Candidates for the Slichter Triplet

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Abstract: In the last 55 years, there have been many attempts to measure the three frequencies of the 1S1 self-resonance, the vibration of the Earth's core. One reason for this failure could be the insufficient reduction of the background noise, caused by the earthquakes. This is just another attempt to solve the ancient mystery. This time, perhaps with more success ;-)

Introduction

Very strong earthquakes can probably move the inner, solid core, which is surrounded by the liquid outer core. With the known data, one can calculate that a full linear oscillation around its rest position should take about five hours. The existence of this eigenresonance was postulated 55 years ago by Slichter[1], but could never be confirmed by measurements despite intensive search[2].

One can specify multiple causes for this failure. Obviously, the signals from the Earth's core have very low amplitude and can hardly be detected in the noise of the frequent earthquakes. Additionally, the (three) expected frequencies of ${}_1S_1$ are in the same range as the very strong tides, which deform the surface of the earth. The friction of the sea water on solid ground leads to non-linearities, which produce several disturbing intermodulation products in the same frequency range. In this noisy environment, it is not easy to identify oscillations when you do not even know their frequencies. If the putative oscillation deep inside the earth can be triggered by changes on the surface, the search should focus on the period after a strong earthquake. The strongest event since the invention of the Superconducting gravimeter was on 2004-12-26.

The Preparation of the data records

The 2004/2005 CORMIN records of all available SG stations were linked to long data chains. A barometric admittance of 3.8 nm/(s^2 hPa) decreases the influence of atmosphere pressure variation on the gravity data. Previous studies have shown that this "magic number" minimizes the noise level around 70 µHz. Then, the data reduction is carried out through a series of filters. Their order greatly affects the signal-to-noise ratio of the weak signals.

First, a triple narrow-band notch filter at 0.8 µHz, 11 µHz and 22 µHz reduces the amplitude of the strongest spectral lines (calculated with quadruple precision) and a windowed-sinc filter limits the bandwidth of the recorded data to the range 33 µHz – 600 µHz. After increasing the sampling time to 10 minutes, a second windowed-sinc filter narrows the bandwidth to the range 35 µHz – 160 µHz, creating the final data string for every SG station.

The reduction of the noise

Extremely weak signals require that any unnecessary noise is removed by narrowband filters. A reduction of the bandwidth to zero is technically impossible and would mean that the amplitude remains fixed for all time. As the bandwidth increases, the noise increases and the signals gets more and more unreadable. A careful analysis of the frequencies in question showed that at up to a distance of ± 0.1 µHz, weak sidebands might belong to the signal and they should not be neglected.

Such a narrow bandwidth is possible only at a very low frequencies. Therefore, the signals from the Earth's core are mixed down to 9 µHz and the sampling period is extended to 100 minutes. With these values, it is possible to construct a filter with a passband from 8.84 µHz to 9.16 µHz. All other frequencies are regarded as undesirable noise and are suppressed.

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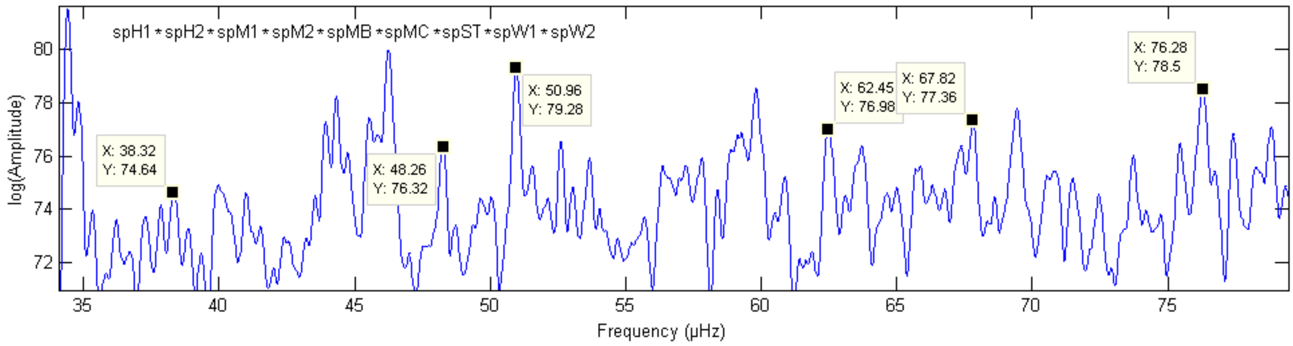
Criteria to be fulfilled

Between 35 µHz and 80 µHz, the spectrum of every station shows several hundred anomalies and it means endless work to detect coincidences between different stations. Is there an easier way to find candidates for the Slichter Triplet?

