the flow of fluids and obtaining its parameters including the pressure field. One example of using the Navier-Stokes equations to obtain the axial pressure is the case of the one-dimensional form of the Navier-Stokes flow in distensible tubes where an implicitly defined axial pressure field can be obtained analytically (see [2] and Equations 7). However there are many limitations on the validity and direct use of the Navier-Stokes equations. One reason is the limiting assumptions on which the Navier-Stokes flow is based; for example the fluid is assumed Newtonian and hence any extension beyond this requires an extension to the Navier-Stokes model.

There are also many practical difficulties in obtaining analytical solutions from the Navier-Stokes equations due to their non-linear nature associated potentially with other mathematical difficulties originating for example from a complex flow geometry. So even if the Navier-Stokes flow is valid in principle for describing the problem the subsequent mathematical difficulties may prevent obtaining a solution. However, numerical methods can in most cases bridge such gaps and hence numerical solutions can be obtained. Another difficulty is that even if an analytical solution can be obtained it may not be possible to obtain an explicit functional form of the axial pressure in terms of the axial coordinate and hence a numerical method is needed to obtain the pressure field numerically from its implicit definition. However, obtaining a solution numerically from an implicit form of an analytical expression is easy through the use of a simple numerical solver based for instance on a bisection method.

2.2 Using Flow Relation

This method is based on the exploitation of an available relationship correlating the volumetric flow rate to the two pressure boundary conditions at the inlet and outlet. An equation can then be formed by using the boundary conditions and an arbitrary axial point with the elimination of the flow rate by equating the flow rate in one part of the tube to the flow rate in the other part. For example if we have the following general relation

$$Q = f(p_i, p_o, L) \tag{1}$$

where Q is the volumetric flow rate, f is a function with a known form, p_i and p_o are the inlet and outlet pressure respectively, and L is the tube length, then we can form two equations each one with one boundary condition and an arbitrary axial point, i.e.

$$Q = f(p_i, p_x, x) \qquad \text{and} \qquad Q = f(p_x, p_o, L - x) \qquad (2)$$

where p_x is the axial pressure at an arbitrary axial coordinate $0 < x < L$. Because the flow rate is constant, due to the incompressible steady state assumptions, then by eliminating Q between the above two equations a single equation can be formed which links explicitly or implicitly p_x to p_i and p_o and hence p_x can be obtained either directly, if it is explicitly defined, or by the use of a simple numerical solver based for instance on the bisection method if it is an implicit function.

Alternatively, the value of the volumetric flow rate Q is computed from the two given boundary conditions and then substituted in the p-Q relation with one of the given boundary conditions. The second boundary condition is then replaced with the condition at an arbitrary axial point $0 < x < L$ and the resulting equation is solved, usually numerically, to obtain the unknown "boundary condition" which is the axial pressure at the internal point with x coordinate.

We clarify this with an example from the one-dimensional Navier-Stokes flow in elastic tubes where a linear elastic relation between the axial pressure and the corresponding cross sectional area of the following form is used to describe the mechanical response

$$p = \gamma (A - A_o) \qquad (3)$$

where p is the axial pressure, γ is a stiffness coefficient, A is the tube cross sectional area at the axial pressure p, and A_o is the reference area corresponding to the reference pressure which, in this equation, is set to zero for convenience without affecting the generality of the results.

For the distensibility mechanical relation of Equation 3, the following formula that links the volumetric flow rate to the inlet and outlet boundary conditions is derived and validated previously [1]

$$Q = \frac{L - \sqrt{L^2 - 4\frac{\alpha}{\kappa} \ln (A_{ou}/A_{in}) \frac{\gamma}{3\kappa\rho} (A_{in}^3 - A_{ou}^3)}}{2\frac{\alpha}{\kappa} \ln (A_{ou}/A_{in})} \qquad (4)$$

where α is a correction factor for the axial momentum flux, κ is a viscosity friction

coefficient, ρ is the fluid mass density, and A_{in} and A_{ou} are the tube cross sectional area at the inlet and outlet respectively. If this equation is solved for two given boundary conditions to obtain a value for the volumetric flow rate Q_v, then the following equation can be formed using the inlet boundary condition

$$\frac{x - \sqrt{x^2 - 4\frac{\alpha}{\kappa}\ln(A_x/A_{in})\frac{\gamma}{3\kappa\rho}(A_{in}^3 - A_x^3)}}{2\frac{\alpha}{\kappa}\ln(A_x/A_{in})} = Q_v \qquad (5)$$

where A_x is the tube cross sectional area at an arbitrary axial point $0 < x < L$. The latter equation can then be solved numerically for A_x and the pressure at x is then obtained from A_x by using Equation 3. Alternatively, the outlet boundary condition can be used with an arbitrary axial point representing an inlet boundary, that is

$$\frac{(L-x) - \sqrt{(L-x)^2 - 4\frac{\alpha}{\kappa}\ln(A_{ou}/A_x)\frac{\gamma}{3\kappa\rho}(A_x^3 - A_{ou}^3)}}{2\frac{\alpha}{\kappa}\ln(A_{ou}/A_x)} = Q_v \qquad (6)$$

