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Particle Physics and Cosmology in the Microscopic Model

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Abstract

This review summarizes the results of a series of recent papers[1, 2, 3, 4, 5], where a microscopic structure underlying the physics of elementary particles has been proposed. The 'tetron model' relies on the existence of an internal isospin space, in which an independent physical dynamics takes place. This idea is critically reconsidered in the present work. As becomes evident in the course of discussion, the model not only describes electroweak phenomena but also modifies our understanding of other physical topics, like gravity, the big bang cosmology and the nature of the strong interactions.

1 Introduction

The Standard Model of elementary particles is very successful on the phenomenological level. The outcome of (almost) any particle physics experiment can be predicted accurately within this model, and where not, by some straightforward extension. For example, one may introduce right handed neutrinos to account for tiny neutrino masses[6].

Nevertheless, it is widely believed that the SM is only an effective low-energy theory valid below a certain energy scale, which is supposed to be larger than 1 TeV. This view is based on the fact that the SM has many unknown parameters with hitherto unexplained hierarchies. Furthermore, there is one rather mysterious component, the so-called Higgs field, needed for the spontaneous symmetry breaking (SSB) to take place in the model.

In recent papers a microscopic model has been developed[1, 2, 3, 4, 5], whose central assumption is the existence of a 3-dimensional *internal tetrahedral structure* attributed to each point of Minkowski space, in which an independent physical dynamics takes place.

Under this assumption spacetime originally is 6+1 dimensional, and at the time when the tetrahedrons are formed, it fibers into internal space and Minkowski space as $R^3 \times R^{3,1}$.

The sites $i = 1, 2, 3, 4$ of the tetrahedron are populated by 6+1 dimensional spinor fields ψ, called **tetrons**. The tetron on site i will be denoted ψ_i.

The fundamental spinor representation in R^{6+1} is of dimension 8. It decomposes as[38]

$$8 \rightarrow (1, 2, 2) + (2, 1, 2) = ((1, 2) + (2, 1), 2) \tag{1}$$

under the fibration $SO(6, 1) \rightarrow SO(3, 1) \times SO(3)$.

Here representations of $SO(3, 1) \times SO(3)$ are denoted by a set of 3 numbers (a, b, c), where (a, b) are representations of the Lorentz group and c is the dimension of a $SO(3)$-representation. For example, c=2 corresponds to a non-relativistic Pauli spinor in internal space, whose 2 spin orientations are identified with the SU(2) flavors U and D. It should be noted that (1,2,2) and (2,1,2) are complex conjugate with respect to each other, so one is the antiparticle representation of the other.

Eq. (1) means that each tetron is an isospin doublet $\psi = (U, D)$ of 3+1 dimensional Dirac fermions U and D. One may write it as a 2-index object ψ^a_α, where $\alpha = 1, 2, 3, 4$ is the Dirac index and $a = 1, 2$ the internal index. The internal spin will be called isospin.

Using the triplet $\vec{\tau}$ of internal Pauli matrices an isospin (pseudo)vector

$$\vec{Q} = \psi^\dagger \vec{\tau} \psi \tag{2}$$

may be defined for any tetron ψ. It fixes a direction in the internal space and, up to an overall constant, can be interpreted as the internal angular momentum vector of the tetron ψ.

Since the tetrons are Dirac fermions on Minkowski space, $\vec{Q}$ can be written in terms of creation and annihilation operators of a tetron and an antitetron as

$$\vec{Q} = \psi^\dagger \vec{\tau} \psi = a^\dagger \vec{\tau} a - b^\dagger \vec{\tau} b \tag{3}$$

For the calculation of the quark and lepton masses the chiral iso-vectors

$$\vec{S} := \vec{Q}_L = \frac{1}{2} \psi^\dagger (1 - \gamma_5) \vec{\tau} \psi \qquad \qquad \vec{T} := \vec{Q}_R = \frac{1}{2} \psi^\dagger (1 + \gamma_5) \vec{\tau} \psi \tag{4}$$

turn out to be of particular importance. For simplicity of notation they are called $\vec{S}$ and $\vec{T}$ in the following. Obviously, they fulfill $\vec{Q} = \vec{S} + \vec{T}$.

In fig. 1 the *local* ground state of the model is drawn, a configuration with 4 tetrons on the 4 sites of a tetrahedron, their isospin vectors $\vec{Q}$ pointing in radial directions away from the origin. These internal vectors fulfill the commutation relations of a system of decoupled internal angular momenta. In other words, they play the role of angular momentum observables corresponding to rotations of the internal R^3 space.

While the coordinate symmetry is S_4, the arrangement of isospin vectors in fig. 1 respects the Shubnikov symmetry[9, 11, 12]

$$G_4 := A_4 + S(S_4 - A_4) \tag{5}$$

where $A_4(S_4)$ is the (full) tetrahedral symmetry group and S the internal time reversal operation that changes the direction of internal spin vectors. This is equivalent to saying that S interchanges the role of the internal spinors in the following way

$$S : (U, D) \to (-D^*, U^*) \tag{6}$$

As shown later in (2.4.14), one may actually use charge conjugation instead of the concept of an internal time to define the Shubnikov group G_4.

Note the arrangement fig. 1 does not respect S or internal parity R, but only the product SR. Furthermore it is chiral, the configuration with opposite internal chirality being given when the isospin vectors would point inwards instead of outwards. As will be shown in (2.1.21), this internal chirality is dynamically related to the $V - A$ nature of the q/l interactions.

As for the *global* ground state the set of all tetrahedrons forms a flat 3-dimensional crystal structure within the original R^6, similar to what is shown in fig. 2. This structure may be called a **hyper-crystal**. It is our world, the space in which all physical processes take place. Actually it will turn out to resemble an elastic or even a fluid system, so that it may as well be called a hyper-plastics or, within the Lorentz covariant cosmological framework to be developed later, the discrete micro-elastic spacetime continuum, the **DMESC**.

Contrary to what is drawn, the tetrahedrons extend into internal space alone, not into physical space. In other words, physical space is *defined* to be the 3-dimensional subspace of R^6 orthogonal to the 3 dimensions spanned by the aligned tetrahedrons.

There is not only a coordinate alignment of tetrahedrons in fig. 2, but also a parallel alignment of isospins of neighboring tetrahedrons. Before the appearance of this structure the internal spins U and D, which are the building blocks of the isospin vectors, can freely rotate, and thus there is an internal spin SU(2) symmetry group. In ordinary magnetism this group is usually called Heisenberg's SU(2); in the present context it can be interpreted as the Standard Model SU$(2)_L$ gauge group.

The point is that these transformations are local symmetries in the sense that the isospin vectors can be rotated independently over each point of Minkowski space. The group gets broken to G_4 when the internal arrangement fig. 1 is formed. It may be given the index L, because this arrangement is chiral and because there is a dynamical relation between internal and external chirality, as explained in (2.1.21).

The mixing with the electromagnetic $U(1)$ symmetry has not been introduced at

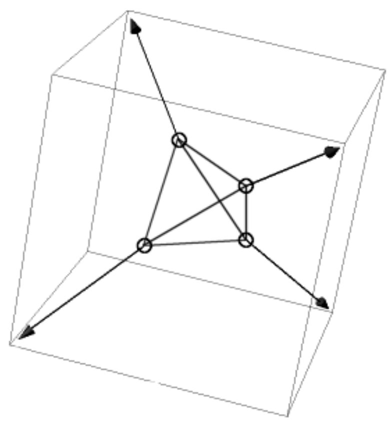

Figure 1: The local ground state of the model, living in a 3-dimensional isospin space called the 'fiber'. Shown are the tetron locations (open circles) and the 4 ground state isospin vectors $\langle \vec{Q}_i \rangle$. The origin of coordinates is taken to be the center of the tetrahedron, and is identical to the base point of the fiber in Minkowski space. The tetrahedron itself has the tetrahedral group S_4 as point group symmetry. However, due to the pseudovector property of the spin vectors the whole system has the Shubnikov point symmetry (5). The Shubnikov group is chiral, the configuration with opposite chirality being given when the 4 isospin vectors would point inwards instead of outwards. Before the formation of the chiral tetrahedron, the internal spins U and D, which according to (2) are the building blocks of the isospin vectors, can freely rotate and thus there is an internal spin SU(2) symmetry group, which however is broken to G_4 when the chiral tetrahedron is formed. Note there are actually 2 tetrahedrons in this figure, one with respect to the internal coordinates (tetron locations) and the other one with respect to isospin vectors, and both tetrahedrons are 'aligned', in the sense that the coordinate vectors and the isospin vectors point into the same (radial) direction. This 'alignment' of coordinate and isospin vectors within one fiber has to be distinguished from the alignment of isospin vectors with respect to the isospins of neighboring tetrahedrons, as shown in fig. 2. The latter forms the basis for the electroweak phase transition, while the coordinate alignment of neighboring tetrahedrons is relevant for crystal formation at big bang temperatures.

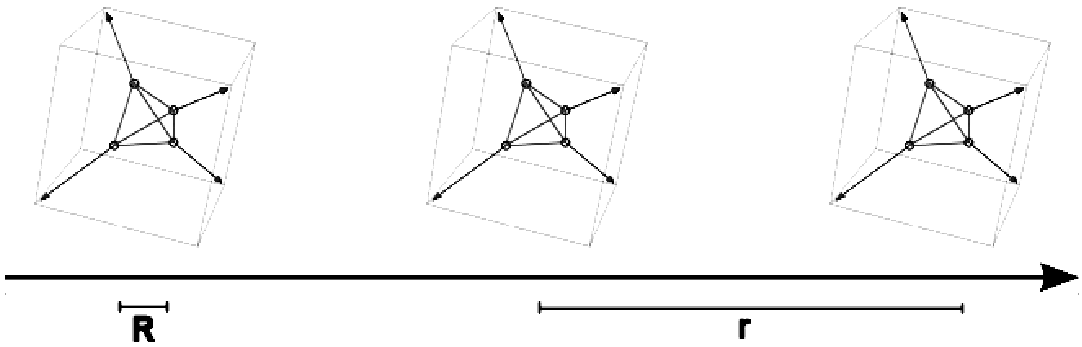

Figure 2: The global ground state of the model after the electroweak SSB consists of an aligned system of chiral tetrahedrons over physical space (the latter represented by the long arrow). R is the internal magnitude of one tetrahedron and r the distance between two of them. The figure is a bit misleading, not only because the tetrahedrons do not have an extension into physical space, but also the relative magnitudes are not correctly drawn. While r and R are tiny (of the order of the Planck length), the tetrahedrons formed by the isospin vectors are much larger, of the order of the Fermi scale (in inverse energy units). Actually there are 2 kinds of alignment in this figure: the alignment of neighboring coordinate tetrahedrons and the alignment of isospin vectors in neighboring tetrahedrons. The isospin vector alignment is associated to the electroweak symmetry breaking, because at temperatures above the Fermi scale (before the SSB) the isospins in each tetrahedron are oriented randomly (not shown) and there is a corresponding *local* $SU(2)$ symmetry which gets broken when the isospin vectors align. The figure also shows how the universe looks like in the tetron model. It is a 3-dimensional 'monolayer' of internal tetrahedrons whose average distances are given by the Planck length $\langle r \rangle = L_P$. Gravity is due to the elasticity of the coordinate bonds between the tetrahedrons and corresponds to tiny deviations from this average in the vertical or horizontal direction. Finally, the coordinate alignment of the tetrahedrons is related to crystal formation and cosmic inflation after the big bang. These latter issues will be discussed in detail on pages 14-22 and in section 2.5.

this point. This omission will be clarified later in (2.1.8) and (2.1.12), together with the tetron model interpretation of the electroweak mixing angle.

One wants to interpret the 3 generations of quarks and leptons as isospin wave excitations of the internal isospin structure. These excitations will be called **mignons**. They behave as quasi-particles while they travel through Minkowski space and can be grouped according to representations of G_4.

G_4 is a finite symmetry group which remains intact to the lowest energies. As shown in [9] it has only 1- and 3-dimensional representations. To generate all possible excitations describing the quarks and leptons one has to consider the vibrations of $\vec{S} = \vec{Q}_L$ and $\vec{T} = \vec{Q}_R$ for each of the 4 tetrons separately, cf. (2.3.1) and (2.4.14).

The isospin vibrational excitations are described by deviations δ from the ground state fig. 1, i.e.

$$\vec{S}_i = \langle \vec{S}_i \rangle + \delta \vec{S}_i \qquad \vec{T}_i = \langle \vec{T}_i \rangle + \delta \vec{T}_i \tag{7}$$

or, more precisely, by certain linear combinations of them – the eigenmodes of the isospin Hamiltonian to be discussed later in (12) and (13).

The resulting 24 mignon states can be arranged in six singlet and six triplet representations $A_{\uparrow,\downarrow}$ and $T_{\uparrow,\downarrow}$ of G_4 to yield precisely the multiplet structure of the 24 fermion states of the 3 generations, not less and not more:

$$\begin{aligned} A_\uparrow(\nu_e) + A_\uparrow(\nu_\mu) + A_\uparrow(\nu_\tau) \quad &+ \quad T_\uparrow(u) + T_\uparrow(c) + T_\uparrow(t) + \\ A_\downarrow(e) + A_\downarrow(\mu) + A_\downarrow(\tau) \quad &+ \quad T_\downarrow(d) + T_\downarrow(s) + T_\downarrow(b) \end{aligned} \tag{8}$$

The SM quantum numbers can be recovered from this spectrum in the following way:
–the $\uparrow$ representations can be obtained from the $\downarrow$ ones by the transformation $\delta\vec{S} \leftrightarrow \delta\vec{T}$ for any of the tetrons, i.e. by interchanging left and right. As shown in (2.4.14) this is precisely what is needed for an isospin transition on the level of mignons.
–singlet and triplet Shubnikov states have a different U(1) charge. The corresponding symmetry can be interpreted as gauged fermion oder $B - L$ number. Details will be given in (2.1.8) and (2.2.3). The mixture with the photon and the appearance of the Weinberg angle will be discussed in (2.1.12).

–the 3 states within each triplet T in (8) are always degenerate, because G_4 remains unbroken. The relation between those triplets and the QCD color triplets of quarks will be further discussed in (2.3.25).

Actually, to obtain the quark and lepton spectrum (8) a discrete structure is compelling only in internal space, not in physical space. Looking at fig. 2, one could try to come along with a continuous model of Minkowski space, i.e. with $r \to 0$. However, it is tempting to assume $r \neq 0$, i.e. that there is a sort of lattice underlying spacetime, with spacings so small that Lorentz symmetry is effectively maintained for all available energies.

Details of this idea will be discussed after (16) and in section 2.5, where it will be shown that the lattice must be (i) elastic and (ii) a Planck lattice, otherwise it would contradict (i) cosmological observations and (ii) Einstein's principle of equivalence[15, 16]. Due to quantum fluctuations it may be a foam[7] or a spin network[8] – although in the tetron model there is no a priori necessity to quantize gravity, cf. section 2.5.

Can the aligned structure fig. 2 be understood heuristically? The answer is yes, if one assumes that the arrangement of isospin vectors follows similar rules than that of spin vectors in a magnetic environment. What matters are value and sign of (internal) exchange integrals J of tetron wave functions as a function of the distance between 2 tetrons, because these integrals will appear as couplings in the Heisenberg isospin Hamiltonian (12).

The behavior of isospins in fig. 2 can then be understood via the so-called Bethe-Slater curve shown in fig. 3. If the tetrons are part of one tetrahedron, their distance is small $\sim R$ and according to the figure J is negative. This corresponds to anti-ferromagnetic behavior and leads to the formation of the frustrated structure fig. 1 with symmetry $A_4 + S(S_4 - A_4)$, because the spin vectors try to avoid each other as far as possible.

In contrast, if the internal spin vectors belong to different tetrahedrons, the distance of the corresponding tetrons is somewhat larger, of order r, and J is positive. This corresponds to ferromagnetic behavior.

Due to the tetrahedral 'star' structure fig. 1 it is appropriate to change the notion of isospin. Usually in an (anti)ferromagnetic environment, the spin vectors align into the + or - orientation of the z(=magnetization) direction, and the corresponding Pauli spinors are given by $U = (1, 0)$ and $D = (0, 1)$. In the present case of a frustrated system the situation is different. The anti-ferromagnetic structure is defined by isospin vectors either pointing outward or inward in the radial direction. Correspondingly, the isospinors U and D are to be understood as 'radial' spinors[22]

$$\begin{aligned} U_\star = \sqrt{\frac{1}{3}} Y_1^0 U - \sqrt{\frac{2}{3}} Y_1^1 D &= \cos\frac{\vartheta}{2}\ U + \sin\frac{\vartheta}{2} e^{i\frac{\varphi}{2}}\ D \\ D_\star = \sqrt{\frac{2}{3}} Y_1^{-1} U - \sqrt{\frac{1}{3}} Y_1^0 D &= \sin\frac{\vartheta}{2} e^{-i\frac{\varphi}{2}}\ U - \cos\frac{\vartheta}{2}\ D \end{aligned} \tag{9}$$

where Y_l^m denote the sperical harmonics and ϑ and φ are the angles of the radial vector w.r.t. some cartesian coordinate system. These new spinors are radial in the sense that they reproduce the unit vector in polar coordinates

$$\vec{e}_r = U_\star^\dagger \vec{\tau} U_\star = -D_\star^\dagger \vec{\tau} D_\star \tag{10}$$

Furthermore they are normalized in such a way that

$$U_\star^\dagger U_\star + D_\star^\dagger D_\star = U^\dagger U + D^\dagger D \tag{11}$$

The iso-spinor corresponding to an isospin vector pointing outward is denoted by $U_\star$. According to figs. 1 and 2 it is the building block of the hyper-crystal in its ground state. As shown in sections 2.1 and (2.2.7) it is unpolarized and its electric charge vanishes.

Note that this presentation is equivalent to the 'universal' z-axis approach[64, 65] used in the actual mass calculations[2]. Although according to (10) $D_\star$ has as much to do with U as it has with D, I will often leave out the star index in the following for reasons of simplicity and understand that always $U_\star$ and $D_\star$ are meant. I will include the star only in cases this is needed for clarity, e.g. in (2.1.9).