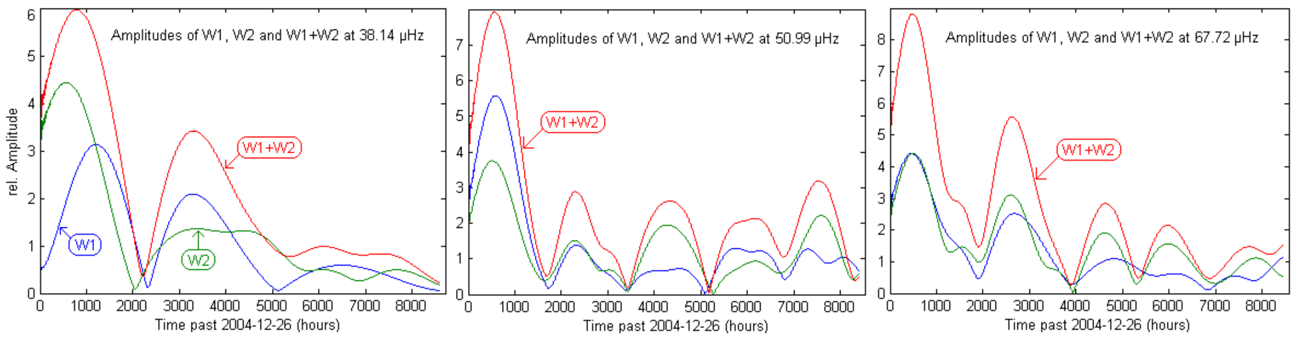
The following method indicates which frequencies should be examined more closely: The addition or multiplication of different spectra reinforces matching spectral lines and levels out random coincidences. The following figure shows the product spectrum of nine neighboring stations with particularly low background noise.



At least six prominent peaks in the frequency range of interest can not be assigned to known causes[3] and were investigated in detail. But the identification of unusual frequencies with striking good SNR is not sufficient, there are additional constraints to convince with the result.

The movement of the Earth's core in the ${}_1S_1$ eigenmode probably corresponds to a damped oscillation, that disappears in the noise after several hours. In contrast, the amplitude of the tides and their combination frequencies should be nearly constant over a sufficiently long period. If a time analysis shows the typical amplitude decay of a spectral line, we may have found a member of the Slichter triplet.

Another criterion is decisive: The gravimeters of some stations contain *two* superconducting spheres and both should measure identical amplitudes *and* phases. In fact, the following pictures (based on the Wettzell records) show that the envelope of the sum (red) is always greater than either of the individual curves. Just three of the six "suspicious" frequencies meet all the requirements and could be members of the long-sought Slichter triplet.



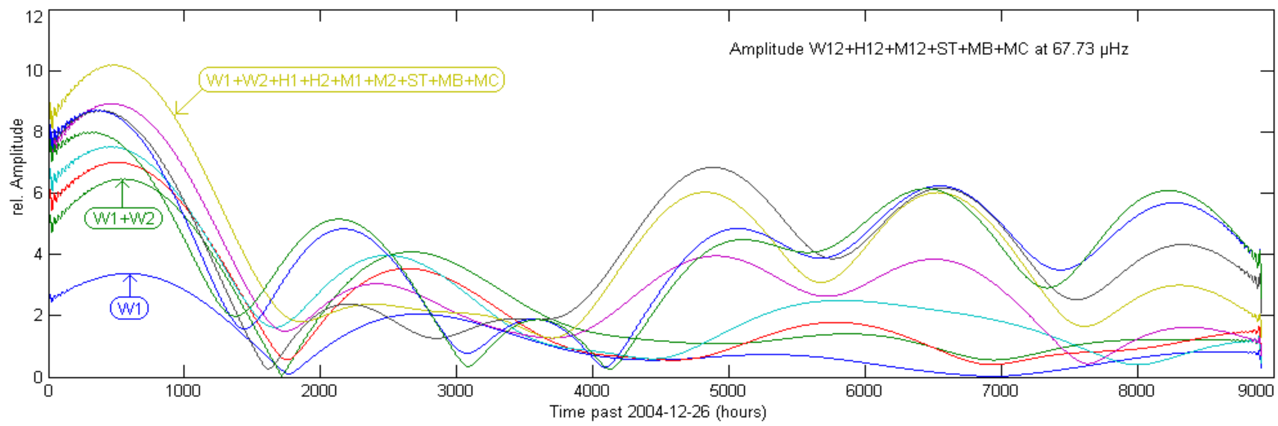
None of the three examples above shows a typical damped oscillation with a monotonically decreasing amplitude, dropping to half its value after about 4000 hours. There is an additional almost-periodicity. If the oscillations are generated by the Earth's core, the frequent but irregular poles might help to describe its shape and motion.