The main requirement for the applicability of this method is that the characteristic flow equation is indifferent to the axial coordinate due to the fact that the tube has a uniform reference cross section, i.e. either rigid with a constant cross section or distensible with a constant cross section under reference pressure.

One limitation of this method is the need for a pressure-flow rate relationship which usually should be obtained analytically. Another limitation is the aforementioned requirement which restricts the method to certain geometries as required by the p-Q expression, and hence it cannot be used for instance to find the pressure field in converging-diverging geometries that have analytical p-Q relations [25, 26]. Also, the method normally requires the employment of a numerical solver because the resulted expression is usually implicit in its definition of the axial pressure as a function of the given conditions; although only a simple numerical solver, such as a bisection solver, is needed.

2.3 Traditional Meshing Techniques

Another method is the use of the traditional meshing techniques such as finite element and finite difference [27–31]. These methods are widely used for solving flow problems although they may not be the best available option for finding the pressure field due partly to the approximations and possible errors and bugs; moreover most of these methods are not easy to implement.

2.4 Residual-Based Lubrication

For any history independent characteristic flow, such as the flow of Newtonian fluids in elastic tubes or the flow of non-Newtonian fluids of a Herschel-Bulkley type into rigid tapered tubes, there is a characteristic flow relation that links the volumetric flow rate through the tube to the two pressure boundary conditions at the inlet and outlet. The residual-based lubrication method exploits such a characteristic flow relation by discretizing the flow conduit into axially-divided thin slices where the flow relation is assumed to equally apply to each one of these slices. These slices either have the same width or have different widths but where the largest width is below a predefined maximum limit.

For a conduit discretized into $(N-1)$ slices there are N axial nodal points: two boundaries and $(N-2)$ interiors. A system of N simultaneous equations based on the two boundary conditions for the boundary nodes and the conservation of flow rate, which is equivalent to the conservation of mass for an incompressible flow, for the interior nodes is then formed and solved by a non-linear solution scheme such as Newton-Raphson method subject to a predefined error tolerance. The result of this solution is the axial pressure at the interior nodes, which defines the pressure field, and the volumetric flow rate. More detailed description of this method can be obtained from [32–34].

The residual-based lubrication method is the most general method for obtaining the pressure field as compared to the other methods in terms of the fluid, i.e. Newtonian or non-Newtonian, and the flow conduit, i.e. rigid or distensible, with a constant or variable cross section along its axial direction. In fact the only requirement for the applicability of the residual-based lubrication method is the availability of a characteristic flow relation that describes the flow on each slice, where this relation can be analytical, empirical or even numerical [35]. Therefore, the flow conduit is not required to have a constant shape or size in the axial direction. In fact the residual-based method can equally apply to conduits with non-circular cross sections. Moreover, it can apply in principle to conduits with more than one characteristic flow relation where these different relations apply to different parts of the conduit, e.g. one apply to a first part with circularly-shaped cross sections and another apply to a second part with squarely-shaped cross sections. However, the residual-based lubrication method can not be applied in some circumstances due to the lack of a characteristic flow relation; e.g. in most cases of flow of non-Newtonian fluids through distensible conduits.

The residual-based lubrication method can provide very accurate solutions if a

proper discretization with a low error tolerance were employed. To demonstrate this fact, in Figure 1 we compare the analytical solution for the pressure field in a distensible tube with a Navier-Stokes one-dimensional flow to the numerical solution as obtained from the residual-based method. The analytical solution, which is derived in [2], is given by

$$
x = \frac{2\alpha Q \ln\left(\frac{\frac{A_o}{\beta}p+\sqrt{A_o}}{\frac{A_o}{\beta}p_i+\sqrt{A_o}}\right)}{\kappa} + \frac{\beta\left[\left(\frac{A_o}{\beta}p_i+\sqrt{A_o}\right)^5 - \left(\frac{A_o}{\beta}p+\sqrt{A_o}\right)^5\right]}{5\rho\kappa Q A_o} \tag{7}
$$

where β is a stiffness coefficient in the presumed non-linear pressure-area elastic relation, while the other symbols are as defined previously. The analytical relation of Equation 7 is based on the following distensibility elastic relation, whose essence is that the axial pressure is proportional to the corresponding radius change with a proportionality stiffness factor scaled by the reference area, that is

$$
p = \frac{\beta}{A_o}\left(\sqrt{A} - \sqrt{A_o}\right) \tag{8}
$$

Other advantages of the residual-based lubrication method include relative ease of implementation, reliable and fast convergence, and low computational costs in terms of memory and CPU time as compared to equivalent methods like finite element.