In the tetron model the SM SSB arises from the 'ferromagnetic' alignment of isospin vectors in neighboring tetrahedrons. As shown in (2.1.9) and (2.3.12), the corresponding order parameter is given by a non-vanishing vacuum expectation value $\langle \bar{U}_\star U_\star \rangle \neq 0$. In other words, there is a pairing active comparable to the case of Cooper pairs in a superconductor, and excitations of this tetron-antitetron pairing

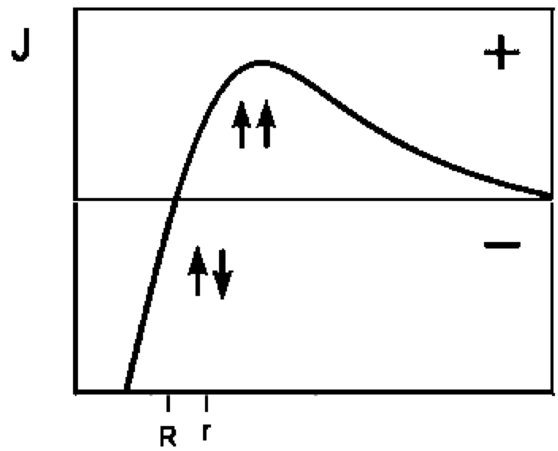

Figure 3: Bethe-Slater curve: the exchange integral coupling J as a function of the distance between 2 tetrons.

will appear as the physical Higgs field and the electroweak bosons. This is the way the SM Higgs mechanism is realized on the microscopic level.

In [2] the masses of the mignons (8) have been calculated, and the observed hierarchy in the quark and lepton spectrum as well as the hierarchy in the CKM and non-hierarchy in the PMNS matrix elements has been reproduced. As described above, mignon masses can be identified with the eigenfrequencies of the vibrations of the isospin vectors $\vec{S}$ and $\vec{T}$. These eigenfrequencies get contributions both from inner- and from inter-tetrahedral interactions.

Firstly, the *inner*-tetrahedral interactions are responsible for the frustrated tetrahedral configuration fig. 1, i.e. for the structure of the local vacuum. They are small distance contributions and relatively simple to treat because they can be described by an internal Heisenberg Hamiltonian for one tetrahedron alone, with corresponding internal spin vector excitations. The most general form of this Hamiltonian is

$$H_H = -J_{SS} \sum_{i \neq j=1}^{4} \vec{S}_i \vec{S}_j - J_{TT} \sum_{i \neq j=1}^{4} \vec{T}_i \vec{T}_j - J_{ST} \sum_{i,j=1}^{4} [\vec{S}_i \vec{T}_j + \vec{T}_i \vec{S}_j] - K_{ST} \sum_{i=1}^{4} \vec{S}_i \vec{T}_i \quad (12)$$

where the couplings J are internal exchange energy densities characteristic for the internal Heisenberg interactions. By introducing K_{ST}, I have allowed that the coupling $\vec{S}_i\vec{T}_j$ is different within a site ($i = j$) than outside of it ($i \neq j$).

Using (12) and (7) one is lead to e.o.m. for $\delta\vec{S}_i$ and $\delta\vec{T}_i$ which can be solved in a similar way as the e.o.m. for magnons in solid state physics. On this basis the contributions from (12) to the eigenfrequencies of the 24 eigenmodes were calculated in [2].

Secondly, the *inter*-tetrahedral interactions are based on the parallel (='ferromagnetic') alignment of isospins between different tetrahedrons fig. 2. Their leading effect turns out to be a contribution of order $O(\Lambda_F)$ solely to the top quark mass[2]. Physically speaking, this interaction handicaps the specific eigenmode describing the top quark, because this mode disturbs the SSB alignment in the strongest possible way.

Mathematically, the effect can be described by adding terms to the inner-tetrahedral Heisenberg interaction with a normal ferromagnetic plus a Dzyaloshinskii-Moriya (DM) component[24]. The sum of the 2 components will yield a quasi-democratic mass matrix which in leading order only contributes a term of order Λ_F to the top-quark mass and nothing to the masses of the other quarks and leptons.

More in detail, the Hamiltonian for the SSB interactions of neighboring tetrahedrons can be derived from the W-mass term of the SM Lagrangian. By considering a $SU(2)_L$ gauge transformation, which removes the longitudinal components of the W-bosons from the Higgs part of the SM Lagrangian, one obtains

$$H_{SSB} = \frac{\mu^2}{4\Lambda_F} \sum_{i,j=1}^{4} [\vec{S}_i\vec{S}'_j \; + \; i\;(\vec{S}_i \times \vec{D}_{ij})\;\vec{S}'_j] \tag{13}$$

to be added to the Heisenberg Hamiltonian (12). μ is the mass parameter of the SM Higgs potential. Only terms involving the left handed isospin vectors $\vec{S} = \vec{Q}_L$ appear, in accordance with the $V - A$ structure of the weak interactions. In (13) the factor $\vec{S}'_j$ denotes the left handed isospin vector of an adjacent tetrahedron. An elaborate connection between (12) and (13) on one side and the SM Lagrangian terms on the other will be developed in (2.1.15).

Eq. (13) contains a ferromagnetic interaction plus the additional DM term which is due to the non-abelian nature of the W-bosons. The overall normalization of the

DM-term is dictated by $SU(2)_L$ gauge invariance, while the relative values of the DM-couplings $\vec{D}_{ij}$ are fixed by the internal $A_4 + S(S_4 - A_4)$ symmetry[2].

Quite in general, a DM component stands for a tendency to form a rotational structure (instead of the ordinary ferromagnetic alignment of neighboring tetrahedrons depicted in fig. 2) simply because the DM-term tends to rotate the spin vectors instead of aligning them. In the present case it appears as a consequence of the non-abelian nature of SU(2). Therefore the DM-term can be interpreted quite naturally, namely by the fact that the $SU(2)_L$ gauge fields induce a curvature of the fiber bundle formed by the system of all tetrahedrons, and the DM-term simply takes care of this curvature effect to effectively maintain the aligned structure.

This argument is supported by the fact that the gauge transformation mentioned above leads to the SM Lagrangian in the so-called 'unitary gauge'. As shown in (2.3.15) this is exactly the gauge where one encounters the aligned isospin structure among different tetrahedrons depicted in fig. 2.

Using (12) and (13) one can derive the e.o.m. for the isospin vectors. With the usual ansatz $\sim \exp(i\omega t)$ one obtains two 24×24 eigenvalue problems, which turn out to be separately diagonalizable[2]. The eigenvalues ω correspond to the quark and lepton masses.

The quantitative results can be found in [2]. In that paper it was explicitly verified that the corresponding 24 eigenstates can be arranged into 6 singlets and 6 triplets as predicted by the Shubnikov symmetry analysis (8), i.e. as 6 lepton and 6 quark flavors. Each triplet (quark flavor) consists of 3 states with degenerate eigenvalues, because the Shubnikov symmetry $A_4 + S(S_4 - A_4)$ is unbroken at low energies.

The dominant contribution from (13) gives the top quark a mass of the order of the Fermi scale while leaving the other quark and lepton masses unchanged. b, c, s and τ get their masses mainly from (12). For details see (2.4.16), (2.4.17) and [2].

In contrast, there are no contributions from (12) and (13) to u, d, e and neutrino masses. These 10 excitations remain massless on this level. To obtain their masses one has to include additional small torsional interactions[2].

The masses of the neutrinos are particularly suppressed because the 3 neutrino modes correspond to the vibrations of the 3 components of the total internal angular momentum vector

$$\vec{\Sigma} := \sum_{i=1}^{4}(\vec{S}_i + \vec{T}_i) = \sum_{i=1}^{4}\vec{Q}_i \tag{14}$$

Whenever this quantity is conserved

$$d\vec{\Sigma}/dt = 0 \tag{15}$$

the neutrino masses will strictly vanish ($\omega = 0$). In fact, the interactions considered so far, i.e. (12) and (13), conserve total internal angular momentum. Therefore, they fulfill (15) and give no contribution to the neutrino masses. Further details can be found in (2.4.10)-(2.4.13) and in [2].

The general solution to the eigenproblem given above does not only yield the energy eigenvalues but (via the corresponding eigenvectors) can also be used to accommodate the CKM and PMNS mixing matrices[2]. The mass eigenstates are the states corresponding to the energy eigenvalues, while the interaction eigenstates naturally correspond to the original vectors $\vec{S}_i$ and $\vec{T}_i$.

Within this framework one can understand[2] why the CKM elements turn out to be small, whereas the PMNS matrix elements are naturally large: the lepton eigenstates (roughly given by $\vec{S} \pm \vec{T}$) are 'far away' from $\vec{S}$ and $\vec{T}$, while the up- and down-type quark eigenstates are relatively small deformations of $\vec{S}$ and $\vec{T}$, respectively. Due to the dominant contribution from (13) the top quark triplet state has the smallest mixing matrix elements with other quarks, because it corresponds to the vibration of $\sum_i \vec{S}_i$ to an accuracy of less than 1%.

In summary, the present model describes the physical world as a huge ordered crystal of internal 'molecules', each molecule of tetrahedral form and arranged in such a way that the internal Heisenberg spin symmetry is spontaneously broken. As shown below, this approach not only provides a nice microscopic understanding of particle physics phenomena but in addition substantially supplements our understanding of the (inflationary) big bang cosmology. In effect, it gives the phase transitions in the early universe a microscopic meaning.

To comprehend this fact, it is appropriate to redevelop the full history of the early universe within the assumptions of the tetron model: before the 'big bang' there were the free tetrons ψ floating around as a Fermi gas in R^{6+1} space at extremely high pressure and temperature. While the universe was cooling down, 3 fundamental transitions occurred:

I. the formation of tetrahedral 'molecules' from tetrons at very high temperature of order Λ_R, where the scale R is roughly given by the extension of one molecule. Although this process is not a phase transition in the strict sense it has certainly released a large amount of energy which has amplified the initial temperature of the universe.
Note that with 4 molecular sites each molecule 'fills' only 3 of the 6 spatial dimensions.

II. the formation of the 'hyper-crystal' from tetrahedrons takes place at somewhat lower temperatures $T \sim \Lambda_r$, where r is the 'lattice spacing', i.e. is roughly given by the distance between 2 tetrahedrons. This alignment of all tetrahedral structures is a coordinate alignment and to be distinguished from the isospin vector alignment (item III) describing the electroweak phase transition. It puts all 3-dimensional molecular structures in parallel thus separating an internal 3-dimensional space from the rest. In other words, the crystal expands into a 3+1-dimensional subspace of R^{6+1}, while the tetrahedrons extent into what becomes the 3 internal dimensions.

Since II corresponds to the process, in which our 3+1 dimensional universe was born, it may rightfully be called the big bang. As a crystallization process it is a first order phase transition associated with the sudden release of a large amount of energy. As will be explained later, the coordinate interactions among the tetrahedrons are of elastic type. Under this condition the outcome of phase transition II is not a crystal in the strict sense, and one may as well call it a condensation of a hyper-plastics instead of a crystallization, cf. (2.5.9). In any case the release of crystallization/condensation energy naturally drives an inflationary expansion of the system and the corresponding metric. Therefore, within the framework of the tetron model, the big bang and the beginning of inflation are more or less identical. As argued in (2.5.9) and further below in this section, the characteristic scale Λ_r can be identified to be of the order of the Planck scale Λ_P.

III. the arrangement of isospins at temperatures of order Λ_F. Above those temperatures the isospin vectors fluctuate randomly with an associated internal 'Heisenberg' $SU_L(2)$ symmetry, but at Λ_F they arrange into the chiral isomagnetic structure figs.

1 and 2. At that point the so far freely rotatable internal spins get ordered and $SU_L(2)$ is broken to the Shubnikov group (5). Note that $SU_L(2)$ is a local symmetry, because isospins can be rotated separately over each point of the Minkowski base space.
A more detailed description of this phase transition is given in (2.3.15) and (2.3.12). The electromagnetic $U(1)$ symmetry will be included in (2.1.8) and (2.1.12).

As discussed below, gravity cannot be described by a rigid crystalline micro-system with constant lattice spacings r and a fixed position of its micro-components (the internal tetrahedrons). It rather resembles an elastic medium, where the distances of the tetrahedrons vary around an average lattice spacing $\langle r \rangle = L_P$.

Furthermore, it could happen that there is no internal coordinate order right after the crystallization, in the sense that the coordinates of tetrons in neighboring tetrahedrons are aligned.[1] In that case there is no global internal tetrahedral symmetry of the hyper-crystal right after the condensation. However, since internal coordinate alignment is a prerequisite for the isomagnetic alignment (= electroweak transition) III, this has to be catched up later at the Fermi temperature, i.e. it takes place at about the same time as the isospin alignment.

This possibility will be called scenario C in later discussions, and in fact it has several benefits. For example, III automatically becomes a first order phase transition (cf. 2.3.16) with an associated second inflationary process that removes domain walls (cf. 2.3.21) from the visible parts of the universe. Furthermore, it is easier to understand that the electroweak phase transition is really spontaneous (cf. 2.3.15) and that the ground state fig. 1 is assumed by both the left- and the right-handed isospin vectors $\vec{Q}_{Li}$ and $\vec{Q}_{Ri}$ (cf. 2.3.3).

Since II happens after I, i.e. at lower temperature, one naturally expects Λ_R larger than Λ_r (i.e. $R < r$) in agreement with fig. 2 and the Bethe-Slater curve fig. 3. As argued in (2.3.17) both scales are $\geqslant \Lambda_P$ and much larger than the scale Λ_F where the isospins align. Note that while Λ_F approximately corresponds to the critical point of

[1] I am not talking about a regular crystal structure in physical space, which for an elastic system is missing anyhow. Instead I am talking about the alignment of the *tetron* coordinates as depicted in fig. 2. See also the discussion at the end of (2.3.15) where the assumption of internal coordinate order is completely given up.

transition III, the values of the exchange integrals J and therefore the iso-magnetic behavior are determined at distances r and R.

To describe II in the framework of the Landau approach to phase transitions one should consider density fluctuations $D\exp(i\vec{p}\vec{x})$ within the gaseous assembly of tetrahedral 'molecules' and use D as the order parameter of the phase transition.

For an ordinary crystal these fluctuations can be identified with phonons; in the general relativistic (GR) framework of a spacetime continuum at least some of them correspond to gravitational waves. This will become clearer below in this section, where elastic deformations of the crystal will be identified with metrical changes. More information about gravitons in the microscopic model can be found in (2.5.35),(2.5.36) and (2.5.37).

Since the density perturbation adds to the uniform density of the tetrahedron gas, there is no symmetry under changing sign of the density wave, and so the Landau free energy expansion allows for a cubic term

$$\Delta F = \alpha(T - T_c)D^2 + \beta D^3 + \gamma D^4 \tag{16}$$

where $T_c \sim \Lambda_r$ is the critical temperature for phase transition II.

The appearance of the cubic term is characteristic for a first order phase transition where a second minimum, which develops in the potential when the temperature is lowered, for some time remains higher than the minimum at $D = 0$ of the gas phase, and furthermore the two minima are separated by a potential wall. When the temperature drops below the critical value, there is a discontinuity which is not present in second order transitions.

The latent heat associated with this discontinuity is released very suddenly and can be used to explain the extreme acceleration needed for cosmic inflation[13, 14]. As shown in (2.5.23) it provides all the necessary ingredients for the inflationary process to start and to eventually stop, once the condensation of tetrahedrons is completed and the energy is exhausted. Most of the initial molecular energy has then been transferred to the elastic energy of the crystal. However, some of it survives in the form of tetron-antitetron excitations (gauge bosons) and is converted into mignons (quarks and leptons), as the temperature decreases further.

In place of quarks and leptons, whose existence is tied to the isospin ordering fig.

2, shortly after crystallization other excitations are more important, like the internal coordinate vibrations discussed in (2.3.22) and [3], or excitations of the tetron-antitetron bonds discussed in (2.1.9) and (2.1.10). Most prominent among the latter are the gauge bosons and the visible and dark scalars of the 2HDM sector as described in (2.1.15) and (2.5.31).

Because of the dominance of the electroweak bosons this cosmological era is often called 'radiation dominated' or 'electroweak'. At temperatures far above the Fermi scale all these excitations are effectively massless states transforming under the local $SU(2) \times U(1)$ symmetry, and they dominate the universe all the way down to the electroweak SSB (=isospin alignment). More details about the tetron model view on this era can be found in (2.1.13), (2.1.14), (2.3.8) and (2.3.9).

As well known, in general relativity a non-vanishing energy momentum tensor leads to a curvature of the spacetime continuum. Many authors have interpreted this on the basis of metric elasticity[77, 83, 84, 86, 85], and some of them have speculated that gravity forces might be explainable from a microscopic structure which in some sense is analogous to the atomic structures responsible for material elasticity in low-energy physics. In such a framework, GR is equivalent to an elastic continuum and the Einstein equations are not a fundamental but only an effective description of the microscopic dynamics, only valid at distances much larger than the Planck length.[2]

In the following I will describe some details of this approach and adapt it to the requirements of the tetron model. The fundamental dynamical quantity in general relativity is the metric which defines the distance between 2 spacetime points. It can be calculated e.g. from the transition function between arbitrary local coordinates x_μ on the manifold and local Lorentz coordinates ξ^α of an inertial system via

$$g_{\mu\nu} = \eta_{\alpha\beta} \frac{\partial \xi^\alpha}{\partial x^\mu} \frac{\partial \xi^\alpha}{\partial x^\mu} \tag{17}$$

where $\eta_{\alpha\beta}$ is the Minkowski metric valid in the inertial frame. Using this definition one can go further ahead and write down the curvature/field strength and the Einstein equations.

[2]Taking this point of view one avoids the main two problems of general relativity: (i) the problems of quantization - as discussed in (2.5.35) and (ii) the existence of singular solutions - because in the micro-elastic interpretation a solution to the Einstein equation makes no sense at distances of the order of one lattice spacing L_P.

In the micro-elastic interpretation a gravitational field induces a deformation in the medium, i.e. a displacement of the internal tetrahedrons within physical space from x_μ to x'_μ. This corresponds to a modification of the metric

$$g'_{\rho\sigma} = g_{\mu\nu} \frac{\partial x^\mu}{\partial x'^\rho} \frac{\partial x^\nu}{\partial x'^\sigma} \tag{18}$$

and corresponding changes in the curvature tensor.

In order for these ideas to work in the tetron model it must be assumed that in addition to the isospin interactions responsible for the phenomena of particle physics there are elastic inter-tetrahedral *coordinate* interactions responsible for the forces of gravity. While the tetrahedral 'molecules' are rigid bodies which align in their internal spaces, the nature of the inter-tetrahedral *coordinate* interactions is elastic. Calling the universe a hyper-crystal therefore is a little misleading. One should rather call it a 'hyper-plastics' which in general can show curvature and torsion when buckling and bulging within the full R^{6+1}.

In principle the elastic forces between tetrahedrons should be derivable starting from a fundamental theory of the tetron dynamics. As long as this is not available one may imagine them to arise as remnant interactions from the saturated tetron bonds within a tetrahedron and to be naturally much smaller than particle physics(=isospin vector) effects.

