The physical meaning of the fundamental concepts of physics

2. The physical nature of 'force'

Guido F. Nelissen

guido.nelissen@gmail.com

Abstract

The concept of 'force', which finds its origin in Newton's laws of motion, is one of the fundamental concepts of classical physics, because it is the basic concept of the fundamental notions of 'work' and 'energy'. The problem is that the present concept of 'force', which is in a general way defined as the momentum transfer per unit time, covers a wide variety of phenomena, which blurs the disclosure of its true nature.

On the basis of the conclusion of my paper part 1, In which I demonstrated that the 'linear momentum' of a mass particle system is a mathematical expression of its total amount of congruent translational motion, I will reveal in this paper the physical meaning of the 'force' exerted between two colliding bodies.

1. The present concept of 'force'

For centuries the problem of motion and its causes was an important subject of Natural Philosophy. In those times, the scientific paradigm was based on Aristotle's view (384 - 322 BC) that a body was in its natural state when it was at rest and that some 'action' was needed to keep a body moving, because otherwise it would naturally come to at rest. This conviction was based on the everyday experience that all moving objects finally came to a stop. During centuries this daily experience has supported the idea that perpetual motion needed a permanent 'action'.

Roughly 2000 years later, on the basis of the extrapolation of the results of experiments with polished blocs and greasy surfaces, Galileo Galilei (1564 - 1642) came however to exactly the opposite conclusion, namely that in order to change the velocity of a body, an external action is needed, and that no action is needed at all to maintain it!

- This principle of Galileo was adopted by Isaac Newton (1642 - 1727) in his "*Principia Mathematica Philosophiae Naturalis*" (1686), as his 'first law of motion', which is also called the 'law of inertia' ^[1]: "*Every body persists in its state of rest or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed on it*".

- In his second law of motion, Newton gives a (mathematical) definition of the concept of 'force' as "the rate of change of momentum" of an object:

$\mathbf{F} = d\mathbf{p}/dt = d(\mathbf{m}.\mathbf{v})/dt = \mathbf{m}.d\mathbf{v}/dt = \mathbf{m}.\mathbf{a}$

- And in his third law of motion, Newton tells us that "the mutual forces of two bodies on each other are always equal in magnitude and opposite in direction", or in other words: action equals reaction.

2. The indistinctness of the present concepts of 'force'

The concept of 'force', as it is defined by Newton's laws of motion, covers a wide variety of different kinds of 'forces' such as contact force, impulsive force, tensile force, static force, dynamic force, friction force, drag force, conservative force, non-conservative force, inertial force, centrifugal force, centripetal force, coriolis force, electromagnetic force, weak force, strong force, gravitational force, etc., in a way that makes it impossible to reveal the real, physical mechanism behind these different kinds of interactions.

The common property of all these 'forces' is that they are all based on Newton's second law of motion, in which force is defined as the time rate of change of motion. This means however that according to Newton's definition, 'change of motion' and 'force' are used as synonyms: if the motion of a body changes, there is a force on it!

In the centuries after Newton, one has however discovered cases in which the motion of objects changes, but in which there is nevertheless no 'force' involved:

- The so-called coriolis 'force' is a well known example of a change of motion without the intervention of any kind of force and it is therefore nowadays called a 'pseudo force', in the same way as the centrifugal and the centripetal forces.
- In the case of gravitation also, the motion of gigantic masses undeniably changes, and therefore Newton, in his 'universal law of gravitation ($F_g = Gm_1m_2/r^2$) has defined it as a 'force'. Newton's law of the gravitational force has allowed us, during centuries, to calculate the motion of falling objects with clockwork precision. In the early years of the twentieth century, Einstein has however demonstrated in his 'general theory of relativity', that falling masses are not at all pulled by gravitational 'forces', but that instead, they meander effortlessly toward each other in a geometric space-time curvature (i).

In this paper, I will accurately reveal the true nature of the so-called 'contact forces' (and the different ways they transfer momentum from one body to another.

3. Force as transfer of momentum flow

In my paper 'The true physical nature of linear momentum' I have referred to the study of fluid dynamics that is used in hydraulic engineering and in which "the amount (of something) that flows across a given section per unit time" is defined as the 'flow' (\mathbf{Q}).

- I have thereby analyzed the concept of 'mass flow' (Q_m) which is a mathematical expression of the amount of 'mass' that moves per unit time across a given section.