2.5 Special Methods

As indicated early, many of the previous attempts to find the pressure field are based on special mathematical and computational methods which are tailored for the given problem with the presumed geometry and flow characteristics. A sample of these attempts and the employed methods have been highlighted in the Introduction. It is out of the scope of the present paper to go through these special methods.

3 Conclusions

In this study we outlined general methods that can be used to obtain the axial pressure as a function of the axial coordinate for the Newtonian and non-Newtonian one-dimensional flow in the rigid and distensible tubes with constant and variable cross sections along their length. As seen, there is no single method that can be used in all cases although the residual-based lubrication method is the most general
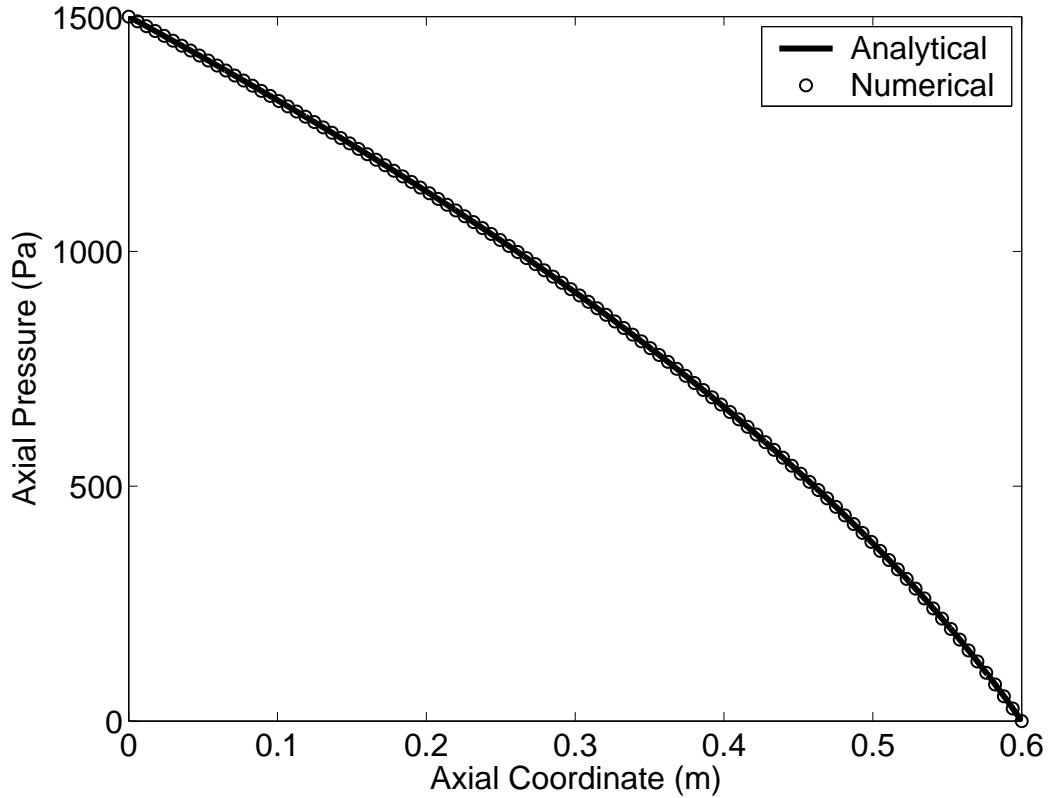
Figure 1: Comparison between the analytical solution given by Equation 7 and the numerical solution obtained from the residual-based lubrication method for the one-dimensional Navier-Stokes flow in an elastic tube. The parameters for this example are: $A_o = 0.0201062$ m^2, $L = 0.6$ m, $\alpha = 1.25$, $\beta = 10635$ Pa.m, $\rho = 1100$ kg.m^{-3}, $\mu = 0.035$ Pa.s, $p_i = 1000$ Pa, $p_o = 0$ Pa, and $Q = 0.101768$ m^3.s^{-1}.

one with respect to the type of fluid (Newtonian or non-Newtonian), and the type of tube (rigid or distensible, having a uniform cross section or axially varying cross section). The residual-based method has also other advantages such as comparative ease of implementation, low computational cost in terms of CPU time and memory, fast and reliable convergence and highly-accurate solutions which compare in their accuracy to any existing or potential analytical solutions if sufficiently fine discretization and low error tolerance are used.