The reader may wonder, what the physical meaning of 'deformations' in the time direction is. In the spatial direction it is rather clear that the distances $\sim L_P$ between neighboring tetrahedrons get modified when a gravitational field is applied. In the time direction it is the 'hopping time', which gets modified, i.e. the time a photon or some other quasi-particle needs to travel from one tetrahedron to its neighbor. These points will become clearer in the example discussed below.

Note we are talking here about an elasticity in the plastics sense. There are no *Rückstellkräfte*, and the distortions are solely driven by energy as described by Einstein's equations. For example, the energy released during crystallization immediately blows up the distance between the tetrahedrons thus inflating the volume of the DMESC in accordance with inflationary cosmology.

Another example arises by considering the newtonian limit. For a spherically symmetric configuration the metric is given via the line element

$$ds^2 = (1 + \frac{2\phi}{c^2})(cdt)^2 - (1 - \frac{2\phi}{c^2})(d\vec{x})^2 \tag{19}$$

and the Newtonian limit is defined by $|\phi| \ll c^2$. For a point mass M the gravitional potential far away from the source is given by $\phi = -GM/|\vec{x}|$. The square root of the coefficients

$$\sqrt{g_{00}} = 1 + \frac{\phi}{c^2} \qquad \sqrt{g_{xx}} = 1 - \frac{\phi}{c^2} \tag{20}$$

give the general relativistic time dilation and length contraction, respectively. In the tetron model these effects are interpreted in the following way:

-The gravitational potential of the point source M modifies the average distance L_P between 2 neighboring tetrahedrons by a factor $1 + \phi/c^2$. As a consequence, any measured length of a physical object is modified by this factor.

-The gravitational potential of the point source M modifies the average hopping time that is needed by a hyper-crystal excitation to move from one tetrahedron to a neighboring tetrahedron by a factor $1 - \phi/c^2$. This applies in particular for the hopping time T_P of a photon defined in (23). As a consequence, any measured time interval between physical events is modified by this factor.

It is clear then that the gravitational potential also can be viewed to modify the speed of light $c = L_P/T_P$ by a factor $1 + 2\phi/c^2$.

More details on this approach as well as on FLRW, gravitational waves and the interpretation of the Newton limit will be given in section 2.5. In general one has to use the ADM formalism[44] or the approach by Carter et al.[45, 46] to describe a general relativistic elastic system which includes arbitrary transformations of the time coordinate. I have chosen to restrict myself to the special cases (19) and (64), because it makes the presentation much simpler and the arguments more transparent.

Furthermore, I have been intentionally vague about which version of GR must actually be chosen. There are generalizations like teleparallel, Poincare or Einstein-Cartan gravity where in addition to Lorentz transformations 4-dimensional translations are gauged. This leads to torsion in addition to curvature as dynamical quantity[79, 87, 88]. Due to lack of experimental information on torsion and of full knowledge of the tetron dynamics it is difficult to say whether one needs a model which describes dislocations or disclinations[85] of tetrahedrons in a flat hyper-crystal or whether 'true' curvature effects are involved, in the sense that the tetrahedrons in fig. 2 are not only shifted by tiny amounts in the horizontal but also in the vertical direction.

In any case, the behavior of the gravitational field in gravity theories is determined by the form of the gravitational action S_G. Since the equations of motion should be of second order in field derivatives, S_G must be at most quadratic in torsion and curvature

$$S_G = -\frac{c^4}{16\pi G} \int d^4x \sqrt{\det(g)}[\mathfrak{R} + O(\mathfrak{R}^2, \mathfrak{T}^2)] \tag{21}$$

where G is the Newton constant and $\mathfrak{R}$ the curvature scalar. The explicit structure of S_G[58] is not given here because it is rather complicated, containing the leading term (formally identical to $\mathfrak{R}$ appearing in the Einstein-Hilbert action) plus 3 independent terms quadratic in torsion and 6 quadratic in curvature, plus possibly the cosmological constant. It can be derived from an analysis which demands consistency with the principle of equivalence and the existence of second order e.o.m. and is an example of a generalized 'f(R,T)' gravity theory[80, 82].

All in all 11 independent coupling constants[58] appear in (21). This large number of free parameters is in accord with the idea that the complete description of gravity must be quite complicated, because it is not more than an effective theory for an elastic system of microscopic entities (the internal tetrahedrons) that fill Minkowski space.

To summarize, the lattice spacing r in fig. 2 is not fixed, but corresponds to an average distance between the tetrahedrons. In addition it varies with time (temperature) during cosmic expansion, simply because the properties of an elastic continuum depend on thermodynamic variables like temperature, pressure etc. For reasons explained in (2.5.6) it is to be identified with a time dependent Planck length, i.e. one has

$$L_P(t) = \langle r \rangle \tag{22}$$

with 1.6×10^{-35} m being its present value. There is clearly a relation of this quantity to the scale factor a(t) of the FLRW universe (64) because cosmic expansion is connected to a timely increase in L_P.

By definition, the Planck length is constructed from c, G and h as one of 3 dimensionful quantities which - in the absence of SM interactions - describe all the basic properties of space[m], time[s] and matter[kg]

$$L_P = \sqrt{\frac{\hbar G}{c^3}} \qquad T_P = \sqrt{\frac{\hbar G}{c^5}} \qquad M_P = \sqrt{\frac{\hbar c}{G}} \tag{23}$$

One may invert these relations to obtain

$$c = \frac{L_P}{T_P} \tag{24}$$

$$\hbar = \Lambda_P T_P \tag{25}$$

$$\kappa = \frac{L_P}{\Lambda_P} \tag{26}$$

where $\Lambda_P = M_P c^2$ is the Planck energy and $\kappa = G/c^4$. κ is called the Einstein constant and according to (21) is the coupling of choice in GR.

Furthermore it has a rather intuitive meaning in the micro-elastic approach. Actually, this statement is true for all 3 quantities (24)-(26), because they reflect the physics of space, time and matter, as inherited in the quantities c, h and G:
(i) since L_P is the average distance between 2 tetrahedrons, then $c = L_P/T_P$ makes T_P the 'hopping' time it needs for a photon excitation to hop from one tetrahedron to the next. The question why T_P is the characteristic time of the whole system valid even for gravitational waves is answered in (2.5.37).
(ii) since $\Lambda_P \sim \Lambda_r$ is the binding energy of a tetrahedron in the DMESC, Planck's constant $\hbar = \Lambda_P T_P$ reflects the action of the binding energy during the characteristic time.
(iii) finally $\kappa = L_P/\Lambda_P$ gives the disclination of a tetrahedron in the DMESC per unit energy, i.e. applying an energy Λ_P to the tetrahedron will displace it by an amount L_P. In other words the gravitational coupling quantifies the elasticity of the ground state tetrahedron material, i.e. its reaction to any kind of mass/energy influx, as described by the Einstein equations.

According to this interpretation the values of h, c and G must be derived and temperature dependent[3] material properties of the DMESC, not valid outside of

[3]It has been claimed that specifying the timely evolution of these dimensional 'constants' is meaningless[51], because the standard rulers also change with time, and that the only thing that counts in the definition of worlds are the values of the dimensionless constants. This claim has been rightfully refuted for various reasons by many authors, see [52].

it, the only fundamental force (valid over full R^{6+1}) being the unknown interaction among the tetrons, from which h, c and G as well as all particle physics effects can in principle be calculated.

In the present model the known particles (quarks, leptons and gauge bosons, cf. (2.5.28), (2.1.2) and (2.1.3)) are interpreted as intrinsic excitations of the hyper-crystal and as such will extend over at least one average lattice spacing $\langle r\rangle$. Therefore measurements involving physical particles can never be more accurate than $\langle r\rangle$. As discussed in (2.5.6), this corresponds to a modification of Heisenberg's uncertainty principle and can be used to fix the value of h as

$$\hbar = \frac{c^3}{G}\langle r\rangle^2 \tag{27}$$

Since the photon is an excitation of the hyper-crystal, its velocity must be an intrinsic property of the crystal, and not of the full R^{6+1} spacetime - just as the speed of sound in an elastic medium is determined by the properties of the medium. According to this picture, c should be derivable from crystalline parameters and should vary with time, temperature, pressure etc. A simple argument will now be given why this effect is not detectable in present day experiments. The point is that photons on a lattice with spacing L_P have a dispersion

$$c(k) = \frac{2c_0}{L_P k}|\sin\frac{kL_P}{2}| \approx c_0 + O(kL_P)^2 \tag{28}$$

i.e. for wavelengths $\lambda = 2\pi/k$ much larger than L_P the speed of light is constant to a very good approximation. Even with the hardest and 'oldest' cosmic gamma rays observed so far deviations from c_0 cannot be tested.

Note that (28) relies on the existence of an equilibrium state and therefore does not control the behavior of c at the time of inflation when the hyper-crystal was formed under non-equilibrium circumstances.

Finally it should be mentioned that (28) is not valid in the tetron model near the boundary of the Brioullin zone, because it is derived assuming a fundamental photon wave which can acquire arbitrarily small wavelengths – which in the tetron model is not possible because photons are lattice excitations and cannot be produced with wavelengths $\leqslant L_P$. For further discussion on this point see the end of (2.5.6).

2 Questions and Answers

In this section a list of questions and answers is presented which arise in connection with the tetron model. Open problems will be specially marked and the more important ones reviewed in the summary section 3.

2.1 Questions about the gauge sector

According to section 1 the universe is interpreted as a fiber bundle over Minkowski space R^{3+1} with fibers given by the iso-magnetic tetrahedrons fig. 1. The electroweak gauge fields are to be interpreted as connections in that fiber space, i.e. they help to define what parallel alignment in between different fibers means.

2.1.1 How can such a model lead to a local gauge theory?

The internal 3-dimensional space which hosts the tetrahedrons is naturally endowed with a $SU(2)_L \times U(1)_F$ symmetry:
–the $SU(2)$ factor arises from the rotational symmetry of the internal spin vectors before their alignment and was explained in section 1.
–the U(1) factor corresponds to tetron number conservation, which on the level of quarks and leptons translates into the $B - L$ quantum number, cf. (2.1.8).
As explained in section 1 these groups act as *local* symmetries, because their elements can be chosen different for different points of the Minkowski base space. Connections can be defined for the $SU(2)_L \times U(1)_F$ bundle, which are to be interpreted as gauge bosons. As shown in question (2.1.12), there is a mixing of the $U(1)_F$ field with W_z, with mixing angle equal to the Weinberg angle. After the mixing the corresponding local gauge fields are given by the observed $W^{\pm}$, Z and γ.

2.1.2 Are the electroweak bosons and the Higgs field composite?

The answer is yes, but one should specify how this works out in detail.
–The most straightforward possibility is that they are composites of mignon antimignon pairs. However, as will be seen in (2.3.25), such a pairing is more appropriate to describe chiral symmetry breaking in QCD. Furthermore, such a construction

would make the top-quark content dominate the boson sector of the SM, similar to top condensate models. Since $m_t \gg m_W$, this is usually not considered a convincing scenario.
–Secondly, one could be tempted to insist they are fundamental objects, because they are connections of the basic $SU(2) \times U(1)$ fiber bundle in the sense of differential geometry. As such they could have been induced by curvature dynamics of the full R^{6+1} geometry. I do not think this is a very attractive option, because the flat R^{6+1} knows nothing about the dynamics within the 'curved' hyper-crystal bundle. Furthermore, the Higgs field as a necessary add-on to account for the SSB has not a simple interpretation in the pure differential geometric framework.
–I adhere to the idea that they must be excitations of tetron-antitetron bound states, i.e. directly arise from the nature of the tetron interactions in the crystal. Even more, the dynamics is such that these excitations can only form and travel inside the hyper-crystal and cannot exist outside of it. In other words, it is assumed that the tetron matter making up the crystal is so strongly bound that tetrons cannot be split off, not even in pairs. This requirement is dictated by the no-dissipation concept, cf. (2.5.28). Namely, one has to take care that such pairs do not leave the crystal and dissipate into R^{6+1}, because otherwise energy would not be conserved inside the crystal.
'Travelling' of such a pairing excitation is meant in the sense of a quasi-particle, i.e. the excitation hops from one tetron antitetron pair to another, while the tetrons themselves stick to their place in the crystal. Equivalently, one may describe it as a density wave bilinear in $\bar{\psi}$ and ψ that travels through the crystal.

2.1.3 Can a stable and massless particle like the photon be an excitation?

Yes. Masslessness of the photon is protected by the U(1) gauge symmetry. As long as this symmetry holds, the photon remains massless.
Note that the masslessness of the photon implies its stability.

2.1.4 Is the photon really an excitation?

The answer is 'no' in the QED7 model advocated in [1], where the ordinary photon is part of a '7-dimensional' photon, responsible for the iso-magnetic interactions. But yes within the 'no-dissipation' hypothesis advocated in this article, cf. (2.5.28). The latter has the advantage that energy is conserved for all processes inside the crystal, so no compactification of internal spaces is needed. The only particles which are not excitations are the tetrons, the building blocks of the crystal. These, however, are bound with energies $> 10^{10}$ GeV.
The photon being an internal excitation cannot be scattered away from the hyper-crystal. Since according to (2.2.3) one has $Q(U) = 0$, the photon is a $\bar{D} - D$ excitation of D-tetrons which are themselves bound within the hyper-crystal

$$A_\mu \sim e\, Q(D) \bar{D} \gamma_\mu D \tag{29}$$

As discussed before, the no-dissipation hypothesis (2.5.28) may have rather challenging consequences. If the photon is not a fundamental particle, it is difficult to believe that the Lorentz symmetry valid within the hyper-crystal is a fundamental property of the original full R^{6+1} spacetime. Lorentz structure comes into being only when the crystal is formed and holds only inside of it. This point is further discussed in (2.5.3).

2.1.5 What happens on the microscopic level when a mignon and an antimignon annihilate into an electroweak gauge boson?

Assume the 2 mignons are in 2 neighboring tetrahedrons. When the gauge bosons are formed, the mignon vibrations disappear and are replaced by an internal density vibration of a bound $\bar{\psi}$-ψ pair involving 2 tetrons from the neighboring tetrahedrons. This is in contrast to the suggestion made in [1] where these pairs were made up of free tetrons floating around. The latter idea has been abandoned because the binding energy of a tetron in the crystal is too large, of order Λ_P, and it would furthermore allow energy in the form of $\bar{\psi}$-ψ pairs to dissipate away from the hyper-crystal.
The excitations of bound pairs of tetrons behave trivially under $A_4 + S(S_4 - A_4)$

Shubnikov transformations, i.e. the information about the discrete tetrahedral structure is washed out, because mignon and antimignon compensate each other in that respect. What remains is the transformation property under $SU(2)_L \times U(1)_F$. Since ψ is an isospin doublet, the product of ψ and $\bar{\psi}$ leads to $2 \otimes 2 = 3 + 1$, i.e. a triplet (the weak bosons) and a singlet (the B-L photon).
These serve as connections in the fiber space. As such they are useful to define what alignment of adjacent tetrahedrons means.

2.1.6 Why can mignon couplings be understood as gauge couplings?

The mignons are dynamical sections in the $SU(2)_L \times U(1)_F$ fiber bundle described above. In order to keep up gauge invariance they are naturally endowed with gauge couplings to the connections. As for the couplings of the fundamental tetron fields $\psi = (U, D)$ I refer to (2.2.3).

2.1.7 What is the meaning of the initial $U(1)_F$ symmetry?

On the tetron level it is tetron number, on the mignon level it is $B - L$. For more details see (2.1.8).

2.1.8 How do the electric charges of mignons arise?

According to (2.1.21) in the tetron model the parity violation of the weak interaction follows from the internal chirality of the tetrahedral 'star' configuration fig. 1. This implies that there are no separate W_R bosons and that all $V + A$ couplings to mignons necessarily vanish, cf. (2.1.19). Still it is possible to formally introduce a right handed isospin quantum number via $I_3 = I_{3R} + I_{3L}$ (with vanishing coupling g_R due to the parity violating effect).
Furthermore, $F = B - L = B + \bar{L}$ is the appropriate fermion number to choose for mignons (8), with $F(l) = -1$ for leptons and $F(q) = 1/3$ for quarks. The mixing among the neutral gauge bosons can then be described by introducing the unbroken generator Q as

$$Q = I_3 + \frac{F}{2} \tag{30}$$

so that

$$Q(u) = \frac{1}{2} + \frac{F(q)}{2} \qquad Q(d) = -\frac{1}{2} + \frac{F(q)}{2} \tag{31}$$

$$Q(\nu) = \frac{1}{2} + \frac{F(l)}{2} \qquad Q(e) = -\frac{1}{2} + \frac{F(l)}{2} \tag{32}$$

2.1.9 What is the tetron content of the Higgs field and of the SM vev?

To answer this question, the same idea is used which has led to the photon equation (29), namely that all observed scalars and vector bosons arise from correlations between tetrons and antitetrons of *neighboring* tetrahedrons, cf. questions (2.1.2) and (2.1.5).

One of these correlations is directly related to the electroweak SSB and is called the Higgs particle. Since it is to support the radial alignment of isospinors in fig. 2 responsible for the SSB, it can be identified as

$$H \sim \bar{U}_\star U_\star \tag{33}$$

where $U_\star$ is the 'radial' iso-spinor introduced in (10) corresponding to an isospin vector $\vec{Q} = U_\star^\dagger \vec{\tau} U_\star$ pointing outward as in fig. 1. The point is that the content of the Higgs particle is in one-to-one correspondence with the vev needed to stabilize the alignment of isospins in fig. 2, and isospin vectors pointing outward correspond to radial spinors $U_\star$ while those pointing inwards corresponds to $D_\star$.

According to these considerations the SM Higgs doublet Φ must be of the form

$$\Phi \sim \tau_2 (\bar{U}_R Q_L)^{\dagger T} \sim \begin{pmatrix} -\bar{D}_\star(1+\gamma_5)U_\star \\ \bar{U}_\star(1+\gamma_5)U_\star \end{pmatrix} \tag{34}$$

i.e. not as in ordinary $SU(2)_L \times SU(2)_R$ symmetric NJL theories[32, 33] but formally similar to top-color models[26] – provided the use of radial isospinors is understood. The implication of (34) on the vev and on the NJL structure inherent in the SM will be discussed in (2.1.15), (2.1.16) and (2.3.12). For use in those sections I include here the definition

$$\widetilde{\Phi} := i\tau_2 \Phi^\star \sim \bar{U}_R Q_L \tag{35}$$

2.1.10 What is the tetron content of the weak gauge bosons?

The answer to this question depends on whether one is talking about the ordered or about the symmetric phase. In the symmetric phase, e.g. shortly after the hypercrystal was formed, there is only the coordinate tetrahedron but no tetrahedral 'star'-configuration of isospin vectors as in fig. 1. Therefore radial spinors (10) should play no role. The photon is given by (29) and the $U(1)_F$ tetron number gauge boson by

$$B_\mu \sim g' \, F(\psi) \, [\, \bar{U}\gamma_\mu U + \bar{D}\gamma_\mu D \,] \tag{36}$$

where g' is the $U(1)_F$ gauge coupling. Similar formulas hold for the $SU(2)$ gauge bosons.