For a steady stream of particles, such as in the case of a fluid or a moving particle cloud that has a length 'L' in its direction of motion and that covers an area 'A' perpendicular to this length and that consists of 'N' unit particles all with unit mass ' m_1 ', that are uniformly spread over its total volume 'V' (= A.L) and that are all moving in a congruent way, with the same velocity 'v', the mass flow can be written as:

$$Q_m = Nm_1/t = m/t = m(\mathbf{v}/\mathbf{L}) = \rho V \mathbf{v}/\mathbf{L} = \rho \mathbf{A} \mathbf{v}$$

in which 'm' (= $N.m_1$) is the total mass and ' ρ ' (= Nm_1/V) is the mass density of the fluid.

⁽i) This will be analyzed in my paper "The physical nature of the gravitation".

- Another important application of the 'flow' concept, which is currently used in hydraulic engineering, is 'momentum flow' (\mathbf{Q}_p) which indicates the amount of 'linear momentum' that moves per unit time across a section with a given area (A) and that is therefore equal to the mass flow (\mathbf{Q}_m) times the velocity 'v':

$$Q_p = Q_m \cdot v = (Nm_1v)/t = (mv)/t = (mv) \cdot (v/L) = mv^2/L = \rho V v^2/L = \rho A v^2$$

In my paper "The true nature of linear momentum", I have demonstrated that the linear momentum is a mathematical expression of the amount of congruent translational motion, so that momentum 'flow' is a mathematical expression of the total amount of congruent translational motion that flows across a given section.

This definition of the 'momentum flow' for a steady stream of particles corresponds exactly to Newton's (mathematical) definition of 'force' as "the rate of change of the linear momentum" of an object. Newton's definition can indeed be written as:

$$F = p/t = m.v/t = (mv).(v/L) = mv^2/L = \rho Vv^2/L = \rho Av^2 = Q_p$$

It means that a particle (system) with a mass 'm' that moves with a velocity 'v' has a momentum flow:

$$\mathbf{Q}_{\mathbf{p}} = \mathbf{m}\mathbf{v}^{2}/\mathbf{L} = \rho\mathbf{A}\mathbf{v}^{2} = \mathbf{F}$$

and that when this particle (system) hits another particle (system) that is at rest in the same reference frame, it will experience a collision, by which its total amount of momentum flow is transferred to that other particle system (ii).

This demonstrates that 'force' is not a basic physical phenomenon, but that it is the consequence of conflicting momentum flows.

This interpretation of 'force' as 'momentum flow' was already demonstrated in 1980 by Andrea A. diSessa of the Division for Study and Research in Education of MIT, who proposed in his paper "*Momentum flow as an alternative perspective in elementary mechanics*" ^[2] to use the notion of 'momentum flow' instead of 'force', because momentum flow analysis allows a better insight in the intrinsic dynamical nature of 'force. According to diSessa, "*force is simply the flow of the conserved momentum, from one place to the other. Technically speaking, force is the rate with which momentum flows*" and in his paper he works out a number of examples to demonstrate this, like e.g. the case in which I hold a weight in my hand. Newton's law 'F = ma' tells me that since the acceleration is zero, the force on the weight must be zero and I conclude that my hand is providing an equal and opposite force. In a momentum flow analysis, the apple is pouring momentum through my hand, through my arm and further to my body and through my legs, into the floor (see further section 6).

4. Impulsive forces

4.1 The average impulsive force

The basic way to transfer momentum from one body to another is by means of an elastic collision (like in the case of e.g. two colliding billiard balls).

⁽ii) This will be analyzed in my paper "The conservation of kinetic energy in elastic collisions".

In the present textbooks, this problem is solved by means of the definition of 'impulse' (J), which is defined as the change of momentum ($\Delta \mathbf{p}$):

$\mathbf{J} = \Delta \mathbf{p} = \int \mathbf{F}_{\mathbf{p}} dt$

In this classic representation, the impulsive 'force' is introduced, which is defined as the change of momentum per unit time ($\mathbf{F}_{\mathbf{p}} = d\mathbf{p}/dt$). In this way, the impulse is defined as the integration of the impulsive force over time, which is represented in the F,t-diagram as the area under the curve of force versus time.