Nomenclature

α correction factor for axial momentum flux

β stiffness coefficient in the non-linear pressure-area elastic relation

γ stiffness coefficient in the linear pressure-area elastic relation

κ viscosity friction coefficient

μ fluid dynamic viscosity

ρ fluid mass density

A cross sectional area of elastic tube at given axial pressure

A_{in} cross sectional area of elastic tube at inlet

A_o cross sectional area of elastic tube at reference pressure

A_{ou} cross sectional area of elastic tube at outlet

A_x cross sectional area of elastic tube at x coordinate

L tube length

p pressure

p_i inlet boundary pressure

p_o outlet boundary pressure

Q volumetric flow rate

Q_v numeric value of volumetric flow rate

x tube axial coordinate

References

[1] T. Sochi. Navier-Stokes Flow in Cylindrical Elastic Tubes. *Journal of Applied Fluid Mechanics (Accepted)*. 1, 5

[2] T. Sochi. The Yield Condition in the Mobilization of Yield-Stress Materials in Distensible Tubes. *Submitted*, 2013. arXiv:1311.6513. 2, 4, 8

[3] S. Miekisz. The Flow and Pressure in Elastic Tube. *Physics in Medicine and Biology*, 8(3):319, 1963. 2

[4] S. Oka. Pressure development in a non-Newtonian flow through a tapered tube. *Rheologica Acta*, 12(2):224–227, 1973. 2

[5] F.T. Degasperi; S. Verardi; M. Martins; J. Takahashi. Pressure Field Distribution in a Cylindrical Geometry with Arbitrary Cross Section. *9th European Particle Accelerator Conference, Lucerne, Switzerland, 5-9 July 2004*, pages 1597–1599, 2004. 2

[6] F.T. Degasperi; M.N. Martins; J. Takahashi; S.L.L. Verardi. Pressure field in a tube with a general and arbitrary time- and position-dependent gas source. *The Journal of Vacuum Science and Technology A*, 22:2022, 2004. 2

[7] F.T. Degasperi; V. Baranauskas. Mathematical Physics Formulation to Pressure Field in a Tube with an Arbitrary Axisymmetric Cross Section. *16th International Vacuum Conference - IVC-16, Venice, Italy*, 2004. 2

[8] F.T. Degasperi; M.N. Martins; J. Takahashi; S.L.L. Verardi. Pressure Field Distribution in a Conical Tube with Transient and Outgassing Gas Sources. *Proceedings of 2005 Particle Accelerator Conference, Knoxville, Tennessee*, 2005. 2

[9] S. Naili; M. Thiriet. Pressure field in flow through uniform straight pipes with varying wall cross curvature. *Computers in Biology and Medicine*, 35(8):645–663, 2005. 2

[10] A.C.L. Barnard; W.A. Hunt; W.P. Timlake; E. Varley. A Theory of Fluid Flow in Compliant Tubes. *Biophysical Journal*, 6(6):717–724, 1966. 2

[11] A. Lahbabi; H-C. Chang. Flow in periodically constricted tubes: Transition to inertial and nonsteady flows. *Chemical Engineering Science*, 41(10):2487–2505, 1986. 2

[12] M.F. Letelier; D.A. Siginer; C. Caceres. Pulsating flow of viscoelastic fluids in straight tubes of arbitrary cross-section-Part I: longitudinal field. *International Journal of Non-Linear Mechanics*, 37(2):369–393, 2002. 2

[13] S.H. Momeni-Masuleh; T.N. Phillips. Viscoelastic flow in an undulating tube using spectral methods. *Computers & fluids*, 33(8):1075–1095, 2004. 2

[14] D. Tripathi. Peristaltic Flow of a Fractional Second Grade Fluid Through a Cylindrical Tube. *Thermal Science*, 15(2):S167–S173, 2011. 2

[15] X. Descovich; G. Pontrelli; S. Melchionna; S. Succi; S. Wassertheurer. Modeling Fluid Flows in Distensible Tubes for Applications in Hemodynamics. *International Journal of Modern Physics C*, 24(5):1350030, 2013. 2