2.1.11 How does the transition between $(U_\star, D_\star)$ and (U, D) work?

or in other words, what happens to the gauge bosons at the electroweak phase transition?

In the ordered phase, i.e. at temperatures below Λ_F, equations like (29) and (36) formally keep their validity, U and D naturally being replaced by $U_\star$ and $D_\star$. The phase transition is the point, at which Higgs and W bosons attain their non-zero masses, and this is accompanied by a redefinition of what is meant by the isospins U and D. While in the symmetric phase U and D are defined by spanning the SU(2) isospin space, in the ordered phase one switches to an effective coordinate system spanned by $U_\star$ and $D_\star$.

2.1.12 Can γ-Z mixing and the measured value of the Weinberg angle be understood in the tetron model?

Yes. In (2.2.3) it is shown that $F(\psi) = -1$ and $Q(D) = -1$. Using this input one can directly infer from (29) and (36) that the weak mixing angle at the unification/crystallization point Λ_r must be 45 degrees, i.e. $\sin^2(\theta_w) = 1/2$. The form of the Z-boson is

$$Z_\mu \sim -\frac{e}{\sin(\theta_w)\cos(\theta_w)} \, [I_3(U)\bar{U}\gamma_\mu U + I_3(D)\bar{D}\gamma_\mu D + Q(D)\sin^2(\theta_w)\bar{D}\gamma_\mu D \,] \tag{37}$$

which at Λ_r reduces to $Z \sim \bar{U}\gamma_\mu U$, i.e. at the unification point the Z consists only of U-tetrons. Note I do not distinguish left- and right-handed tetrons in these equations, because in the philosophy followed in this paper the SU(2) gauge bosons a priori contain lefthanded as well as righthanded tetrons. It is only the internal chirality of the configuration fig. 1 that prevents the $V + A$ component to become active, cf. (2.1.18), (2.1.21), (30) and [1].
In the next subsection (2.1.13) the prediction $\sin^2(\theta_w) = 1/2$ at Λ_r will be shown to agree with the present experimental value provided one uses 3 ingredients: (i) the evolution of the SM beta function as given in [59], (ii) eq. (38) and (iii) a value of the unification scale relatively close to Planck scale.

2.1.13 What is meant by 'unification scale' in the framework of the tetron model?

In the tetron model the natural electroweak unification scale is given by the energy Λ_r at which the hyper-crystal is formed from tetrahedral 'molecules' via the phase transition II. As argued in sections 1 and (2.5.6) this scale corresponds to the average distance between 2 tetrahedrons in fig. 2 and is naturally of the order of the Planck scale. As shown in (2.1.12), at Λ_r the value of the Weinberg angle must be 45 degrees. This corresponds to a relation between the $U(1)$ and $SU(2)_L$ gauge couplings

$$g'(\Lambda_r) = g(\Lambda_r) \tag{38}$$

Note that (38) goes beyond the SM because the gauge group $SU(2)_L \times U(1)_F$, even in the form of a U(2) group, is not simply connected and therefore no relation between the values of g and g' can be predicted within the SM. In contrast, in the tetron model a prediction is possible and given by (38). This is based on the observation that the original U(1) gauge symmetry is tetron number and that the photon according to (29) should be of D-content only.
Using the SM beta funtions[59] one can extrapolate g and g' from their measured values at m_Z to ultrahigh energies in order to see for which values of Λ_r eq. (38) can be satisfied. Since there is no diminishing factor 3/5 in (38) like in typical GUT models[89], Λ_r comes out to be nearly equal to the Planck scale instead of $\Lambda_{GUT} \approx 10^{15}$GeV.

One may rightfully ask whether the SM beta functions are really applicable up to such high energies or whether they get appreciable corrections from other crystal excitation like the 2HDM Higgs partners discussed in (2.1.15), or from phinons and isospin density waves which appear at higher energies, cf. (2.3.22).

2.1.14 Is there a connection between the tetron model unification scale and the scales relevant for the standard cosmological model?

Yes. In the usual cosmological terminology there is the scale at which inflation ends, and this scale is usually identified as the temperature below which the radiation dominated epoque starts. This era can be described as an equilibrium state of effectively massless electroweak gauge bosons.
In the tetron model, inflation is associated with the release of latent heat at crystal formation time. The end of inflation is the time when crystallization(=the inflation period) has finished, and the unification of the electromagnetic and the weak interactions is naturally interpreted as happening at this point. It is the time at which our 3+1 dimensional universe started to exist. According to the analysis in (2.1.13) and (2.5.6) this roughly corresponds to Planck scale energies, and therefore in the present model the electroweak era starts already at the Planck scale.

2.1.15 Is there a connection between the isospin interactions (12)+(13) of the tetron model and the SM Lagrangian?

Yes, there is. To explain this in detail, one first has to notice that while the mignon vibrators are supposed to 'live' within one tetrahedron, the Higgs excitations according to the philosophy discussed in (2.1.2) extend over two of them.
In accordance with this observation, two types of internal vectors should be distinguished:
–isospin vectors $\vec{Q}$ of type (2) and (4) which are the carriers of the isospin waves (mignons). They correspond to the internal angular momentum of tetrons *within one tetrahedron* and are the 'charges' of the conserved internal Noether currents. Their excitation spectrum leads to the quark and lepton flavors because they arrange according to the Shubnikov group within one tetrahedron.

–fields like the gauge bosons or the $\vec\pi$ component of the Higgs doublet, which involve $\bar\psi$ instead of $\psi^\dagger$. Together with the vev $\langle\bar\psi\psi\rangle \neq 0$ and the Higgs particle $H = \bar\psi\psi - \langle\bar\psi\psi\rangle$ they are important for the pairing process between tetrons and antitetrons of neighboring tetrahedrons which in the tetron model is responsible for the SSB.

To understand this in more detail consider the SM Higgs potential with one doublet

$$V_{SM}(\Phi) = -\mu^2\Phi^+\Phi + \lambda(\Phi^+\Phi)^2 = -\frac{1}{2}\mu^2(\sigma^2 + \vec\pi^2) + \frac{1}{4}\lambda(\sigma^2 + \vec\pi^2)^2 \tag{39}$$

where $\sigma = \Lambda_F + H$. This potential naturally describes the alignment of neighboring tetrahedrons and anti-tetrahedrons in fig. 2, although in fig. 2 not the $\vec\pi_i$ are drawn but the $\vec Q_i$. The point to notice is that two of the $\vec\pi_i$ are in parallel iff all the corresponding $\vec Q_i$ are. Therefore the pairing force $\sim -\mu^2\vec\pi_i\vec\pi_j$ implied by (39) exactly corresponds to a 'ferromagnetic exchange coupling' of strength μ^2 in the SSB interaction (13).

There is one drawback in this argument, and this concerns the number of d.o.f. While the SM Higgs doublet only has 4 real d.o.f., the isospin vibrators in the form of $\vec Q_L$ and $\vec Q_R$ contain 8. According to (4) these can be given as

$$\psi^\dagger\psi \qquad \psi^\dagger i\gamma_5\vec\tau\psi \qquad \psi^\dagger i\gamma_5\psi \qquad \psi^\dagger\vec\tau\psi \tag{40}$$

I have included $\psi^\dagger\psi$ and $\psi^\dagger i\gamma_5\psi$ in this list albeit their vibrations do not correspond to mignons, but to isospin density fluctuations, cf. (2.3.22).

The expressions (40) are adapted to the $SU(2)_L \times SU(2)_R$ symmetric limit. In more general cases when this symmetry does not hold one may write

$$U^\dagger U \quad D^\dagger D \quad U^\dagger D \quad D^\dagger U \quad U^\dagger\gamma_5 U \quad D^\dagger\gamma_5 D \quad U^\dagger\gamma_5 D \quad D^\dagger\gamma_5 U \tag{41}$$

By comparing with the Higgs doublet (34) one sees that half of the d.o.f. are missing in (34). To account for the other half one should add a second scalar doublet to the dynamics, e.g. in the form[4]

$$\Phi' \sim \bar D_R Q_L \sim \begin{pmatrix} \bar D(1-\gamma_5)U \\ \bar D(1-\gamma_5)D \end{pmatrix} \tag{42}$$

This corresponds to adding a pseudo-scalar iso-scalar particle η and a scalar iso-vector triplet $\vec v$ to the theory.

[4]The radial star indices (10) in these expressions are left out for simplicity.

Together, Φ and Φ' form the basis for an extended SM with 2 Higgs doublets. Such models are usually abbreviated as 2HDM, and have been extensively discussed in the literature[61, 62, 63].

The argument about the ferromagnetic alignment induced by the negative mass term $-\mu^2\vec{\pi}\vec{\pi}$ in the potential can be extended to the 2HDM model where the potential contains a term $\sim \vec{v}\vec{v}$ in addition. This term, however, must not give an appreciable contribution to the SSB interaction (13), because otherwise the b-quark mass would come out to be of order Λ_F. In other words, Φ' must not take part in the SSB; the 'mass term' $\sim \vec{v}\vec{v}$ has to have a positive coefficient and correspondingly

$$\langle \Phi' \rangle = 0 \tag{43}$$

for the second Higgs doublet and therefore no q/l mass contributions from Φ'.

This feature smartly agrees with the property of the inert version[62, 63] of the 2HDM model. It is interesting to note that in that model the η or the v_z (depending on which mass is smaller) is a serious dark matter candidate. For further details see (2.1.16)and (2.5.31).

2.1.16 What precisely is the argument in favour of the inert version of the 2HDM model?

The most general quark Yukawa Lagrangian in a 2HDM model is given by

$$-L_Y = \bar{q}_L(\Gamma\Phi + \Gamma'\Phi')d_R + \bar{q}_L(\Delta\widetilde{\Phi} + \Delta'\widetilde{\Phi}')u_R + c.c. \tag{44}$$

where 3x3 matrices of Yukawa couplings Γ, Γ', Δ and Δ' in family space have been introduced. The resulting quark mass matrices are then given by

$$M_d = \Gamma\langle\Phi\rangle + \Gamma'\langle\Phi'\rangle \qquad\qquad M_u = \Delta\langle\Phi\rangle^\star + \Delta'\langle\Phi'\rangle^\star \tag{45}$$

Unfortunately, the diagonalization of M_u and M_d does not in general diagonalize the quark Higgs Yukawa interactions implied by (44) and this leads to the problem that unwanted FCNCs are present in the most general 2HDM model[61]. This is usually handled by the ad hoc introduction of an additional Z_2 symmetry. For example, one may demand the 2HDM Lagrangian to be invariant under the transformation

$$\Phi' \to -\Phi' \tag{46}$$

In this case all Yukawa couplings involving Φ' drop out, and all quarks and leptons couple solely to Φ. Furthermore, symmetry under (46) forbids mixing terms $\sim \Phi^\dagger\Phi' + c.c.$ in the 2HDM Higgs potential so that the Higgs field with vanishing vev can be unambiguously taken to be Φ' in accordance with the representation (42). In the tetron model one has explicit representations (34) and (42) for Φ and Φ' and may therefore ask whether the physical origin of the Z_2 symmetry can be understood. It is easily seen from (34) and (42) that (46) corresponds to the transformation

$$D_R \to -D_R \tag{47}$$

among tetrons D_R. I want to argue that this kind of symmetry naturally arises because mignon interactions with the D_R field in the effective tetron Lagrangian do not appear. The point is that according to fig. 1 the system's ground state is composed of tetrons U alone, and not of D. Since quarks and leptons are excitations of the ground state, it is thus understandable that their couplings to D are strongly suppressed. This reasoning applies more to D_R than to D_L because parity is broken in the tetron model already at hypercrystal formation time, so that D_L couplings are present because D_L appears in an isodoublet with U_L. Note in this and the previous question I am actually always talking about the radial isospinors $U_\star$ and $D_\star$ instead of U and D.

2.1.17 What is the precise tetron content of the additional scalar fields in the 2HDM?

This question is most easily answered in the $SU(2)_L \times SU(2)_R$ symmetric limit of the model. According to the above discussion there are 5 observable Higgs scalars in the 2HDM model given by

$$H \sim \bar\psi\psi \qquad \eta \sim \bar\psi i\gamma_5\psi \qquad \vec v \sim \bar\psi\vec\tau\psi \tag{48}$$

leading to a quadratic part of the potential which can be interpreted in an NJL way[33]

$$-\mu^2\Phi^\dagger\Phi - \mu'^2\Phi'^\dagger\Phi' \sim G[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\vec\tau\psi)^2] + G'[(\bar\psi i\gamma_5\psi)^2 + (\bar\psi\vec\tau\psi)^2] \tag{49}$$

where the terms with coupling G would correspond to the quadratic term $\sim \mu^2$ in the SM Higgs potential (39) and the terms with coupling G' to a quadratic term for

the second Higgs doublet.
Although it is true that the vibrators $\vec{Q}_{L,R}$ are chosen in a $SU(2)_L \times SU(2)_R$ manner (4) in accordance with the discussion in (2.4.7), for the observed particles it is better to rely on the representations (34) and (42) for the doublets Φ and Φ'. Anyway, due to the possibility of mixing, e.g. between η, H and v_z, the actually observed scalar excitations will in general be linear combinations of these states.

2.1.18 How can the chiral nature of the weak bosons be ensured?

The iso-magnetic tetrahedral structure in fig. 1 violates internal parity, the state with opposite internal parity being given by a system where the 4 internal spin vectors show inwards instead of outwards. As shown in [1] and (2.1.21) this internal parity violation triggers the violation of external parity needed for the weak interactions, provided the interaction among tetrons stems from a common interaction in the full R^{6+1} space, cf. (51).

2.1.19 Are there SU(2)$_R$ gauge fields $\vec{W}_R$ in addition to SU(2)$_L$?

No. Z and W are originally connections of an SU(2) bundle. According to the discussion in (2.1.21) it is only the formation of the chiral structure fig. 1 together with the octonion (i.e. R^{6+1}) origin of the interaction which forces them to couple to left-handed mignons only. Without the internal chirality fig. 1 the weak interactions would be vectorlike.

2.1.20 If there is no $\vec{W}_R$, why is there η?

The 2HDM model (2.1.15) naturally accompanies the vibrations of $\vec{Q}_L$ and $\vec{Q}_R$. 2HDM models do not need a $\vec{W}_R$-field.

2.1.21 Is there a tetron interaction which gives rise to such iso-magnetic structures? Can the parity violation of the weak interactions be explained from first principles?

Before I start to discuss this question, note it is not about the fundamental coordinate forces which are responsible for the formation of (tetrahedral molecules and) the hyper-crystal at scale Λ_r, but only about the isomagnetic forces relevant for the isospin vector alignment at the Fermi scale.
The model advocated in this paper consists of 6+1 dimensional spinor fields ψ ('tetrons'), which form tetrahedral structures and a hyper-crystal a la fig. 2. In this world the quarks and leptons propagate as internal spin waves. The Higgs and the observed vector bosons are excitations of tetron-antitetron bound states, and the system as a whole naturally gives rise to the $SU(2)_L \times U(1)_F$ gauge symmetric SM. For the appearance of the strong interactions see (2.3.25), and the role of gravity is discussed in section 2.5.
The isomagnetic tetron interactions are claimed to derive from the octonion structure which is naturally inherent in a 6+1 dimensional space. The octonions form the unique non-associative, non-commutative and normed division algebra in 8 dimensions, and their imaginary units provide for 7 of the 21 generators of SO(6,1). They are closely related to the Dirac matrices Γ_μ in 6+1 dimensions[89]. The 6+1 dimensional vector current $\bar{\psi}\Gamma_\mu\psi$ arises more or less directly from the product of two octonions corresponding to the spinors $\bar{\psi}$ and ψ.
When the hyper-crystal is formed and R^{6+1} spacetime decomposes into Minkowski and internal space, the Γ_μ split into SO(3,1) Dirac matrices γ_μ and a remainder according to[89, 40]

$$\Gamma_{1-7} = (\gamma_{1-4}, \gamma_5\tau_{x,y,z}) \tag{50}$$

where x, y and z denote the internal coordinates. This splitting has its physical origin in the coordinate interactions of the tetrons, which lead to the formation of the hyper-crystal, and mathematically it parallels the splitting of an octonion into 2 quaternions.
Starting from (50) one can try to derive the parity violation of the weak interaction. The important point to notice is the appearance of the product $\gamma_5\vec{\tau}$ in the internal part of (50). In principle, the presence of such a coupling corresponds already to a

parity violating behavior, both in internal and Minkowski space, because γ_5 signals axial behavior in Minkowski space and $\vec{\tau}$ does the same job for the non-relativistic internal fiber.
According to (50), any 6+1 dimensional vector coupling $\bar{\psi}\Gamma_\mu\psi$ reduced to internal space will induce such a term. However, for this to actually become perceivable, an additional appropriate 'chiral situation' has to be provided, again both in internal and Minkowski space. In Minkowski space this can be achieved, for example, by using polarized beams or if there is a second vertex with a γ_5-coupling in the Feynman diagram of the process.
An analogous requirement must be met in the internal space. In other words, a configuration with a handedness must be present, in order to pick up a non-vanishing contribution from the axial coupling, and this in the case at hand is given by the local chiral ground state structure fig. 1.
As a matter of fact, the non-relativistic circumstances of the internal R^3 space make it a similar situation as one has in optical activity of molecules, where in addition to a circularly polarized photon there must be a handed molecule in order to produce a non-vanishing effect. According to (50) a 4-tetron interaction of two vector currents will induce among others a term

$$\vec{\pi}\vec{\pi} = [\bar{\psi}\vec{\tau}\gamma_5\psi][\bar{\psi}\vec{\tau}\gamma_5\psi] \tag{51}$$

which agrees with the quadratic term of the Higgs potential (39) responsible for the alignment of isospin vectors and on the level of isospin vectors reproduces the Heisenberg ansatz (12). As discussed in connection with fig. 3, the sign of the coupling must be anti-ferromagnetic for inner- and ferromagnetic for inter-tetrahedral distances. The latter is in accord with the negative mass term of the Higgs potential.
An important question is whether there is a renormalizable interaction in 6+1 dimensions containing the 6+1-dimensional vector current $\bar{\psi}\Gamma_\mu\psi$ as a necessary ingredient. In [1] a 6+1 dimensional QED model was proposed as a candidate. According to that scenario, 6+1 dimensional photons and tetrons would be the fundamental field content of the original R^{6+1}. This model, however, is not compelling. It may suffer from the problem of energy dissipation into the internal dimensions, cf. (2.5.28). In the present article I advocate the gauge bosons to be tetron-antitetron excitation states which do not exist in the original R^{6+1} but arise only when the (3+1)-dimensional hyper-crystal is formed, cf. (2.1.2) and (2.1.4).