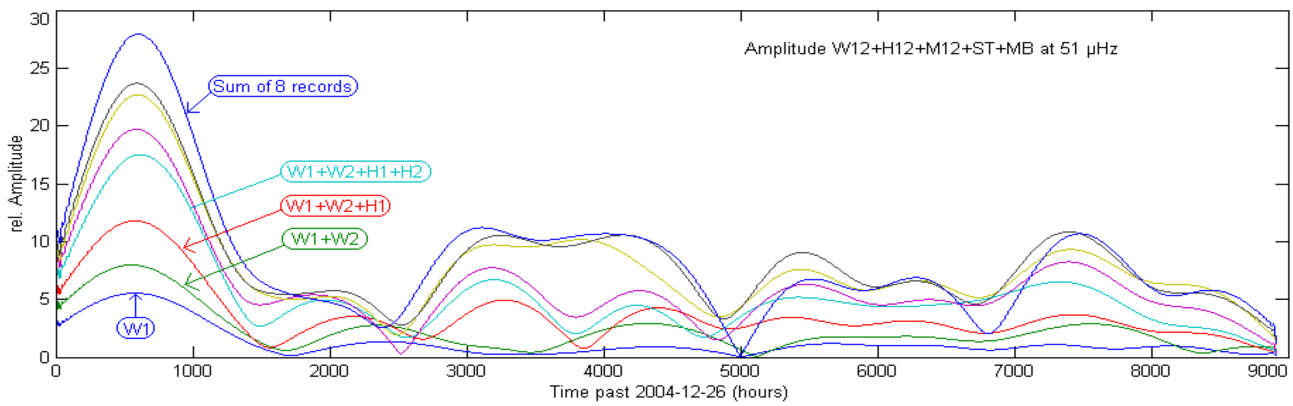
Strangely enough, the oscillation amplitudes reach their maximum value about 400 hours after the earthquake. There are at least two causes for that delay: The earthquake overrides the gravimeters and there is a data gap of several days. Secondly, all filters calculate the average of a certain time window and abruptly onset signals are displayed with a finite rise time. The sinc filters used herein have an extremely low bandwidth ($BW \approx 0.33$ µHz), to raise the signal-to-noise ratio to an acceptable level. This leads inevitably to a particularly large rise time of about $t_r \approx 290$ hours.