One must however realize that it are not the (equal and opposite) impulsive forces that cause the momentum transfer, but that it is in fact the other way around, namely that it is the momentum transfer, together with the elasticity of the colliding bodies, that generates the (equal and opposite) impulsive forces. In other words: the impulsive forces are a consequence of the change of the momentum and not its cause. The magnitude of the impulse force ' $\mathbf{F_p}$ ' during the collision, depends indeed on the momentum change 'd**p**' and the time interval 'dt' during which that momentum change takes place.

The problem thereby is that the impulsive force has a very short duration time (in the range of milliseconds) and changes in that very short time from zero to a very high peak value, so that it is difficult to measure the exact value of the impulsive force ' $\mathbf{F}_{\mathbf{p}}$ '. In the present textbooks of physics ^[3], the impulse is therefore represented as the product of the time interval of the collision ' Δt ' and the average force ' \mathbf{F}_{pav} ' during that time interval:

$$\mathbf{J} = \Delta \mathbf{p} = \mathbf{F}_{pav} \Delta t$$

In order to calculate that average force, the present textbooks of physics make an estimation of the time interval ' Δt ' during which the momentum change takes place and calculate on the basis of that estimation the average impulsive force between the colliding bodies: $\mathbf{F}_{pav} = \Delta \mathbf{p} / \Delta t$.

4.2 The exact calculation of the duration and the peak force during collisions

It is clear that this guessing of the peak force is not a very accurate method to determine the real peak force. For homogeneous materials, the peak force can be calculated by means of the stress/strain graph of the colliding bodies.

For the working stages of loading of most materials, this stress/strain graph is a straight line, which means that the strain (load per unit area 'F/A') is proportional to the stress (stretch per unit gauge length ' Δ L/L'). This linear relationship for most construction materials is generally known as "Hooke's law".

The slope of the stress-strain graph, which is called 'the modulus of elasticity' or 'Young's modulus' for the given material and is usually designated by 'E' (expressed in N/m²):

$$E = f/e = (F/A)/(\Delta L/L) = tg\alpha.$$

This enables us to calculate the elongation or compression of a material under a given load (e.g. for steel, $E = 200.10^9 \text{ N/m}^2$).

These engineering considerations allow us also to calculate the exact value of the peak force that is generated during an elastic collision between e.g. two identical steel blocks, each with mass 'm' of which one has a velocity ' \mathbf{v} ' towards the other, who is at rest in the same reference plane. In this case the kinetic energy of the moving block will be gradually used to produce an elastic deformation of the steel blocks along the stress-strain graph.

In this case the force 'F' is proportional to the deformation: $F = (AE\Delta L/L, so that:$

$$mv^2/2 = (AE/L)\int\Delta LdL = AE\Delta L^2/2L = F\Delta L/2$$

or: $mv^2 = F\Delta L$

From Hook's law we know that the material will deform along a straight line with a steep sloop ($E = tg\alpha$), so that:

 $E = e/g = FL/A\Delta L$

or: $FL = EA\Delta L$

We have two equations with two unknowns, F and ΔL , which give us:

$$F = v\sqrt{mEA/L}$$
 and $\Delta L = v\sqrt{mL/AE}$

For homogenous blocks with a constant section, the velocity of the deformation will gradually decrease, so that $v_{av} = v/2$ and

$$\Delta t = \Delta L/v_{av} = 2\Delta L/v = 2\sqrt{mL/AE}$$

This example demonstrates that for homogenous materials the peak force and the duration of the impulse during collision are calculable and that even in the case of hard physical impacts, the momentum transfer is not instantaneous, but progresses gradually between the individual particles of the colliding bodies (iii).

5. Repetitive impulsive forces

5.1 Introduction

In the former section I have demonstrated that if we want to increase the speed of a perfectly elastic body, like e.g. billiard ball with mass 'm' from '0' to 'v', we can do that by means of an appropriate impulse, e.g. by shooting a second identical billiard ball to it with the required speed 'v'. When the second billiard ball (which has a linear momentum with magnitude 'mv' and a total amount of momentum flow 'mv²') hits the target, it will come to a complete standstill and by doing so it will transfer its total translational motion to the target billiard ball, which will now move with the speed 'v' (and which consequently has a linear momentum with magnitude 'mv' and a total amount of momentum flow 'mv²').

This way of transferring momentum flow from one body to another by means of an elastic collision may be okay for billiard balls, but for fragile, composite macroscopic structures, the accelerations and the consecutive impulsive forces are much too high and will cause serious deformations and even a breakdown of those structures.