[16] D. Tripathi; O. Anwar Bég. Peristaltic propulsion of generalized Burgers' fluids through a non-uniform porous medium: A study of chyme dynamics through the *diseased* intestine. *Mathematical Biosciences*, 2013. DOI: 10.1016/j.mbs.2013.11.006. 2

[17] M.P. Paidoussis. Pressure Waves in Horizontal Liquid-Filled Flexible Tubes. *Journal of Mechanical Engineering Science*, 7(4):380–390, 1965. 2

[18] J.C. Misra; B.K. Sahu. Propagation of pressure waves through large blood vessels: A mathematical model of blood viscoelasticity. *Mathematical and Computer Modelling*, 12(3):333–349, 1989. 2

[19] Y-Y.L. Wang; C.C. Chang; J.C. Chen; H. Hsiu; W.K. Wang. Pressure Wave Propagation in Arteries: A Model with Radial Dilatation for Simulating the Behavior of a Real Artery. *IEEE Engineering in Medicine and Biology*, 16(1):51–56, 1997. 2

[20] K. Berkouk; P.W. Carpenter; A.D. Lucey. Pressure Wave Propagation in Fluid-Filled Co-Axial Elastic Tubes Part 1: Basic Theory. *Journal of Biomechanical Engineering*, 125(6):852–856, 2003. 2

[21] P.W. Carpenter; K. Berkouk; A.D. Lucey. Pressure Wave Propagation in Fluid-Filled Co-Axial Elastic Tubes Part 2: Mechanisms for the Pathogenesis of Syringomyelia. *Journal of Biomechanical Engineering*, 125(6):857–863, 2003. 2

[22] N.N. Kizilova. Pressure Wave Propagation in Liquid-Filled Tubes of Viscoelastic Material. *Fluid Dynamics*, 41(3):434–446, 2006. 2

[23] T. Kitawaki; M. Shimizu. Effect of the Blood Vessel Viscoelasticity on Periodic Blood Pressure Wave Propagation. *Journal of Fluid Science and Technology*, 4(1):234–245, 2009. 2

[24] M. Saito; Y. Ikenaga; M. Matsukawa; Y. Watanabe; T. Asada; P.Y. Lagrée. One-Dimensional Model for Propagation of a Pressure Wave in a Model of the human Arterial Network: Comparison of Theoretical and Experimental Results. *Journal of Biomechanical Engineering*, 133(12):121005, 2011. 2

[25] T. Sochi. The flow of power-law fluids in axisymmetric corrugated tubes. *Journal of Petroleum Science and Engineering*, 78(3-4):582–585, 2011. 6

[26] T. Sochi. Newtonian Flow in Converging-Diverging Capillaries. *International Journal of Modeling, Simulation, and Scientific Computing*, 04(03):1350011, 2013. 6

[27] N.P. Smith; A.J. Pullan; P.J. Hunter. An Anatomically Based Model of Transient Coronary Blood Flow in the Heart. *SIAM Journal on Applied Mathematics*, 62(3):990–1018, 2002. 6

[28] L. Formaggia; D. Lamponi; A. Quarteroni. One-dimensional models for blood flow in arteries. *Journal of Engineering Mathematics*, 47(3/4):251–276, 2003. 6

[29] S.J. Sherwin; V. Franke; J. Peiró; K. Parker. One-dimensional modelling of a vascular network in space-time variables. *Journal of Engineering Mathematics*, 47(3-4):217–250, 2003. 6

[30] P. Reymond; F. Merenda; F. Perren; D. Rüfenacht; N. Stergiopulos. Validation of a one-dimensional model of the systemic arterial tree. *American Journal of Physiology - Heart and Circulatory Physiology*, 297(1):H208–H222, 2009. 6

[31] T. Sochi. One-Dimensional Navier-Stokes Finite Element Flow Model. *Technical Report*, 2013. arXiv:1304.2320. 6

[32] T. Sochi. Pore-Scale Modeling of Navier-Stokes Flow in Distensible Networks and Porous Media. *Computer Modeling in Engineering & Sciences (Accepted)*, 2014. 7

[33] T. Sochi. Flow of Navier-Stokes Fluids in Converging-Diverging Distensible Tubes. *Submitted*, 2013. arXiv:1310.4221. 7

[34] T. Sochi. Flow of non-Newtonian Fluids in Converging-Diverging Rigid Tubes. *Submitted*, 2013. arXiv:1310.7655. 7

[35] T. Sochi. Using the Euler-Lagrange variational principle to obtain flow relations for generalized Newtonian fluids. *Rheologica Acta*, 53(1):15–22, 2014. 7