2.2 Questions about the fundamental fermion ψ

According to (1) the tetron matter fields ψ which form the sites of the lattice fig. 2 transform as a spinor 8 under SO(6,1) and decompose into an isospin doublet when the hyper-crystal is formed. In this section some further properties of the tetrons are elucidated.

2.2.1 What is the use of introducing an additional level of matter?

There are several good reasons to do so:
1. the existence of 3 families of quarks and leptons with altogether 24 states (plus the corresponding mass and mixing values) strongly suggests that they are not truly elementary objects.
2. a material origin for the observed internal symmetry groups of the SM is highly desirable. Traditionally, they are pasted into the theory as purely abstract groups, corresponding to rather static goings-on in the representation spaces. This line of thinking started with Heisenberg's invention of isospin SU(2), went over to SU(3) of color and ended with the (SUSY) GUT groups. The present model works different, isospin symmetry being obtained by extending spacetime by 3 internal spatial dimensions in which an independent dynamics takes place.
3. spontaneous symmetry breaking is introduced in the SM in a more or less ad hoc way by adding a scalar field to a system made up of fermions and gauge bosons. This is similar to and extends the Ginzberg-Landau model for super-conductivity to a local relastivistic non-abelian symmetry. However, as well known from many branches of physics, a material background is required for a phase transition and SSB to occur. For example, in superconductivity the scalar field is provided by electrons bound to Cooper pairs.
In the tetron model the breaking of $SU(2)_L$ is associated to the alignment of the (material) internal spins over Minkowski space as shown in fig. 2[1]. For the interpretation of the Higgs field as a Cooper-pair like tetron-antitetron correlation see (2.1.9).

2.2.2 What is so interesting about the 8 of SO(6,1)?

This question is discussed in (2.1.21) and (2.5.11).

2.2.3 What are the couplings / charges of the tetrons?

After the hyper-crystal is formed a tetron ψ decomposes into its isospin components U and D. This fact fixes the weak charges

$$I_3(U) = +\frac{1}{2} \qquad I_3(D) = -\frac{1}{2} \tag{52}$$

Using (30) one finds

$$Q(U) - Q(D) = 1 \qquad F(\psi) = Q(U) + Q(D) \tag{53}$$

where F is the U(1) charge and given by tetron(=fermion) number. Therefore it must be the same for both types of tetrons, i.e.

$$F(\psi) = F(U) = F(D) \tag{54}$$

Tetrons do not have a color charge because they are not involved in interactions of triplets of the Shubnikov group. Therefore, it is appropriate to normalize F in analogy with leptons instead of quarks, i.e. to put

$$F(\psi) = -1 \tag{55}$$

Eq. (53) then leads to

$$Q(U) = 0 \qquad Q(D) = -1 \tag{56}$$

In other words, the normalization (55) is equivalent to defining the U direction in iso-spinor space to be the one which is electrically neutral. This is the reason why there is no U component in the representation (29) of the photon.

Looking at (55) one may suspect that the result (56) is just a question of normalization and therefore cannot have much physical impact. However, there are only 2 possibilities for the tetron number of tetrons, +1 or -1. The first choice leads to Q(D)=0, the other one to Q(U)=0. This means the only freedom one has is which tetron one wants to call U and which one is called D. In this paper the electrically neutral tetron is called U and gives the dominant contribution to the Higgs particle (33).

2.2.4 What are the charges of the tetrons relevant for the ground state fig. 1?

According to the discussion in section 2.1 the ground state is formed by tetrons $U_\star$ whose electric charge vanishes according to (2.2.3).

2.2.5 How large is the binding energy of a tetron ψ within the hyper-crystal. Can it be ionized?

No. The binding energies are extremely large, of the order of $\Lambda_r \lesssim \Lambda_P$, cf. (2.5.6) and the discussion after (16).

2.2.6 How large is the mass of a single tetron?

Difficult to say. If gauge bosons and Higgs scalars would be $\bar{\psi}$-ψ bound states, the natural guess for m_ψ would be in the range of 40 to 60 GeV. However, in truth they are not bound states but correlations of the $\bar{\psi}$-ψ bonds within the hyper-crystal. Tetrons are so tightly bound within this environment with an extremely large binding energy $\approx E_P$ which has been released during inflation/crystallization that their mass can only be given as an 'effective mass' within the crystal, with value $m_{eff} \approx M_P$.

2.2.7 What is the spin/helicity of a single tetron within the internal tetrahedral ground state fig. 1?

Figures 1 and 2 contain all necessary isospin information for the hyper-crystal ground state. Since the tetrons U and D are ordinary Dirac fermion in 3+1 dimensions, one may also ask what their spin direction within the hyper-crystal is.

First of all, the total spin of the 4 tetrons in a tetrahedron should add up to zero, because otherwise the vacuum state (i.e. the unexcited hyper-crystal) would be polarized. I do not know exactly how strong the limits are, but I am quite sure that a polarized vacuum is not a desirable option.

Now assuming the spins add up to zero, a first guess could be that they do so in a similar way as isospins do in fig. 1. However, we are working in a relativistic

framework, and such a spin configuration clearly is not Lorentz covariant. Actually, it is better to consider helicities L and R instead of Pauli spins, and the appropriate question then is, what the values of $\langle (U, D)_{Li} \rangle$ and $\langle (U, D)_{Ri} \rangle$ are for a tetron on site i of the tetrahedron.
The answer to this question is related to the discussion in (2.3.3) about how the isospin vibrators look like for the 2 helicities in the ground state, because the polarized tetron fields $(U, D)_{L/R}$ appear in the definition of the vibrators (4), and these have to arrange according to the Shubnikov symmetry. The most straightforward solution is to choose $\langle \gamma_5 \psi_i \rangle = 0$. This is obtained from (57) and amounts to the statement that the vacuum values of the left and right handed isospin vectors are identical.

2.2.8 Are there anomalies in the tetron model, which could possibly make it inconsistent?

There are no anomalies in the fundamental theory of tetrons, because there are no γ_5 couplings. Such couplings arise only in the effective description (the Standard Model) due to the existence of the iso-*chiral* tetrahedron fig. 1, cf. (2.1.21). On the level of the effective theory the familiar anomaly cancellations among the quarks and leptons apply.

2.2.9 *How can a crystal system out of tetrons and anti-tetrons be stable?*

In other words: why is there no annihilation between its particle and antiparticle components?
I have no complete answer to this question. One observation is that according to (29) the tetrons $U_\star$ making up the ground state of the hyper-crystal have vanishing electric charge and therefore cannot annihilate into a photon. However they can still annihilate into a Z-boson. Since the Z is massive this maybe is prevented by the strong binding between tetrons in the crystal.
Another line of argument is that tetrons as a fundamental form of matter cannot annihilate into lattice excitations like the photon or the weak gauge bosons and

that writing down (29), (33) and (37) refers to *excitations* of tetron-antitetron pairs rather than the tetrons themselves.
Thirdly, one could assume that the hyper-crystal actually consists only of tetrons and not of anti-tetrons. This would make problems like the baryon asymmetry of the universe easier to solve. However, then there is the question where the excited antitetrons in (29), (33), (34) and (37) come from and how antiparticles can arise at all in this framework. Furthermore, the setup of the calculations would be different, because the vibrators would have no antitetron components and it is then difficult to see how a Dirac mass term for quarks and leptons can arise.

2.2.10 Why not use an internal molecular model instead of a crystal?

Even though Lorentz invariance can be established for lattice spacings of order L_P, it may seem difficult to imagine that the world is a kind of crystal, with every spacetime point occupied by 4 tetrons – so why not use a model where excitations of tetrahedral molecules in an otherwise empty space are used to explain the q/l spectrum? The 'molecules' would extend into internal dimensions and have a frustrated anti-ferromagnetic structure as in fig. 1. Even an explanation of the SSB as a re-arrangement within the molecules below a certain temperature is feasible.
However, with such a picture one would run into all the known problems of classic composite models[31]. The killing argument certainly is, how one and the same molecule can sometimes have a mass larger than 100 GeV and sometimes be as light as neutrinos.

2.2.11 Why not use a tetrahedral lattice in ordinary space, without any internal dimensions?

Since higher dimensions have never been observed experimentally, it is important to critically scrutinize their introduction. In this subsection I follow the idea that the tetrahedrons extend into ordinary space and only mimic the existence of internal symmetries by forming closed, neutral and ordered systems a la figs. 1 and 2 in which mignons can be excited just as in the picture with internal dimensions. As before, the extension of the tetrahedrons would have to be tiny.

The spin-$\frac{1}{2}$ nature of the mignons would be ensured by the spin-$\frac{1}{2}$ nature of the tetrons just as described in (2.4.2), while the internal quantum numbers arise from the relative angular momentum of the tetrons inside the tetrahedron, i.e. from the Shubnikov symmetry.

There are then several advantages of this approach as compared to the model with internal d.o.f.:

-there is no problem with the dissipation of energy into internal dimensions, and therefore the photon could remain an elementary particle.

-parity violation of the weak interaction would work just like optical activity in molecules, without recurrence to an octonion structure.

However, there is one serious drawback. It is related to the fact that a rotation in physical space would not only flip the ordinary spin of a mignon, but also transform the internal quantum numbers. At first sight this argument seems to kill the idea. However, it should be noted that color has only been observed in singlet states which in no case are sensitive to spatial rotations. As for internal SU(2), weak isospin partners like e and ν_e appear in (8) as singlets, i.e. do not transform into each other by rotations either. Weak isospin transitions in the tetron model are constructed rather indirectly as transition between excitations of $\vec{Q}_L$ and $\vec{Q}_R$, cf. the discussion in (2.4.14), and this construction can in principle be taken over to the scenario discussed in this subsection.

As before, the electroweak bosons would be related to $U(1) \times SU(2)$ gauge transformations referring to tetron number and the rotations of the tetron spin vectors, respectively.

2.2.12 How large is the internal extension R of one tetrahedron? How large is the average spacing $\langle r(t) \rangle$ between 2 tetrahedrons?

In most parts of the paper the following scenario is considered: R is (much) smaller than r, and r is of the order of the Planck length. This is a reasonable assumption because R is the scale of tetrahedral 'molecule' formation, these internal molecules being rigid strongly bound objects, while the condensed system of tetrahedrons is elastic and its 'lattice spacing' r grows with cosmic time[5].

Since $\langle r(t)\rangle$ was identified with the Planck length L_P in (22), the relation $R < r$ may look problematic. However as discussed in (2.5.6) and in connection with (27), the quantum nature of matter arose when the hyper-crystal was formed during the big bang / via crystallization, i.e. *after* the time of tetrahedral molecule formation.

2.2.13 Is the quantum theory of angular momentum used in (12)-(13) applicable in case $R < L_P$?

Yes, because the isomagnetic interactions responsible for the Standard Model physics take place at energies much smaller than Λ_P and Λ_r.

2.2.14 Why should the distance between 2 adjacent tetrahedrons be identified with the Planck scale?

This question is answered in (2.5.6).

2.2.15 *Why are the scales Λ_R and Λ_r so much larger than the quark and lepton masses? Why is gravity so weak as compared the mignon interactions? Why is the Planck scale so large in comparison to the Fermi scale?*

These questions are variation of why gravity is so weak as compared to QED and the other particle physics interactions. Not knowing the precise nature of the fundamental interaction among tetrons, one can only say that gravity is some kind of remnant elastic interaction among the tetrahedrons which otherwise are strongly bound to the hyper-crystal structure. According to this philosophy, the forces of gravity are so weak because elastic deformations of spacetime do not cost the system too much energy.

On the other hand, the energies $\sim \Lambda_F$ involved in the isospin alignment are much

[5] Although I did not follow this route, there is also the possibility that r und R are much larger than the Planck length. In that case tetrons would have nothing to do with general relativity, and one should forget all reasoning about gravity and cosmology presented in this work. Only the particle physics sections would apply.

smaller than the energies $\Lambda_r \approx \Lambda_P$ needed for the coordinate formation of the crystal. R and r are the length scales at which the iso-magnetic exchange integrals J have to be calculated, i.e. they are on the *abscissa* of the Bethe-Slater curve fig. 3, while the values of J are drawn on the *ordinate* of fig. 3 and always $\leq \Lambda_F$.

2.3 Questions about the local tetrahedral structure and the nature of the SSB

To obtain the correct mass spectrum for quarks and leptons not only the internal geometry but also other features of the model have to fixed.

2.3.1 Are the tetrahedrons formed by 4 or 8 tetrons, i.e. how many vibrators are needed on each lattice site?

4 tetrons are enough. Naively, one would think that 4 tetrons can give rise to only $4 \times 3 = 12$ excitations of their isospin vectors. However, there are 2 independent vibrators $\delta\vec{Q}_L$ and $\delta\vec{Q}_R$ for each tetron, and for 4 tetrons this gives the desired $2 \times 4 \times 3 = 24$ states of the form (8).
In case there are 8 tetrons on the tetrahedron one would consider unpolarized isospin vectors $\vec{Q}_{1-8}$, which appear in pairs $\vec{Q}_i$ and $\vec{Q}_{i+1}$ on each tetrahedral site i=1-4. Physically this means the tetrons i and i+1 must be very close to each other (with a tiny but non-vanishing internal distance d_8), i.e. more tightly bound than to the others. Mathematically it corresponds to a coordinate ground state symmetry $A_4 \times Z_2$ instead of S_4. Assuming ground state isospins $\langle\vec{Q}_i\rangle = \langle\vec{Q}_{i+1}\rangle$ on each site to be parallel, the isomagnetic ground state will again be symmetric under $G_4 = A_4 + S(S_4 - A_4)$. It is then straightforward to see that a spectrum of the same form (8) as before is obtained from the vibrations of the $\vec{Q}_{1-8}$.
How can the 2 scenarios be distinguished? First of all, isospin of quarks and leptons is interpreted differently in the 2 cases. According to (2.4.14) the transition $L \leftrightarrow R$ can be chosen to be accompanied by an isospin transformation. In contrast, in the case with 8 tetrons the Z_2 exchange ($i \leftrightarrow i+1$) has to provide for an isospin transition. This seems consistent, because d_8 is a distance in isospin space, not in physical space.

However, there is another, stronger disadvantage of using the approach with 8 tetrons, because the intimate connection between left-handed vibrators and the top-quark gets lost, cf. (13) and (2.4.16), i.e. the understanding why m_t is of the order of the SSB scale while all other quarks and leptons, in particular the b-quark, have much smaller masses, cf. (2.4.17).

2.3.2 What exactly is the local ground state of the system? Do all isospin vectors point outwards?

Yes, they all point outwards, as shown in fig. 2. There is no questioning about the direction of antitetrons because the relativistically covariant isospin vectors (3) are used which comprise both particles and antiparticles. As pointed out in (2.1.9) the definition of 'pointing outwards' is tied to the definition of $U_\star$ in (10).

2.3.3 How do the ground state values $\langle \vec{Q}_i \rangle$ fig. 1 decompose into helicity contributions $\langle \vec{Q}_{Li} \rangle$ and $\langle \vec{Q}_{Ri} \rangle$?

This question is important in several respects. First of all one wants to know how the vectors in fig. 1 decompose into left and right handed vibrators, because this is needed for the initial conditions for the time evolution $d\vec{S}_i/dt$ and $d\vec{T}_i/dt$ following from the Heisenberg Hamiltonian (12)and leading to the quark and lepton masses. Secondly, it is related to the fact that the tetrahedrons are unpolarized in physical space, c.f. (2.2.7).

The actual quark and lepton mass calculations[2] have been done on the basis of

$$\langle \vec{Q}_{Li} \rangle = \langle \vec{Q}_{Ri} \rangle = \frac{1}{2} \langle \vec{Q}_i \rangle \tag{57}$$

because I have found no other configuration fulfilling the Shubnikov symmetry G_4. The point is that in order to obtain the Shubnikov mass spectrum (8) it is necessary that the 2 sets of ground state vectors $\langle \vec{S}_i \rangle = \langle \vec{Q}_{Li} \rangle$ and $\langle \vec{T}_i \rangle = \langle \vec{Q}_{Ri} \rangle$ must be arranged identically as in fig. 1.

The fact that the 2 sets are not rotated w.r.t. each other, is particularly easy to understand within what was called scenario C in section 1, i.e. in case of the simultaneous alignment of coordinate and isospin vectors. In that case the $\langle \vec{Q}_{Li} \rangle$

and $\langle \vec{Q}_{Ri} \rangle$ cannot avoid to point into the same radial directions as the corresponding coordinate vectors of the tetrons.

2.3.4 Is the binding which makes up the Higgs field due to isospin or is it due to tetron coordinate interactions?

It is due to both. The Higgs particle (33) relies on the alignment of $U_\star$ isospinors and is therefore natural part of the iso-magnetic interactions. On the other hand, the Higgs (as well as all other scalar and vector fields) is an excitation of the tetron-antitetron bonds in the crystal, and therefore controlled by the coordinate interactions.

2.3.5 What are the unbroken symmetries of the model?

The Shubnikov group (5) and $U(1)_Q$. The unbroken Shubnikov group has only singlet and triplet representations and leads exactly to the observed flavor spectrum of 3 families of quarks and leptons (8).

2.3.6 Can one calculate the Fermi scale, the Weinberg angle and W/Z and Higgs mass from first principles?

The origin of the Weinberg angle was discussed in (2.1.12). The Fermi scale and the Higgs mass arise from iso-magnetic exchange and pairing interactions, as discussed in (2.1.9). Therefore if one would know the fundamental tetron interactions, these quantities would be calculable from 6-dimensional exchange integrals.

2.3.7 Did gauge bosons exist at temperatures above the Fermi scale?

Yes, they did. In the tetron model gauge bosons are particle-antiparticle correlations of crystally bound tetrons. Shortly after the crystal was formed at temperature Λ_r they came into being and made up for the bulk of particles in the 'radiation dominated epoque', cf. (2.1.14).

2.3.8 Did quarks and leptons exist at temperatures above the Fermi scale?

No, because from their very nature they require the existence of the iso-magnetically ordered state fig. 2. When the tetron gas cooled down and the hyper-crystal began to form, there was only the coordinate alignment of tetrahedrons but no alignment of isospin vectors. In that stage, at temperatures above the Fermi scale, no quarks and leptons existed, because iso-magnetic waves could not travel through the crystal. Only phinons (2.3.22) and scalar and vector bosons (as excitations of the tetron-antitetron bonds) existed. This era is usually called radiation dominated, cf. (2.1.14).