In the case of composite bodies that are made up from individual elements, the momentum transfer between both colliding particle systems takes place by means of successive collisions between their constituent particles. In that way a second time interval comes into play, which is the average time interval between two successive collisions (T) between the particles of

⁽iii) This will be analyzed in my paper "The true nature of work and kinetic energy".

both particle systems. This can also be expressed by its inverse, as the frequency (f = 1/T) of the successive collisions.

This form of momentum flow is similar to the case in which we fire a steady stream of bullets/particles to a target/body. When each individual particle has a mass ' m_1 ' and a velocity 'v', the frequency 'f' with which the particles proceed to the body is equal to the total number of fired particles 'N' divided by the total firing time 't'. Since the total length 'L' of the particle stream is equal to the common velocity of the particles times the total firing time, the frequency (f) with which the particles proceed to the body can be expressed as:

f~=~1/T~=~N/t~=~Nv/L

In that way, the linear momentum that is transferred per unit time or i.e. the momentum flow towards the body is equal to the linear momentum of one particle (\mathbf{p}_1) times the frequency (f) with which the successive particles hit the particles of the (other) body.

 $Q_p = p_1.f = (m_1.v).(Nv/L) = (Nm_1)v^2/L = mv^2/L = mAv^2/V = \rho Av^2 = Q_m.v$

This means that the force (**F**) on the body or i.e. the transfer of momentum flow (ΔQ_M), is then equal to the linear momentum, or i.e. to the amount of congruent translational motion, that is transferred per impulse ' Δp_1 ' times the number of impulses per unit time (or i.e. the frequency) 'f':

$$\mathbf{F} = \Delta \mathbf{p_1}.\mathbf{f} = \Delta \mathbf{Q_p}$$

This viewpoint demonstrates clearly that 'force' in general and even static forces in particular and consequently also 'pressure', are intrinsically dynamic characteristics because they are the consequences of repetitive collisions of the molecules of a force particle system with the molecules of a body. This is as a matter of fact what happens between the moving molecules of a gas and the walls of the pressure tank or the combustion cylinder in which it is enclosed, or between the jiggling molecules of our hand and the surface of an object when we exert a pressure on it or when we push it away, etc. (see section 6).

To analyze this further we have to make a distinction between the momentum transfer to a immovable rigid wall (the 'tensile force') and the momentum transfer to a body that is free to move (the 'driving force').

5.2 Tensile force (F_t)

5.2.1 The case of an elastic wall

This is the case of the transfer of linear momentum (or i.e. of congruent translational velocity/motion) by means of repetitive collisions of unit particles with mass ' m_1 ' and velocity 'v' with an immovable, but perfectly elastic rigid wall.

The classic equation of the final velocity for an elastic collisions give us in this case: $\mathbf{v_{1f}} = -\mathbf{v}$ so that the transfer of momentum of the elastic wall to the particle is: $\Delta \mathbf{p_{1p}} = -2m_1 \mathbf{v}$.

This means that the impulse at each collision on an elastic wall is equal to twice the linear momentum (\mathbf{p}_1) of the particles that are moving to the wall: $\Delta \mathbf{p}_{1\mathbf{w}} = 2\mathbf{p}_1 = 2\mathbf{m}_1\mathbf{v}$.

The (tensile) force on that wall, which is per definition the amount of linear momentum (or i.e. of congruent translational motion) that is transferred per unit time to that wall, is then

equal to:

$$F_t = \Delta Q_p = \Delta p_1 f = (2m_1 v)(Nv/L) = 2(m_1 N)v^2/L = 2mv^2/L = 2\rho Av^2 = 2Q_p$$

It follows from this that the (tensile) force (\mathbf{F}_t) on a perfectly elastic wall is equal to twice the momentum flow (\mathbf{Q}_p) to the wall. It is thereby important to stress the fact that in these equations of the tensile force, 'L' is the total length of the moving 'force' cloud (particle system) in its direction of motion ($\mathbf{L} = \mathbf{v}t$).

