A NEW FORMULATION OF SPECIAL RELATIVITY

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This article presents a new formulation of special relativity whose kinematic and dynamic quantities are invariant under generalized Lorentz transformations.

Introduction

From an auxiliary point object (called free-point) can be obtained kinematic quantities (such as absolute time, absolute position, etc.) that are invariant under generalized Lorentz transformations.

The free-point is a point object (massive particle) that must always be free of internal and external forces (or the net force acting on it must always be zero)

The absolute time $(\breve{t})$, the absolute position $(\breve{\mathbf{r}})$, the absolute velocity $(\breve{\mathbf{v}})$ and the absolute acceleration $(\breve{\mathbf{a}})$ of a particle relative to an inertial reference frame S are given by:

$$\breve{t} \doteq \gamma \left(t - \frac{\mathbf{r} \cdot \boldsymbol{\psi}}{c^2} \right)$$

$$\breve{\mathbf{r}} \doteq \left[\mathbf{r} + \frac{\gamma^2}{\gamma + 1} \frac{(\mathbf{r} \cdot \boldsymbol{\psi})\, \boldsymbol{\psi}}{c^2} - \gamma\, \boldsymbol{\psi}\, t \right]$$

$$\breve{\mathbf{v}} \doteq \left[\mathbf{v} + \frac{\gamma^2}{\gamma + 1} \frac{(\mathbf{v} \cdot \boldsymbol{\psi})\, \boldsymbol{\psi}}{c^2} - \gamma\, \boldsymbol{\psi} \right] \frac{1}{\gamma\, (1 - \frac{\mathbf{v} \cdot \boldsymbol{\psi}}{c^2})}$$

$$\breve{\mathbf{a}} \doteq \left[\mathbf{a} - \frac{\gamma}{\gamma + 1} \frac{(\mathbf{a} \cdot \boldsymbol{\psi})\, \boldsymbol{\psi}}{c^2} + \frac{(\mathbf{a} \times \mathbf{v}) \times \boldsymbol{\psi}}{c^2} \right] \frac{1}{\gamma^2\, (1 - \frac{\mathbf{v} \cdot \boldsymbol{\psi}}{c^2})^3}$$

where $(t,\, \mathbf{r},\, \mathbf{v},\, \mathbf{a})$ are the time, the position, the velocity and the acceleration of the particle relative to the inertial reference frame S, $(\boldsymbol{\psi})$ is the velocity of the free-point relative to the inertial reference frame S and (c) is the speed of light in vacuum. $(\boldsymbol{\psi})$ is a constant. $\gamma = (1 - \boldsymbol{\psi} \cdot \boldsymbol{\psi}/c^2)^{-1/2}$

The Dynamics

If we consider a particle with rest mass m_o then the linear momentum $\mathbf{P}$ of the particle, the net force $\mathbf{F}$ acting on the particle, the work W done by the net force acting on the particle, and the kinetic energy K of the particle, for an inertial reference frame, are given by:

$$\mathbf{P} \doteq \frac{m_o\,\breve{\mathbf{v}}}{\sqrt{1-\frac{\breve{v}^2}{c^2}}}$$

$$\mathbf{F} = \frac{d\mathbf{P}}{d\breve{t}} = \frac{m_o\,\breve{\mathbf{a}}}{\sqrt{1-\frac{\breve{v}^2}{c^2}}} + \frac{m_o\,\breve{\mathbf{v}}}{\left(1-\frac{\breve{v}^2}{c^2}\right)^{3/2}}\,\frac{(\breve{\mathbf{v}}\cdot\breve{\mathbf{a}})}{c^2}$$

$$\mathrm{W} \doteq \int_1^2 \mathbf{F}\cdot d\breve{\mathbf{r}} = \Delta\,\mathrm{K}$$

$$\mathrm{K} \doteq m_o\,c^2\left(\frac{1}{\sqrt{1-\frac{\breve{v}^2}{c^2}}} - 1\right)$$

Forces and fields must be expressed only with absolute quantities. For example, the Lorentz force must be expressed with the absolute velocity $\breve{\mathbf{v}}$, the electric field must be expressed with the absolute position $\breve{\mathbf{r}}$, etc.

Observations

§ In this article, the quantities $(\,\breve{t},\ \breve{\mathbf{r}},\ \breve{\mathbf{v}},\ \breve{\mathbf{a}},\ \mathbf{P},\ \mathbf{F},\ \mathrm{W},\ \mathrm{K}\,)$ are invariant under generalized Lorentz transformations.

§ In arbitrary inertial reference frames ($\breve{t}_{\mathrm{fp}} \neq \tau_{\mathrm{fp}}$ or $\breve{\mathbf{r}}_{\mathrm{fp}} \neq 0$) (fp = free-point) a constant must be included in the definition of absolute time such that the absolute time and the proper time of the free-point are the same ($\breve{t}_{\mathrm{fp}} = \tau_{\mathrm{fp}}$) and another constant must also be included in the definition of absolute position such that the absolute position of the free-point is zero ($\breve{\mathbf{r}}_{\mathrm{fp}} = 0$)

§ Finally, this article considers, on one hand, that it would also be possible to obtain kinematic and dynamic quantities $(\,\breve{t},\ \breve{\mathbf{r}},\ \breve{\mathbf{v}},\ \breve{\mathbf{a}},\ \mathbf{P},\ \mathbf{F},\ \mathrm{W},\ \mathrm{K}\,)$ that would be invariant under transformations between inertial and non-inertial reference frames and, on the other hand, that the dynamic quantities $(\,\mathbf{P},\ \mathbf{F},\ \mathrm{W},\ \mathrm{K}\,)$ would also be given by the above equations.

Appendix

Generalized Lorentz Transformations

The time (t'), the position ($\mathbf{r}'$), the velocity ($\mathbf{v}'$) and the acceleration ($\mathbf{a}'$) of a particle relative to an inertial reference frame S' are given by:

$$t' = \gamma\left(t - \frac{\mathbf{r}\cdot\mathbf{V}}{c^2}\right)$$

$$\mathbf{r}' = \left[\mathbf{r} + \frac{\gamma^2}{\gamma+1}\frac{(\mathbf{r}\cdot\mathbf{V})\,\mathbf{V}}{c^2} - \gamma\,\mathbf{V}\,t\right]$$

$$\mathbf{v}' = \left[\mathbf{v} + \frac{\gamma^2}{\gamma+1}\frac{(\mathbf{v}\cdot\mathbf{V})\,\mathbf{V}}{c^2} - \gamma\,\mathbf{V}\right]\frac{1}{\gamma\,(1-\frac{\mathbf{v}\cdot\mathbf{V}}{c^2})}$$

$$\mathbf{a}' = \left[\mathbf{a} - \frac{\gamma}{\gamma+1}\frac{(\mathbf{a}\cdot\mathbf{V})\,\mathbf{V}}{c^2} + \frac{(\mathbf{a}\times\mathbf{v})\times\mathbf{V}}{c^2}\right]\frac{1}{\gamma^2\,(1-\frac{\mathbf{v}\cdot\mathbf{V}}{c^2})^3}$$

where (t, $\mathbf{r}$, $\mathbf{v}$, $\mathbf{a}$) are the time, the position, the velocity and the acceleration of the particle relative to an inertial reference frame S, ($\mathbf{V}$) is the velocity of the inertial reference frame S' relative to the inertial reference frame S and (c) is the speed of light in vacuum. ($\mathbf{V}$) is a constant. $\gamma = (1 - \mathbf{V}\cdot\mathbf{V}/c^2)^{-1/2}$

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