2.3.9 Then why can quarks and leptons be produced at energies above the Fermi scale?

In collider experiments they can exist at energies much larger than 1 TeV because the alignment of isospins is stable much beyond Λ_F in the fully ordered hyper-crystal, i.e. in our universe. This is due to a collective hysteresis effect in which the crystal stabilizes itself by the concerted action of all aligned tetrahedrons to maintain the isomagnetic ordering. As a result, quarks and leptons can be produced and propagate normally, even in cases where energies locally exceed the critical temperature Λ_F.

2.3.10 Why is $\bar{\psi}$ involved in the order parameter and not $\psi^\dagger$, whereas the total 'iso-magnetization' $\vec{\Sigma}$ eq. (14) is defined just like in ordinary magnetic models?

$\vec{\Sigma}$ (with its factors ψ and $\psi^\dagger$) refers to one single tetrahedron. It is an inner-tetrahedral quantity which only indirectly influences the SSB. In contrast, the SSB is described by the alignment of two neighboring tetrahedrons over physical space in a relativistically covariant way. This means, the order parameter should be built from ψ and $\bar{\psi}$.
Furthermore, one has $\langle\vec{\Sigma}\rangle = 0$ for the ground state fig. 1, so it is no candidate for the order parameter anyhow.

2.3.11 Why not use $\langle\vec{\pi}\rangle$ as an order parameter?

To answer this question consider 2 neighboring tetrahedrons A and B with tetrons ψ_{Ai} and $\bar{\psi}_{Bi}$, and $i = 1, 2, 3, 4$ counting the tetrahedral sites. The aligned tetrahedral 'star' configuration of these 2 tetrahedrons not only implies $\langle\vec{\Sigma}_{A,B}\rangle = 0$ for each tetrahedron separately, but also $\langle\vec{\Pi}\rangle = 0$, where $\vec{\Pi}$ is defined as $\vec{\Pi} = \sum_i \vec{\pi}_i$ and $\vec{\pi}_i = \bar{\psi}_{Bi}\vec{\tau}\psi_{Ai}$. This is because one can show that $\vec{\pi}$ vectors of 2 adjacent tetrahedrons are parallel, iff the corresponding isospin vectors $\vec{Q}$ are.

2.3.12 Is the vev $\langle\bar{U}U + \bar{D}D\rangle$ or $\langle\bar{U}U\rangle$ or what?

The vev is given by $\langle\bar{U}_\star U_\star\rangle$ in accordance with (33) where $U_\star$ is the 'radial' iso-spinor introduced in (10) corresponding to an isospin vector pointing outward as in fig. 1. Such a vev is precisely what is needed to stabilize the alignment of isospins in fig. 2.

2.3.13 So what is the microscopic interpretation of the Higgs particle?

As discussed in (2.1.2) the Higgs is neither fundamental nor a bound state of mignons, but an excitation of tetron-antitetron pairs which are themselves bound within the hyper-crystal.

2.3.14 Should one consider separate tetrahedrons for anti-tetrons, with isospin vectors pointing inward?

This question is justified, because anti-fermions usually react to magnetic forces with an opposite sign. However, using isospin vectors (2) one is treating the problem in a relativistically covariant way. As can be seen in (3), the isospin vectors contain particle as well as well as antiparticle contributions, and the antiparticle contributions have a negative sign.

2.3.15 Is there a difference between the SM SSB and a ferromagnet, apart from the fact that the SM SSB takes place in internal space? Is the symmetry breaking in the tetron model really spontaneous?

Both cases (ferromagnet and tetron structure) are similar in that at high energies / temperatures the directions of (iso)spins are oriented randomly with an associated SU(2) Heisenberg symmetry, and this defines the symmetric state.
In an uni-axial ferromagnet an accidental magnetization axis usually appears spontaneously, based on a thermodynamic potential

$$V_{FM}(\vec{M}) = -a\vec{M}^2 + b\vec{M}^4 \tag{58}$$

where $\vec{M}$ is the total magnetization and the minimum of the potential is at $\langle \vec{M}^2 \rangle = a/2b$.
In the case at hand the crystallization process at scale Λ_r is accompanied by a coordinate alignment of all tetrahedrons, i.e. there is a spontaneous selection of one global internal coordinate system for all tetrahedrons. This coordinate alignment, however, happens much prior to the alignment of isospins and has not much to do with it.
When the temperature decreases towards Λ_F, the anti-ferromagnetic tetrahedral 'star' configurations fig. 1 appear where the isospin vectors within one tetrahedron avoid each other as far as possible. Note there is an infinite SU(2) symmetric set of such 'star' configurations just as in a ferromagnet there is an infinite set of possible magnetized states corresponding to all possible magnetization directions in R^3. The difference as compared to a ferromagnet is that not only the stars over one tetrahedron have to be included but also those over all the other tetrahedrons over Minkowski space, with their independent SU(2) degeneracies, and this makes the problem a local gauge symmetric one.
The SSB consists in the simultanous selection of one among all the possible star configurations over all Minkowski base points – namely the one with $\langle \Phi \rangle \sim (0,1)$. According to (34) this corresponds to a vev for the $U_\star$ and $\bar{U}_\star$ isospinor component, i.e. the one with an isospin vector pointing outward in radial direction and the corresponding isospin vector in the neighboring 'anti'-tetrahedron pointing in the same direction. The choice of $(0,1)$ - and of U - is notational convention and, in

the framework of the gauge theory, corresponds to choosing a certain gauge (the so called unitary gauge). There is again a similarity to the situation in a ferromagnet where some axis is selected by the spontaneous magnetization, and the coordinate system is then 'gauged' in such a way that this axis is called the z-axis $\sim (0, 0, 1)$.
One may ask what role the coordinate alignment of tetrahedrons at the crystallization point $\Lambda_r \sim \Lambda_P$ plays in this game, because it seems plausible that the state, where the tetrahedrons of coordinate and isospin both point in the same radial directions, is energetically preferred. (This geometry is in fact depicted in figures 1 and 2.)
A similar situation is sometimes encountered in ordinary uni-axial ferromagnets in cases when the coordinate backbone of the crystal prefers one specific magnetization direction, so that the ferromagnetic phase transition is not really spontaneous. This effect can be modeled by adding a tiny explicit symmetry breaking contribution to the potential (58) by hand. At high temperatures due to thermal fluctuations this structural / coordinate effect is not important. But it becomes relevant near the Curie temperature where it fixes the magnetization direction.
In the present case, however, this possibility needs no consideration. The reason is that $\Lambda_R \sim \Lambda_P$ is so large as compared to Λ_F, that the granular internal coordinate structure is not noticed by the isospin vectors (nor by any human experiment). From the perspective of the isospin vectors it looks as if they are sitting on an internal coordinate structure which is rotationally invariant. They only feel the anti-ferromagnetic aversion towards their 3 fellows within one tetrahedron.
As a consequence of these considerations all 'star' configurations are energetically equivalent, and the symmetry breaking is spontaneous.
One can even go as far to say that there could be no coordinate alignment among the tetrahedrons at all. This would be in accord with the idea that the inter-tetrahedral coordinate (=gravitational) interactions are elastic and therefore can give rise to any relative orientation between neighboring tetrahedrons. The alignment of isospins could live with this option, because the only thing which matters for the SM SSB is that the 4 isospin vectors point in radial direction and build up the isomagnetic tetrahedral configuration, irrespective of what the coordinates of the tetrons are.
Further it may be noted that the question whether the SB is really spontaneous, is much easier to answer in what was called scenario C in section 1. In that case the

electroweak phase transition III consists in a simultaneous alignment of coordinate and isospin vectors as shown in fig. 2. In other words the tetrahedral star configuration consists in a coordinate and an isospin star where the coordinate and the associated isospin vector always point into the same radial directions. The transition to the ordered state is then necessarily spontaneous because all rotated (coordinate + isospins) tetrahedral star configurations are energetically equivalent.

2.3.16 Is the electroweak phase transition first or second order?

Lattice calculations in the SM with one Higgs doublet give no definite answer to this question. The transition seems to be second order for $m_H \lesssim 120$GeV, while for $m_H \gtrsim 130$GeV one obtains a first order transition[47]. In the intermediate region it may be a cross-over[48]. By contrast, in 2HDM models the situation is clearer; there the electroweak phase transition turns out to be first order[49], i.e. terms of order $\sim \Phi^3 T$ arise in the temperature dependent Higgs potential.
Since the 2HDM model (2.1.17) arises as the low energy approximation of the tetron model, one may be content with this result, in particular because a first order transition is preferable for various reasons, cf. (2.3.21) and [48]. However, it should also be possible to directly determine the nature of the phase transition in the tetron model without recurring to an effective theory. To achieve this aim, a calculation in the isomagnetic framework figs. 1 and 2 should be performed. If one looks at fig. 2, such a magnetic alignment is normally expected to be of second order. However, first order magnetic transitions are also known, in particular in connection with deformable structures[50].

2.3.17 What are the relevant scales in the model?

Naively, there are only 2 scales: the Fermi scale Λ_F and the Planck scale Λ_P.
The crystallization process with coordinate alignment of tetrahedrons but erratic directions of isospin vectors corresponds to scales Λ_r and Λ_R both or order Λ_P, while Λ_F is the scale where the isospin vectors align.
On a more sophisticated level 2 other scales might seem reasonable:
-if one looks at fig. 2 and assumes that in the process of the elastic expansion of

the universe the size R of the internal tetrahedrons does not grow together with r, then $\Lambda_R \gg \Lambda_r = \Lambda_P$ will be a third independent energy scale. It can be associated to the formation of tetrahedral molecules from a tetron gas in 6 dimensions.
-it is conceivable that the crystallization and the coordinate alignment of tetrahedrons do not happen at the same temperature, i.e. there is Λ_P for the crystallization and another scale Λ_A for the coordinate alignment fulfilling $\Lambda_P > \Lambda_A > \Lambda_F$. However, this would imply another phase transition in the early universe for which there is no indication. Therefor as explained in section 1 and (2.3.15) it is best to assume that $\Lambda_A = \Lambda_P$ or $\Lambda_A = \Lambda_F$ or that there is actually no coordinate alignment at all, only isospin alignment.

2.3.18 What is the geometrical meaning of these scales?

According to fig. 2, r and R can be interpreted as spacings in the discrete structure under consideration, or as lengths of certain tetron bonds. The Fermi scale measures the 'length' of isospin vectors of the ground state fig. 1.

2.3.19 Do GUT theories play any role?

No. It is difficult to imagine why there should be other SSBs in the tetron model besides those described in section 1. I see no reason for the proliferated Higgs sector characteristic for most GUT theories.

2.3.20 Is there a unification of electroweak and strong couplings?

No. In the tetron model the QCD forces are on a less fundamental footing than the electroweak interactions. For further details see (2.5.29) and (2.3.25). The unification scale of electromagnetism and weak interactions has been discussed in (2.1.13).

2.3.21 What about domain walls?

Phase transitions in physics are usually associated with the formation of domains. However, domain structures have never been observed in cosmology.

In the tetron model, cosmological domains either appear in the form of different universes, or if they appear they have long disappeared beyond our event horizon. To understand this in detail, one should first realize that one has to distinguish (i) domains arising at the crystallization point (coordinate alignment of tetrahedrons) from (ii) those arising at the electroweak phase transition (tetrahedral alignment of isospins).

(i) In an ordinary crystal, one expects the appearance of domains with different ordering directions arising from concurrent nucleations of crystal germs in different points of space. In principle, this is also true for the R^{6+1} space under consideration. The reason why domains are not present in the case at hand, has to do with the fact that the hyper-crystal grows into and occupies only a quasi 3-dimensional subspace of R^6. Therefore it intersects with other hyper-crystals from concurrent nucleation points, which grow into other 3-dimensional subspaces of R^6, in at most 1 point (because the intersection of 2 almost flat 3-dimensional submanifolds in R^6 in general is just 1 point). This means the result of the other nucleations will be different hyper-crystals, i.e. they correspond to different worlds whose intersection with our universe consists of at most one point, showing up as a point defect within our hyper-crystal structure, cf. (2.5.33).

(ii) In an ordinary ferromagnet, one expects the appearance of 'Weiss domains' with different ordering directions of spin vectors. In the present case such domains can in principle exist and would differ by a global rotation of the isomagnetic tetrahedral 'star' configuration fig. 1. However, as discussed in (2.3.16), the phase transition is first order, i.e. associated with a sudden release of latent heat, which in the interpretation of the universe as a DMESC blows up the micro-elastic continuum, i.e. triggers an inflationary process which in turn shifts domain walls outside the visible part of the cosmos.[6]

[6]Models with inflation near the electroweak scale have been discussed extensively in the literature[66, 67, 68, 69].

2.3.22 Are there excitations besides the known quarks, leptons and scalar and vector bosons?

Yes. An incomplete listing includes:
–phinons. They are the analogs of phonons in a solid and have been described under more general circumstances in [3]. In the case at hand there are 12 phinon states, that can be arranged according to representations of the permutation group S_4. Just as mignons they travel as quasi-particles through the hyper-crystal. Phinon masses are expected to be much larger than mignon (quark/lepton) masses. While the mignon spectrum is lying at and below Λ_F, the phinon spectrum is concentrated towards the crystallization energy Λ_P. Note, this is not a very accurate characterization, in view of the fact that neutrino masses are so tiny with respect to the Fermi scale. Note further, phinons are internal coordinate vibrations, and thus have to be distinguished from gravitational waves. An interesting question is whether mignon-phinon scattering is possible.
–isospin density waves: they are to be distinguished from phinons and from mignons. The vibrators in this case are similar to the isospin vectors (4), however without the factor of $\vec{\tau}$, i.e. given by $\psi^\dagger(1 \pm \gamma_5)\psi$. As far as I can judge these excitations correspond to a fourth family of fermions, i.e. a lepton-like and a quark-like isospin doublet, probably higher in mass, because they are not related to the other families by the Z_3 family quantum number inherent in the Shubnikov group $A_4 + S(S_4 - A_4)$. In particular, the fourth 'neutrino' is expected to be much heavier than the known neutrinos, because its mass is not suppressed by internal angular momentum conservation, cf. (2.4.10).
–one should mention scalar fields other than the Higgs bosons. They are the components of the second Higgs doublet (42) and the most promising candidates for dark matter, cf. (2.5.31). In contrast to mignons, phinons and isospin density waves they are pairing involving 2 tetrahedrons (see the explanations in section 2.1).

2.3.23 What about vibrations of $\bar{\psi}\gamma_\mu\vec{\tau}\psi$, $\bar{\psi}\sigma_{\mu\nu}\vec{\tau}\psi$ etc?

These are other examples of higher mass excitations of the hyper-crystal.

2.3.24 Could quark and leptons be phinons?

or in other words: what is the advantage of using mignons with Shubnikov symmetry $A_4 + S(S_4 - A_4)$ over phinon excitations with symmetry group $A_4 \times Z_2$ as advocated in [3]?
The answer is that many of the attractive features of mignons are absent, like the explanation of the Higgs mechanism, of why $m_t \gg m_b$, of tiny neutrino masses etc.

2.3.25 *Is it possible to understand the dynamics of the strong interactions from within the tetron approach?*

I don't have a final answer to this question. From its very construction the tetron model is concerned mainly with the symmetries and interactions of electroweak physics. The colors of quarks arise merely as a byproduct, because they are interpreted as the 3 d.o.f. of the Shubnikov triplet representation. It is therefore clear that QCD with a color gauge group and SU(3) color triplets does not directly arise in the tetron model.
In section 1 it was argued that the phase transitions of a 6+1 dimensional spacetime filled with a condensing tetron gas supplies all relevant physics for the early universe and it may even account to understand the forces of gravity. As for the latter the suggestion is that it may arise from elastic forces between tetrahedrons which are the remnants of the fundamental tetron-tetron coordinate interactions and induce curvature and/or torsion effects on Minkowski space.
One would like to interpret the strong interactions in a similar spirit, namely starting with the paradigm that there are no other interactions in the universe besides the ones among tetrons. The strong interaction structure does not correspond to any part of the $SU_2 \times U_1$ bundle connection, because this allows only for the 4 electroweak gauge bosons.
One may note, however, that a Shubnikov invariant mignon-mignon interaction is to be expected among the triplets, transforming as

$$T \times T = A + A' + A'' + 2T \tag{59}$$

As shown in [5] this structure can be embedded into a $SU_c(3)$ algebra where the color indices correspond to the 3 d.o.f. of the triplet representations in (8). Unfortunately, it is not clear whether gluons and QCD gauge interactions can really arise from this line of reasoning.

2.3.26 Is color a part of isospin?

This questions is justified in view of the fact that the tetron color triplet is the Shubnikov group (5) which arises from transformations in the same internal space as isospin. States of a triplet can be transformed into each other by a discrete rotation which take place in the 3-dimensional internal isospin space; and this could lead one to the conclusion that color in some sense is part of isospin. However, it must be noted that these are completely different symmetries. While the Shubnikov group is unbroken to the lowest energies, isospin symmetry corresponds to the free rotations of isovectors on each tettron site and is completely broken in the ordered state.

2.3.27 Is there a connection between the QCD vacuum and the electroweak condensate?

Not really. Chiral symmetry breaking is the appearance of a non-vanishing quark condensate $\langle \bar{u}u + \bar{d}d \rangle \approx -(0.1GeV)^3$ which breaks $SU(2)_L \times SU(2)_R$ to the diagonal isospin $SU(2)_V$ group. The main difference as compared to the electroweak case is that all these groups are global, not local symmetries. Furthermore, the value of the quark condensate is much smaller than that of the Higgs condensate. While the former extends over the strong interaction scale $\sim$ 1 GeV, the latter is related to the Fermi scale.
According to (33) the Higgs condensate is related to $U_\star$, i.e. the tetrahedral star configuration pointing outside as in fig. 1. In contrast, the quark condensate is a singlet w.r.t. the relevant quantum number (in this case color, while for the Higgs condensate it is isospin). In other words

$$\langle \bar{q}q \rangle = \langle \bar{q}_1 q_1 + \bar{q}_2 q_2 + \bar{q}_3 q_3 \rangle \tag{60}$$

does not correspond to a preferred direction or orientation in color space, but has the property of a color-internal density.

2.4 Questions about the quark and lepton mass spectrum and the CKM and PMNS mixing matrices

Quark and lepton masses can be calculated as excitation frequencies of mignons – a rather straightforward procedure where the results are obtained quite naturally by considering isomagnetic interactions among the tetrons of one or two tetrahedrons only. Analytic expressions for masses[2] can be given in terms of internal Heisenberg and Dzyaloshinskii-Moriya exchange couplings.