5.2.2 The case of an inelastic wall

In the specific case of a completely inelastic wall: $\Delta p_1 = m_1 v$ so that in this case the force on the inelastic rigid wall is equal to the momentum flow (Q_p) to that wall: $F_t = \Delta Q_p = mv^2/L = \rho Av^2 = Q_p$

5.2.3 General solution by means fluid mechanics

In fluid mechanics (hydraulics) the problem of the force exerted by the steady flow of a fluid on an obstacle (a wall, a deflector, etc.) is generally solved by means of 'the momentum equation', which for a steady, uniform flow is written as:

$$\Sigma \mathbf{F} = \rho_2 \mathbf{A}_2 \mathbf{v}_2 \mathbf{v}_2 - \rho_1 \mathbf{A}_1 \mathbf{v}_1 \mathbf{v}_1$$

For a stationary obstacle, this leads to a force \mathbf{F} on the wall which is equal to:

$$\mathbf{F} = \rho A v^2 (1 - \cos \alpha)$$

In which ' α ' is the angle between the direction of the flow (which we take to be horizontal, that is along the x-axis) and the direction of the obstacle (see figure 1).

- For an obstacle that stands vertical on the direction of the flow:

 $\alpha = 90^{\circ}$, so $\cos \alpha = 0$ and $F = \rho A v^2$

This corresponds to the case of an inelastic wall (section 5.2.2) by which the horizontal component of the flow disappears completely

- For an obstacle that has a circular form and that deflects the flow over 180 degrees: $\alpha = 180^{\circ}$, so $\cos \alpha = -1$ and $F = 2\rho Av^2$ This corresponds to the case of an elastic wall (section 5.2.1) by which the flow is completely reversed. This is the case of turbine blades such as the so-called 'Pelton turbines', which completely reverse the momentum flow and produce in that way a force

that is twice that on a flat blade.

5.3 Driving force (F_d)

In this case the repetitive collisions do not take place against a rigid wall, but against a body that is free to move. In section 2 we have seen that it is possible to change the motion of such a body by means of an impulse. This means that we could e.g. apply one big impulse in order to give an object the desired speed. This may be okay for billiard balls, but for fragile composite structures, the accelerations and the consecutive impulsive forces are much too

high and will cause serious damage to the material structure of the body.

Therefore we use forces that consist of a large number (N) of very small successive impulses. This is exactly what is done by the heated or pressurized gas in e.g. the cylinders of a combustion engine. When a steady, quasi continuous momentum flow from the molecules of a 'force' particle system is transferred in a reversible way to a body with a mass 'm' that is free to move, that body will experience a steady 'driving force' or 'thrust' (\mathbf{F}_d). This driving force will increase the speed of the body when it acts in its direction of motion, it can also decrease the speed of the body when it acts against its direction of motion (in which case it is called a 'braking force'), or it can change the direction of motion of the body when it acts perpendicular to its direction of motion (in which case it is called a 'steering force'). In this case the linear momentum 'm.v' is transferred to a free moving body, while this body covers a distance 'L'. During this displacement, the velocity is gradually increasing from '0' to 'v'. The time interval 't' necessary to transfer the total momentum 'm.v' is consequently:

$$t = L/v_{av} = L/(v/2) = 2L/v$$

so that the average frequency with which the particles hit the body is:

 $\mathbf{f} = \mathbf{N}/\mathbf{t} = \mathbf{N}\mathbf{v}/2\mathbf{L}$

This means that the steady transfer of momentum flow from the force particle system to the body, which is necessarily equal to the increase of the momentum flow of the body while both particle systems are covering the same distance 'L' is then equal to:

$$\mathbf{F}_{d} = \Delta \mathbf{Q}_{p} = \Delta \mathbf{p}_{1}.\mathbf{f} = (m_{1}.\mathbf{v})(N\mathbf{v}/2\mathbf{L}) = (Nm_{1})v^{2}/2\mathbf{L} = m.v^{2}/2\mathbf{L} = \rho \mathbf{A}v^{2}/2 = \mathbf{Q}_{p}/2$$

This leads to the conclusion that if we want to increase the speed of a mass 'm' from zero to 'v' over a distance 'L', we have to maintain a steady transfer of momentum flow (or i.e. a steady transfer rate of congruent translational motion) ' ΔQ_p ' which is equal to only half the momentum flow ' Q_p ' of that body when it steadily proceeds with that velocity 'v':

$$\mathbf{F_d} = \Delta \mathbf{Q_p} = \mathrm{m.v^2/2L} = \mathbf{Q_p/2}$$
 (iv)

6. The fundamentally dynamic nature of 'force'

These finalizations of the concept of 'force' clearly underline the fact that Newton's definition of 'force' is not a basic physical phenomenon, but that it is a consequence of conflicting momentum flows by which momentum is transferred from one particle system to another. This causal connection was already developed in 1980 by Andrea A. diSessa of the Division for Study and research in Education of MIT who, mainly for pedagogical reasons, proposed in his paper "Momentum flow as an alternative perspective in elementary mechanics" to use the notion of 'momentum flow' instead of 'force', because this underlines the fundamentally dynamic character of 'force', as the consequence of repetitive collisions of the molecules of the particle system of the 'force', with the particle system of the 'body'.