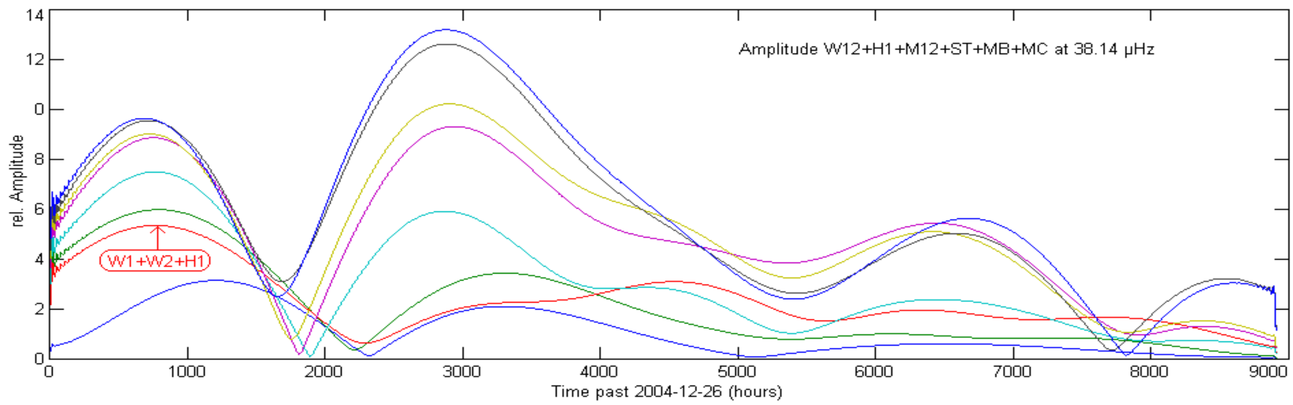
The only way to shorten the rise time is an enlargement of the filter bandwidth, since both values are linked by the formula $BW \cdot t_r \approx 0.34$. But then the desired signals are likely to disappear in the noise because the barn door is open too far. Searching the weak signals using a broadband instrument like FFT without any selectivity is probably not very successful. The narrow bandwidth of the signal processing means that all modulation frequencies above $BW/2$ are cut off. As a consequence, no signal changes can be observed that take place in less than 70 days.

Checking the phase coherence

In Central Europe, there is a dense net of superconducting gravimeters. The following figures show the gradual increase of the amplitude when the records of eight European stations are summed *before* the time analysis, proving the phase coherence at least during the first 1500 hours. However, the overlay also shows how difficult it is to estimate a half-life. Does the vibration disappear 3000 hours after the earthquake, followed by noise? Or it is excited again? Probably, this question can be resolved only after a much stronger excitation of ${}_1S_1$ eigenmode.







Frequencies and Q-Factors

The following table shows the frequencies and *Q* values of all analyzed stations. Despite the very weak signals, there were no problems to identify the spectral lines. Negative *Q* values indicate that the amplitude increases over time – against all expectations.

Station	Q	f/μHz	Q	f/μHz	Q	f/μHz
CB	-2714	38,1297	??	50,982	-17000	67,7778
H1	17500	38,1349	2859	50,9815	1869	67,7327
H2	9100	38,1389	2039	50,9886	3585	67,728
H1+H2	5026	38,1571	3180	50,9785	2840	67,7462
KA	-13200	38,1411	2563	50,9993	1577	67,7569
M1	1791	38,1407	5287	50,9863	2517	67,7446
M2	1172	38,1448	2668	50,9846	3608	67,722
M1+M2	1493	38,143	2542	50,9861	2801	67,7194
MA	1460	38,1376	12600	51,0086	2564	67,7268
MB	964	38,1359	1901	51,0028	2509	67,7158
MC	1675	38,1462	1908	50,9472	2113	67,7373
ME	1291	38,1525	1596	50,9808	2317	67,69
S1	3873	38,1551	2765	50,9982	7727	67,678
S2	-2825	38,1413	3148	51,0036	-8147	67,681
S1+S2	-5930	38,1215	2903	51	4772	67,6791
ST	2578	38,1501	6955	50,9788	2910	67,7302
TC	-16600	38,1427	2014	51,0037	2479	67,7341
W1	1381	38,1435	2621	50,993	2373	67,7354
W2	1104	38,1485	3324	50,9901	3773	67,7314
W1+W2	1450	38,1493	2155	50,9908	2989	67,7215
Average		38,14272 ± 0,0019		50,9892 ± 0,0030		67,7244 ± 0,0058
Period (h)		7,2826		5,4478		4,1016

The means were calculated with the jackknife method.

Acknowledgments

Thanks to the operators of the GGP stations for the excellent gravity data. The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[4].

[1] Slichter, L., 1961. The fundamental free mode of the Earth's inner core. Proc. Natl. Acad. Sci. U.S.A. 47, 186–190.
[2] S. Rosat, A Review of the Slichter Modes: an Observational Challenge, in: The Earth's Core: Structure, Properties and Dynamics, Nova Science Publishers, pp. 63-77, 2011
[3] T. Hartmann, H. Wenzel, The HW95 tidal potential catalogue, 1995
[4] The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html