2.4.1 Using exchange couplings instead of Yukawas – isn't one replacing one set of unknown parameters by another set and one effective theory (the Standard Model) by another one (the internal Heisenberg model)?

No, because the internal couplings can in principle be calculated from first principles as exchange integrals over internal space, just as in ordinary magnetism the exchange couplings of the Heisenberg model are in principle calculable from exchange integrals of electronic wave functions over physical space. What one needs to know is the underlying 6+1 dimensional dynamics of tetron interactions.

2.4.2 Why are mignons spin-$\frac{1}{2}$ particles?

After all, they are constructed from excitations of 'bosonic' isospin operators (2), and this could lead one to believe that they are bosons, just like magnons in ordinary ferromagnets are bosonic quasi-particles.
However, it is important to distinguish the behavior in internal space from that in Minkowski space. While mignons transform as Shubnikov singlets A and triplets T (i.e. *not* as projective representations) w.r.t. internal space, it is not hard to see that they are Dirac fermions w.r.t. Minkowski space.
The point to note is that mignons are not bound states of tetrons but eigenmodes of their excitations. As such they are not tensor products but linear combinations of (fluctuations of) tetron fields ψ_a^α (where $a = 1, 2$ is the internal and $\alpha = 0, 1, 2, 3$ the Dirac index of the tetron). Since each mignon is a vibration of *one* isospin eigenmode, one concludes that it must be a Dirac particle w.r.t. the Minkowski

base space. Using the 'bosonic' isospin vectors is merely a tool to separate the proper isospin triplet vibrations of $\vec{Q} = \psi^\dagger \vec{\tau} \psi$ from singlet density vibrations $\psi^\dagger \psi$, cf. (2.3.22). Looked at from 'below', i.e. from the Minkowski base space, the tetron excitations are Dirac fermions. If such an excitation travels through the crystal as a quasi-particle, it can be either L or R, particle or anti-particle.

2.4.3 Why are there exactly the quark and lepton states of the 3 generations?

The unbrocken (Shubnikov) symmetry group has only singlet and triplet representations, and the 8 independent isospin vectors on a tetrahedron lead exactly to the flavor spectrum of 24 quarks and leptons as given in (8).

2.4.4 Are there other ground states than the tetrahedral one which yield the appropriate quark and lepton spectrum (8)?

No. I scanned other geometries with 8 iso-magnetic vibrators and found that for most systems mignons appear in 2-dimensional representations[9, 10], not useful for the q/l spectrum of particle physics.

2.4.5 Should one really use covariant isospin vectors (3) containing both particle and anti-particle contributions as vibrators?

Yes, one should. It is important for the vanishing electric charge of the hyper-crystal, for the formation of gauge bosons out of tetron-antitetron pairs, and in general to maintain relativistic covariance. Technically it is important for the mass calculations corresponding to mignon mass terms $\langle 0|T(\bar{q}q)|0\rangle$.

2.4.6 Why not use only $\vec{Q} = \vec{Q}_L + \vec{Q}_R$ as vibrators instead of $\vec{Q}_L$ and $\vec{Q}_R$ separately?

In order to obtain the 3 families of quarks and leptons one would have to consider systems with 8 instead of 4 tetrons. This possibility has been discussed in (2.3.1).

The main counterargument is that the $SU(2)_L$ vibrators $\vec{Q}_L$ are needed to inherit the nature of the SSB and the top mass in the model, cf. (13).

2.4.7 Is it okay to start the mass calculations using chiral $SU(2)_L \times SU(2)_R$ symmetry quantities $\vec{S} = \vec{Q}_L$ and $\vec{T} = \vec{Q}_R$?

Yes. This is just the way the SM works. Non-zero fermion masses are developed via SSB starting from a massless, i.e. $SU(2)_L \times SU(2)_R$ chirally symmetric theory.

2.4.8 Can the simple Heisenberg interaction (12) really explain the full q/l mass spectrum with its extreme hierarchies?

No. As shown in [2] the masses of some of the fermions get contributions from other physical sources, namely
–the top mass is dominated by a contribution of order Λ_F which stems from the symmetry breaking *inter*-tetrahedral interactions (13). Physically it arises because the top quark corresponds to the 3 eigenmodes which 'disturb' the global ground state in the strongest possible way. This disturbance is also responsible for the hierarchy observed in the CKM matrix elements.
–only strange-, charm- and muon-mass are dominated by anti-ferromagnetic exchange couplings within one tetrahedron, and thus can be obtained from the *inner*-tetrahedral Heisenberg exchange couplings alone.
–down-quark, up-quark and electron are left massless by the Heisenberg and DM interactions (12) and (13). They get their relatively small masses from energetically favored torsion contributions[2], i.e. vibrations of the internal spin vectors of the generic form $d\vec{Q}/dt \sim \vec{Q}$.
–neutrino masses are protected by internal angular momentum conservation, i.e. by the internal rotational symmetry, cf. (2.4.10).

2.4.9 Is there a mismatch between the internal Heisenberg interaction (12) for a single tetrahedron and the DM interaction (13) involving neighboring tetrahedrons?

This is a technical question concerning the calculation of quark and lepton masses presented in [2], and the answer is that technically one can treat the isospin vectors from the neighbors as if they were vectors of the original tetrahedron.
Starting with 2 neighboring tetrahedrons and interactions of the form $\sim \vec{S}_i \vec{S}'_j$ one would expect the doubling of the number of eigenmodes. However, due to the symmetry between the 2 tetrahedons, i.e. between the $\vec{S}$ and $\vec{S}'$, the modes arrange in pairs of identical energy. In other words, the doubling of modes is a trivial one.
To understand this in more detail consider the first term (Heisenberg contribution) in the inter-tetrahedral interaction (13) and assume that the inner-tetrahedral distances are much smaller than the inter-tetrahedral ones. This is in accord with arguments given above that after the long time of cosmic expansion one expects $r \gg R$ in fig. 2. This assumption implies that the couplings J_{SSB} of an $\vec{S}_i$ to all isospin vectors in the neighboring tetrahedron are identical (as anticipated in (13)) and one obtains

$$\frac{d\vec{S}_i}{dt} = J_{SSB} \quad \vec{S}_i^0 \times \sum_{j=1}^{4} \vec{S}'_j \tag{61}$$

where the superscript 0 denotes ground state values. A second set of equation is obtained for $d\vec{S}'_i/dt$ by exchanging the role of the 2 tetrahedrons, i.e. the primed and unprimed quantities. An obvious set of solutions to these equations fulfills $\vec{S}_i = \vec{S}'_i$, i.e. the vibrations are completely in step, and one obtains the trivial doubling of modes mentioned above.
As for the DM part (second term) in (13) a similar argument can be given. Following the results in [2] this leads the conclusion that the top quark receives the overwhelming mass contribution from the inter-tetrahedral interaction (13).
It is interesting to note that in a cosmological situation where one would have $r \ll R$ the mignon mass spectrum would turn out quite different because in that case it is natural to assume that the coupling of $\vec{S}_i$ to $\vec{S}'_i$ is much larger than to the other $\vec{S}'_j$. Therefore $d\vec{S}_i/dt \sim \vec{S}_i\vec{S}'_i$ corresponding to a much different mass spectrum of mignons.

2.4.10 How can the smallness of neutrino masses be understood?

Neutrinos are interpreted as internal Goldstone modes of the breaking of internal SO(3) by the formation of the discrete structure fig. 1. The associated conserved Noether charge is given by the total internal angular momentum $\vec{\Sigma}$ defined in (14) which implies the existence of 3 zero-frequency modes. All this is analogous to how magnons are interpreted as Goldstone modes in ordinary magnetism, except that here one is considering physical processes in the internal spaces.
While in ordinary ferromagnets after magnetization a U(1) symmetry about the z-axis survives, in the given frustrated configuration fig. 1 all three SO(3) generators give rise to Goldstone bosons, to be identified as the internal magnons corresponding to the 3 neutrino species.

2.4.11 Neutrinos are fermions. How can they be Goldstone modes?

One has to distinguish the dynamics in internal from that in physical space. In physical space the neutrinos are fermions, but neutrinos are Goldstone bosons w.r.t. the dynamics in internal space, because in internal space they are described by (bosonic) excitations of the total internal angular momentum $\vec{\Sigma}$ defined in (14) which is the conserved quantity associated with the internal rotational symmetry.
As discussed before, the representations A and T in (8) are ordinary representations of the Shubnikov group, not projective representations, so all quarks and leptons are 'bosonic' w.r.t. the internal dynamics, cf. (2.4.2).

2.4.12 How do neutrinos obtain their tiny non-zero masses?

As Goldstone modes neutrinos are strongly protected to getting masses. However, as proven in [2] the observed non-zero neutrino masses can be generated on the phenomenological level by tiny torsional interactions which violate (15). These can also be used to accommodate appropriate PMNS mixing values. Physically the existence of such interactions is a signal for the activity in isospin space of small anisotropic forces. Possible cosmological applications of these anisotropies are discussed in (2.5.21).

2.4.13 Are neutrinos Dirac or Majorana particles?

They are Dirac fermions. As all other quark and lepton flavors they inherit this property from the tetrons.

2.4.14 How is isospin realized on the mignon level?

Calculating the masses of quarks and leptons from vibrations $\delta\vec{S}$ and $\delta\vec{T}$ of the ground state fig. 1, the isospin of mignons essentially corresponds to an exchange of the roles of $\vec{S}$ and $\vec{T}$, i.e. of the $SU(2)_L$ and the $SU(2)_R$ sector. This is because a mass term is the form $\bar{q}_R q_L + c.c.$, and a charge conjugation transition (instead of parity) may be used to accomplish this exchange. Such a transformation is necessarily accompanied by an exchange of isospins U and D via[78]

$$(U, D) \to (-D^*, U^*) \tag{62}$$

The Dirac structure has not been made explicit in (62), so that one can see directly that this formula agrees with the behavior under internal time reversal (6), arising in connection with the Shubnikov transformations in sections 1. One concludes that the Shubnikov group G_4 can be defined by using charge conjugation and without introducing the concept of an internal time, cf. (2.5.17).

To be more explicit, the transition between 2 isospin partners can be obtained by $\vec{S}_i \to -\vec{T}_i$ and $\vec{T}_i \to \vec{S}_i$ on the tetrahedral sites i. In the actual calculation of mignon eigenstates, it turns out that the top-quark is predominantly an $\vec{S} = \vec{Q}_L$ excitation while the b-quark is $\vec{T} = \vec{Q}_R$. On the other hand the neutrinos are given by vibrations of the conserved quantity $\sum_i(\vec{S}_i + \vec{T}_i)$, while charged leptons are essentially given by vibrations of the $\sum_i(\vec{S}_i - \vec{T}_i)$ combination.

The connection between the attributes 'left-handed' and 'up-type' (and similarly 'right-handed' and 'down-type') is of fundamental importance in the tetron model. It relies on the chiral structure of the internal ground state fig. 1 and the octonion induced form of the tetron interactions (2.1.21). Furthermore, it is at the heart of the tetron model explanation of weak parity violation (2.1.21) and of the large value of the top quark mass from (13) as compared to the other q/l masses, cf. (2.4.16).

It is interesting to note that according to (6) the internal reflection operators which exchange the elements of A_4 and $S_4 - A_4$ comprise isospin transformations, charge

conjugation as well as transitions between left and right.
Note further that for the actual construction of quarks and leptons as mignons one has to use radial spinors (10) in (62).

2.4.15 Do pairs of ordinary and true Shubnikov representations form isospin doublets?

No. A true Shubnikov representation is a representation of $A_4 + S(S_4 - A_4)$ which is not a representation of S_4. True Shubnikov representations are labeled by an index s in the following. Analyzing the representations appearing in (8) one finds that A and A_s, T and T_s arise in the combination

$$\begin{aligned} A(\nu_e) + A(\nu_\mu) + A_s(\nu_\tau) &+ T(u) + T(c) + T_s(t) + \\ A(e) + A(\mu) + A_s(\tau) &+ T(d) + T(s) + T_s(b) \end{aligned} \tag{63}$$

According to this result an isospin doublet is not given by a pair (A, A_s) or (T, T_s) of an ordinary representation and a Shubnikov representation. Instead, the rows A, A, A_s and T, T, T_s correspond to particles of the 1., 2. and 3. family.

2.4.16 Why is top so heavy, why not bottom?

The top quark corresponds to the vibrations of $\vec{\Sigma}_L = \sum_i \vec{Q}_{Li}$. This vector plays a special role due to the nature of the SSB. Basically this is so because of the relation between internal and external parity[1] and because nature has chosen to break internal parity, i.e. prefers the state fig. 1 with all internal spins pointing outwards over the one with isospins pointing inwards. The top-mignon is the rotational vibration where all isospin variations act against the SSB alignment in the strongest possible way.

2.4.17 Why is m_b much smaller than m_t and from where does it get its predominant contributions?

In contrast to the top quark, the b-quark is mostly an excitation of the $\vec{Q}_{Ri}$, and therefore does not get a contribution from the Dzyaloshinskii-Moriya interaction.

However, there are non-leading inter-tetrahedral SSB interactions in addition to (13), which provide contributions to m_b. Those terms typically involve right-handed isospin vectors and are correlated to the existence of the second Higgs doublet (42).

2.4.18 If up type quarks arise mainly from vibrations of $\vec{Q}_L$, how can their right-handed version be produced?

As shown in [2], lepton states originate dominantly from vibrations of the form $\vec{S}\pm\vec{T}$, while up and down quark states are related to vibrations of $\vec{S}$ and $\vec{T}$, respectively. Therefore one might suspect that the helicities of quarks and leptons generated in this way are restricted, too. However, it must be noted that an excitation δ of a left-handed isospin vector can in principle vibrate into any chiral direction. The same is true for right handed vectors $\vec{Q}_R$.

2.5 Questions about the global crystal structure and cosmology

This section relies on the interpretation of the universe as an elastic system of internal tetrahedrons as described in section 1. As evident from fig. 2, there is an internal discrete tetrahedral structure at smallest distances which is able to build up the global (elastic) structure of the universe by exact repetition. The following list of questions and answers shed light on the nature of this repetition and furthermore shows how the tetron model may be incorporated within a larger, cosmological framework.

Throughout it will be assumed that after the crystallization the spacetime metric shows a Friedmann / Robertson-Walker (FLRW) behavior

$$ds^2 = dt^2 - a^2(t)[\frac{(d\vec{x})^2}{1-k\vec{x}^2} + \vec{x}^2 d\Omega^2] \tag{64}$$

where a(t) is the scale factor and k=0,+1 or -1 for a euclidean, spherical or hyperbolic universe. Rewriting this as

$$ds^2 = dt^2 - a^2(t)\gamma_{ij}dy^i dy^j \tag{65}$$

one has

$$\gamma_{ij} = \delta_{ij} + k\frac{y_i y_j}{1-ky_n y^n} \tag{66}$$

with $i, j = 1, 2, 3$. The y^i are the 'comoving' coordinates, from which the physical coordinates can be obtained as $x^i = a(t)y^i$. Accordingly, the physical velocity of an object can be decomposed as

$$v^i = \frac{dx^i}{dt} = a(t)\frac{dy^i}{dt} + \frac{da}{dt}y^i = w^i + Hx^i \tag{67}$$

where the second term introducing the Hubble parameter is called the 'Hubble flow' and the first term w^i defines the 'peculiar velocity' of an object, i.e. its velocity with respect to the Hubble flow.
Note that the FLRW metric fulfills $g_{00} = 1$ and $g_{0i} = 0$, i.e. it corresponds to a metric in a 'synchronous gauge'. Such a metric has a particular simple interpretation in the tetron model, because the only non-trivial elements are the spatial g_{ij}. These can be identified with modifications of distances in the 3-dimensional elastic system of tetrahedrons described in section 1.

2.5.1 Is the ground state figures 1 and 2 stable or metastable?

In other words, are there isomagnetic states X with a lower energy? If yes, this might threaten our world when such a state would be produced in high energy collisions. More precisely, what could happen is that at collider energies of order Λ_F the ordered state fig. 2 is locally destroyed and in the process of re-ordering of isospin vectors a germ of the state X appears. Energy would then be released, which would destroy the ordering in the neighborhood of the original collision, so that a chain reaction would start at the end of which the metastable ground state would be completely replaced by the stable one.
In the microscopic model it is possible to compare the energies of all isomagnetic configurations with 4 isovectors to the energy of fig. 1. Essentially, one deals with a system of four $\vec{Q}_i$ interacting via an 'anti-ferromagnetic' coupling $J_A > 0$. The local energy of any state X is given by

$$E(X) = J_A\,[\,\vec{Q}_1\vec{Q}_2 + \vec{Q}_1\vec{Q}_3 + \vec{Q}_1\vec{Q}_4 + \vec{Q}_2\vec{Q}_3 + \vec{Q}_2\vec{Q}_4 + \vec{Q}_3\vec{Q}_4\,] \tag{68}$$

With this input one may run over all possible configurations of isospin vectors. As a result, one finds 2 minima with exactly the same energy. One minimum (m1) corresponds to fig. 1 while the other (called m2) is characterized by the *4 isospins* *arranged into 2 pairs of opposite orientation.*

This conclusion remains unchanged, if one includes a left- and a right-handed isospin vector on each tetrahedral site as in (12). However, it may get changed, if the *inter*-tetrahedral energies are different. In general, one expects identical inter-tetrahedral energies, because in both cases (m1) and (m2) the same number of isospin pairs are aligned in a ferromagnetic way, with identical exchange couplings J_F. However, in reality there may be small differences between the J_F for the (m1) and the (m2) configuration due to the different geometries of the two ground states.

2.5.2 Is physical space discrete, i.e. is there a granular structure of physical space in addition to the discrete tetrahedral structure in internal space?

Most probably yes. Although the discrete structure of physical space is not compelling and the distance r between two tetrahedrons could be identically zero, the discussions in section 1 suggest that r has a tiny non-vanishing value of the order of the Planck length. Note, due to the elasticity of the system only an average $\langle r\rangle$ can be given, in accord with (22).

2.5.3 How can such a granular structure be compatible with Lorentz invariance? How can the elastic continuum of tetrahedrons be Lorentz invariant?