As is demonstrated by diSessa, this is exactly what happens in a pressure tank between the gas molecules and the walls of the tank or between the jiggling molecules of an object lying on the floor surface. This is as a matter of fact also exactly what happens between the jiggling molecules of our hand and the jiggling molecules of a wall while stretching ourself against that wall, while holding ourself up on a branch of a tree or while holding a barbell stationary

⁽iv) This will be analyzed in my paper "The true nature of work and kinetic energy".

above the ground. The present textbooks tell us that in those cases no 'work' has been done, because nothing is displaced. But we surely get exhausted by doing it.

I doubt however whether diSessa's plea has had any success, because the role of 'momentum' and consequently of 'momentum flow' is in the present textbooks still completely minimized and restricted to some peripheral phenomena such as the equations for the final velocities of elastic collisions and the equation for rocket propulsion.

In this paper I have demonstrated that di Sessa's representation of 'force' as the transfer of momentum flow doesn't only allow a better comprehension of the observed phenomena, but that it is the more exact description of the real, physical phenomenon that we are used to call 'force'.

The bald underestimation of the fundamentally dynamical nature of 'force' as the transmission rate of linear momentum is strikingly demonstrated by the fact that 'momentum' and 'momentum flow' are completely ignored in the present SI-system, so that in those rare occasions that linear momentum is occasionally used, it must be expressed in 'N.s', there where, as we have demonstrated, 'momentum' (expressed as $M_o = kg.m/s$) is the true fundamental unit and 'force' which represents the transfer of momentum transfer per unit time is the derivative unit. In that way 'force' ought to be expressed as "momentum transfer per unit time" ($M_o/s = N$) which clearly indicates that the unity 'N' has a fundamentally dynamic character and that there is no such things as a 'static' force or a 'static' pressure, because both are the consequences of repetitive momentum transfers between the particles of the force object and the particles of the body.

7. Pressure

Pressure, which is generally defined as the force per unit area, is just like 'force' a derivative unit. In a certain way it is also a mathematical unit, because the chosen 'unit' area (m²) is after all a fortuitous area and the force on 'a unit' area is therefore of exactly the same nature as the force on any other area 'A', which are however all expressed in 'N'.

This mathematical notion of 'pressure' is therefore mainly an engineering convenience that allows the direct calculation of the force on a given surface in function of the force on a 'unit' area of that surface.

Since a 'unit' area is the same for all kinds of surfaces, this means however that for a given material it stands for the same number of molecules, so that the 'pressure' is in that case a useful indication of the force per molecule (and its mutual bonds) and therefore of the physical stress in the given material.

8. Conclusion: The true nature of 'force'

In this paper I have demonstrated that 'force' (which is classically defined as the transfer of linear momentum per unit time) is a mathematical expression of the transfer rate of congruent translational motion.

In the light of the intrinsic discrete, particle nature of matter, this means that the 'force' exerted between two particle systems can be expressed as the transfer of congruent translational motion per impulse, times the impulse frequency of the collisions between their mutual particles.

It follows naturally from this that 'force' has fundamentally a dynamic character and that

therefore all forces that consist of a momentum transfer, are dynamic 'impulsive' forces.

REFERENCES

- ^[1] Resnick & Halliday "Physics Part 1" p 81 88, John Wiley & Sons Inc. (1966).
- ^[2] Andrea A. diSessa "Momentum flow as an alternative perspective in elementary mechanics", American Journal of Physics volume 48 nr 5 May 1980 p 365 369.
- ^[3] Resnick, Halliday, Krane "Physics" p 121-123, John Wiley & Sons, Inc.(2002). Fishbane, Gasiorowicz, Thornton "Physics" p 213, Pearson Education Inc. (2005).