The point to note is that in the tetron model all physical objects that we know are superpositions of excitations which travel as quasi-particles through the hyper-crystal. This holds true even for photons and also for ourselves as well as for all experiments and 'reference frames' we can preparate. In a quantum mechanical framework such excitations always have a wave nature and are to be described by (generalizations of) the d'Alembert wave equation. This fact alone fixes the system of waves, which constitutes our physical environment, to be Lorentzian, because the d'Alembert operator

$$\Box = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \qquad (69)$$

leaves the squared 4-momentum p^2 invariant. In particular, if one wave packet is emitted from another one, their velocities add up according to the rules of Lorentz transformations.
For massive particles which move at velocities $< c$ the operator is modified to $\Box - m^2c^2/\hbar^2$ corresponding to a 'dispersion relation' $p^2 = m^2c^2/\hbar^2$. It is important for this argument (and in general to retain the Lorentz structure) that in all those wave equations there is a universal maximum speed c that fixes the relation between space and time. This is defined to be the speed of the massless excitations and according to (24) is given by the ratio of 'lattice constant' L_P and 'hopping time' T_P.
The value of c is universal for all SM particles because all of them arise from the same isomagnetic interactions introduced in section 1. This is discussed in more detail in (2.5.13), and the question of (metrical) velocities larger than c will be discussed in (2.5.38). The question why gravitational waves propagate at c will be answered in (2.5.37).
It should be stressed, that in particle physics interactions usually only the excitations move. The tetrahedrons stay fixed at their location in the hyper-crystal. They only move in connection with metrical changes induced by gravity, e.g. when the universe expands after the crystallization or on a much tinier level in any kind of gravitational interaction, cf. the discussion after (20) and in (2.5.38).

2.5.4 Is time discrete?

Time is a complicated construct induced by the superposition of many elementary thermodynamics processes. If one likes one may interpret it to have a granularity of order T_P, as defined in (23), because this is the minimum time it takes one of the quasi-particle excitations to jump from one tetrahedron to the other. In our world of quasi-particles the duration of any observable physical process can never fall below this value.

2.5.5 Is quantum mechanics just a material property of the hyper crystal?

My claim is yes. The proof will be given in the following question (2.5.6). It relies on the fact that ordinary matter (including the photon) consists of internal excitations traveling as quasi-particles on a discrete structure with Planck length lattice spacing.

2.5.6 Why is the Planck length the natural lattice spacing for the hyper-crystal?

The short answer: the Planck length L_P arises as a lower limit on Δx in the 'generalized' Heisenberg uncertainty relation (71), which includes the effects of gravity. On the other hand in the tetron model *all* known particles including the photon are interpreted as excitations with an extension of at least one lattice spacing r, i.e. they are quasi-particle waves with wavelength $\lambda > L_P$. Since every physical experiment necessarily makes use solely of these quasi-particles, its resolution cannot be better than r. This strongly suggests $r \approx L_P$. Due to the extremely small lattice spacings, spacetime as we perceive it effectively looks like a continuum. This guarantees local rotational symmetry. The item of invariance under Lorentz boosts is discussed in (2.5.3).

Extended answer: in ordinary quantum mechanics there is a fixed dimensionful quantity h which relates the canonical Fourier variables of frequency to energy and inverse distance to momentum, i.e. it transforms the spacetime quantities x and t into physically 'active' quantities

$$\vec{p} = \hbar\vec{k} \qquad\qquad E = \hbar\omega \tag{70}$$

These relations are the reason why h appears in the uncertainty principle $\Delta x \Delta p > \hbar$ which otherwise would just be the Cauchy-Schwarz inequality known from the Fourier analysis of waves, i.e. $\Delta x \Delta k > 1$ with no dimension on the r.h.s. I consider them as further indication that h is a material property of the hyper-crystal not valid outside of it, cf. the discussion at the end of section 1.

The 'generalized' uncertainty relation includes gravitational effects of the photon on a test particle[55]. These occur because the general relativistic effect from the photon with 'mass' $h\nu/c^2$ adds to the uncertainty about the test particle. The

Heisenberg relation is then modified to

$$\Delta x = \frac{\hbar}{\Delta p} + L_P^2 \frac{\Delta p}{\hbar} = \lambda[1 + (\frac{L_P}{\lambda})^2] \tag{71}$$

where λ is the photon wavelength to be identified with the limit of resolution.

Eq. (71) can be derived e.g. by extending the 'Heisenberg microscope' thought experiment (which imagines a photon to measure x and p of a probe particle) to include gravitational effects of the photon[55]. It implies that there is a minimum value of

$$\Delta x \sim L_P \tag{72}$$

corresponding to a photon with wavelength L_P. Usually, this is interpreted in such a way that at distances/wavelengths smaller than L_P all matter dissolves into quantum fluctuations and the laws of physics do not have a meaning any more.

In the microscopic model the interpretation is a little different. The limitation on distances/wavelengths holds true only inside the hyper-crystal and is restricted to the world of quasi-particles, not of tetrons. For example, since the photon is such a quasi-particle (a correlation among 2 tetrahedrons traveling through space), the minimal wave length of photons, that can be produced and used in experiments, roughly corresponds to the lattice spacing (22), and it is this minimal wavelength which restricts the precision of any experiment.

It is worth mentioning that on the hyper-crystal the value of h is given by (25) where the Planck energy Λ_P corresponds to the binding energy of the tetrahedrons and T_P is the hopping time needed to absorb and re-emit the photon.

At first sight the linear momentum dependence of the gravity contribution in (71) does not fit exactly into this picture. However, it can be interpreted in such a way that the gravitational effect of the photon modifies the average spacing $\langle r \rangle = L_P$ between the tetrahedrons involved in the interaction and thus the resolution $\Delta p = \lambda$ of the photon. The modification factor can be derived from the metric (20) by inserting the photon's gravitational mass $M = h\nu/c^2 = h/c\lambda$ and amounts to

$$1 - \frac{\phi}{c^2} = 1 + \frac{Gh}{|\vec{x}|\lambda c^3} \tag{73}$$

Since the relevant regime of discussion is distances $|\vec{x}| = \lambda$, the factor (73) precisely agrees with the factor in (71).

These considerations may be compared to the Fourier wave analysis which has led to the dispersion relation (28) and which also provides a modified uncertainty relation

$$\Delta x \Delta k = |\langle [x, k] \rangle| = |\langle \cos k L_P \rangle| \tag{74}$$

Eq. (74) may be evaluated in the limit of long wavelengths to give the usual Heisenberg uncertainty relation, i.e. a meaningful result. On the other hand, for wave vectors near the border of the first Brillouin zone, the r.h.s. of (74) vanishes. Thus it would seem that lattice quantum mechanics at Planck scale energies can exhibit classical, non-quantum mechanical behavior. However, in the tetron model, for wavelengths $\lambda \sim L_P$ the excitation nature of the photon spoils the prerequisite of a normal wave with arbitrary small wave vector on which (28) and (74) rely. Apart from that, (74) is a result for a rigid lattice system and not really applicable to the elastic hyper-crystal under consideration.

2.5.7 How is physical space defined within the 6+1 dimensional world? How is it distinguished from the internal dimensions?

The aligned tetrahedrons define a 3-dimensional subspace of R^6. Everything orthogonal gives physical space.

2.5.8 What is the exact lattice structure of the hyper-crystal?

This question is obsolete because of the elastic nature of inter-tetrahedral coordinate bonds. It is an irregular lattice of tetrahedrons in the base 3+1 dimensional spacetime without a discrete symmetry.

2.5.9 Then why is this structure called a 'crystal'?

because there is a rigid tetrahedral structure in the *internal* directions. Concerning physical space it is a disordered system which resembles a plastics or a fluid.

2.5.10 *Why is there no growth of the crystal into the internal directions?*

I have no complete answer to this question. It must have to do with the form of the fundamental tetron interaction and its preference to form the spiky tetrahedral 'star' system fig. 1, which do not allow tetrahedrons to be stacked on top of each other in fig. 2. It is the main reason why internal dimensions need not be compactified, cf. (2.5.27).

The growth and expansion of the elastic hyper-crystal along a lower-dimensional structure is reminiscent of the behavior of superfluids, who can creep along arbitrary surfaces, and leads one to suspect that it shares some properties with them. In low energy physics there are 2 types of superfluids whose typical representatives are He-3 and He-4. While in He-3 there are fermion condensates that populate the macroscopic quantum states, in He-4 superfluidity is a consequence of a normal Bose-Einstein condensation of the He-4 bosons. There are actually speculations that associate gravity to a kind of superfluid[81] similar to He-3. However, the situation in the tetron model resembles more that of He-4, because it is the tetrahedrons and not the tetrons themselves which induce the gravitational interactions. The tetrahedrons are bosons, so that an ordinary Bose-Einstein approach could be useful to describe the growth and expansion of the hyper-crystal.

2.5.11 *Where do the 6 spatial dimensions come from?*

I have only a partial answer to this question. A tetron spinor $\psi = (U, D)$ transforming as 8 under SO(6,1) can be interpreted as an octonion field living in 6+1 dimensions. Octonions form a rather unique mathematical structure[41, 42, 43]. They are the next thing to use when complex numbers (used for amplitudes in quantum mechanics) and quaternions (used for rotations and spinors in physical space) are not ample enough to describe physical phenomena. The octonion nature of tetrons has been used in (2.1.21) to determine the form of the isomagnetic tetron interactions. The splitting of R^6 into an internal space and physical space corresponds to a splitting of an octonion into two quaternions.

2.5.12 How does the value for c come about? Why is it finite?

In the tetron model light is interpreted as an excitation of tetron-antitetron pairs. These pairs are bound in tetrahedrons inside the hyper-crystal. However, like other quasi-particle phenomena, light can propagate through physical space, and its speed has more to do with spacetime properties than with the internal interactions by which it is produced.
Photons are constantly moving in the hyper-crystal. This means, in contrast to massive excitations they cannot cling to one fixed internal tetrahedron. Nevertheless, the speed of light is finite, because even a massless excitation needs a certain 'hopping time' T_P to jump from one tetrahedron to another. Since the tetrahedrons are distributed over the hyper-crystal with average distance L_P, this corresponds to $c = L_P/T_P$ in agreement with (23).

2.5.13 Why is c the universal limiting speed for all particles?

In the tetron model all SM particles are internal excitations whose interactions have the same type of isomagnetic interactions as a universal origin. Therefore it is no surprise that a common maximum speed exists, to be identified with the ratio L_P/T_P, as explained in (2.5.12).
For the interpretation of (metrical) velocities larger than c see (2.5.38).

2.5.14 Is there a rest system of the hyper-crystal?

At first sight the existence of such a system seems to contradict Einstein's principle of equivalence and special relativity - well established concepts which I do not want to question. Still I think the answer to the question is affirmative.
The point is that in the tetron model all material objects and all normal matter and gauge bosons including the photons from their very nature are quasi-particle waves, i.e. entities fulfilling Lorentz covariant Klein-Gordon equations, as discussed in (2.5.3). As such they cannot distinguish an absolute rest system, i.e. they naturally fulfill Einstein's principle of equivalence. By contrast, the original tetron matter, which forms the fixed hyper-crystal ground state, is merely the carrier of those quasi-particles, and according to (2.5.37) lives so to say in a different world than ordinary

matter. This is close to the Fitzgerald interpretation of special relativity[90] and allows for a rest system of the hyper-crystal. More details will be given in the next answers.

2.5.15 Why has a rest system never been observed in Michelson Morley type of experiments?

The essence of the answer to this question has already been given in the last sentence of (2.5.14). An extended version can be found in [90]. In that interpretation of special relativity a Galilean ground state like fig. 2 is not a contradiction to special relativity but rather supplements it. It supplies the 6-dimensional framework which leads to the observed spectrum of physical particles.

2.5.16 Can the hyper-crystal's rest system ever be identified?

It follows from (2.5.14) that it is difficult to experimentally perceive the ground state fig. 2 which forms the rest system of the hyper-crystal - the reason being that our reality (ourselves, our detectors as well as all test particles) consists of quasi particle excitations and not of tetrons themselves.
The puzzle cannot be resolved on the level of excitations. Even gravitational waves propagate at the speed of light, cf. (2.5.37). However, it is well known that the metrical expansion of the universe (and metrical changes in general) can proceed at superluminal velocities, cf. (2.5.24) and (2.5.37).
In section 1 it was shown that metrical changes correspond to displacements of the internal tetrahedrons within the elastic hyper-crystal, cf. the discussion after (17). Applied to the standard model of cosmic expansion, this leads to the idea that *the hyper-crystal's rest system is given essentially by the spatial coordinates of the FLRW metric* (64). Of course one must be aware that due to the expansion factor a(t) this is a 'rest system' permanently changing with (cosmic) time, but at least momentarily it can be considered to be at rest. The time dependence of a(t) defines the expansion of the elastic hyper-crystal, and what comes nearest to the notion of 'matter at rest' are the galaxies on the Hubble flow. The peculiar velocity (67) of stars and galaxies gives their 'true' nontrivial motion with respect to the

hyper-crystal's rest system.
The underlying idea is that to a good approximation mignon-matter forming the galaxies was originally produced 'at rest' (at least on the average) during and shortly after the big bang / crystallization and has since been moving together with the expanding elastic hyper-crystal. This is in accord with the finding that most large lumps of matter, such as galaxies, are nearly comoving with the Hubble flow.

2.5.17 Is there an internal time different from the ordinary time variable?

In magnetism, time reversal is a far-spread concept because it allows to reverse the orientation of the magnetization / of a spin vector. This way it enters the definition of magnetic point groups, for example the Shubnikov group (6).
However, in the case of isomagnetism one may replace the role of internal time reversal by charge conjugation, cf. (2.4.14), so that no internal time is needed.

2.5.18 Is there an absolute time in the hyper crystal?

Yes, it is given by the comoving Hubble time coordinate, i.e. the elapsed time since the big bang according to a clock of a comoving observer.

2.5.19 What is the status of the Copernican and of the Cosmological Principle in the tetron model?

The Copernican Principle states that no place in the universe is 'special' or preferred, while the Cosmological Principle demands that the universe looks the same in all directions (is isotropic) and has roughly the same smooth mixture of material (is homogeneous). These principles are not questioned by the tetron model, assuming a suitable uniformity of the hyper-crystal built from tetrons. However they will *not* be fulfilled at the edges of the DMESC where there may be steadily new accretions of tetrahedrons to the crystal. Those edges are therefore expected to be both 'special' and anisotropic places.

2.5.20 Is the original $R^{6,1}$ Lorentzean oder Galilean?

It was argued in section 1, that the speed of light, which is at the heart of Lorentz symmetry, is an intrinsic property of the hyper-crystal, not valid outside of it. Furthermore, the fact that metric expansion of our universe can proceed with velocities larger than c gives support for Galilean, or equivalently for a SO(6,1) Lorentz symmetry with a limiting speed much larger than c.

2.5.21 Do we know anything about the position of our universe in full R^{6+1}?

To show up in experiments such an information requires *anisotropic* interactions, which would get their anisotropy from 6-dimensional structures which go beyond the hyper-crystal. It cannot be obtained from the dominant isospin interactions (12) and (13), because these are rotationally invariant, i.e. the same mass spectrum is obtained after a global rotation of the isospin axes, and thus they are not useful for the present purpose.
A typical anisotropic Hamiltonian of isospin vectors $\vec{Q}_i$ would look like

$$H_a = -J_z \sum_{i \neq j}^{4} Q_{iz} Q_{jz} - J_{xy} \sum_{i \neq j}^{4} (Q_{ix} Q_{jx} + Q_{iy} Q_{jy}) \tag{75}$$

with $J_z \neq J_{xy}$.
Since they explicitly break internal rotational symmetry, anisotropic interactions like (75) violate internal angular momentum conservation (15). As discussed in (2.4.10), neutrinos are the Goldstone modes corresponding to that symmetry. More precisely, the 3 neutrino species can be identified as the 3 vibrating components of total internal angular momentum. Therefore, the anisotropic Hamiltonian (75) contributes to the neutrino masses, and measuring the neutrino mass matrix has the potential to answer the present question.

2.5.22 Is there a contribution from the tetron ground state configuration to the mass/energy density and to the expansion rate of the universe?

In (2.5.1) the energy of the ground state fig. 2 came into focus and was compared to another possible ground state configuration. According to general relativity the mass/energy density of the universe determines its curvature, its expansion rate and its future. So one might expect that the energy of the tetron ground state should also contribute. However, the forces of gravity have been interpreted in section 1 solely as stress arising from disclinations and/or dislocations within the hyper-crystal, which in turn originate from the presence of quasi-particle states which constitute ordinary matter. By contrast, the DMESC in its ground state is a perfect crystal/fluid, and by definition without any stresses.
The only effect from the tetron ground state configuration is to initialise the big bang process by releasing a vast amount of crystallization energy.

2.5.23 How does all of this fit into inflationary cosmology?

Inflation in the microscopic model is associated to the crystallization process of tetrahedrons with an accelerated expansion due to the elastic nature of the bindings and the sudden release of crystallization energy, cf. (2.1.13) and the discussion after (16).
The major condition for inflation is the exponential increase of the FLRW scale parameter a(t) in (64). The Einstein equations lead to

$$\frac{d^2a}{dt^2} = -\frac{4\pi}{3}G(\rho + 3p/c^2)a \tag{76}$$

where ρ is the mass density of the universe and p its pressure, and the combination $\rho + 3p/c^2$ corresponds to the trace of the energy momentum tensor. Eq. (76) makes it clear that the exponential increase needed for inflation is obtained for constant and negative $\rho + 3p/c^2$.
What kind of matter fulfills such a condition? The immediate answer: matter undergoing a phase transitions. Indeed it is normally assumed in models of inflation that a false vacuum decays in the framework of some abstract phase transition being active in the very early universe.

However, the physical background of this phase transition is never specified. In the tetron model, the situation is better. Namely, for the elastic hyper-crystal of tetrahedrons the trace of the energy momentum tensor can be obtained from the Landau free energy density ΔF given in (16) via $p = -\Delta F$ and $\rho = -p/c^2$, provided one assumes a sufficiently weakly fluctuating density D of tetrahedrons.
Inflation starts almost immediately after the big bang, i.e. the time when the germ of the hyper-crystal comes into being. This point corresponds to the maximum value of the free energy curve (16) from where the system rolls down to its non-trivial minimum, at which point the latent heat is released.
A word of warning: although the qualitative features of inflation are well described by (76), one should be aware that the Einstein equations are not valid close to the crystallization point. According to the discussion at the end of section 1, the least one must expect is a temperature dependence of c and G which will modify the details of the description.

2.5.24 How can the release of crystallization energy lead to metric velocities much larger than the speed of light as needed for inflation?

It is well known that metrical changes larger than c do not contradict general relativity and actually are needed in the standard cosmological model. This is no contradiction to GR because in GR there are rules about matter moving through space, but there is no rule about space expanding faster than light. The Einstein equations describe how the metric evolves after a release of (crystallization) energy, and if this happens fast and is large enough one can even have an inflationary expansion rate.
In the tetron model metric velocities have a special interpretation due to the motion of tetrahedrons in the micro-elastic system, cf. (2.5.38) and section 1.

2.5.25 What are the tetron model answers to the flatness and the horizon problem?

They are similar to those in ordinary inflationary models:
–flatness problem (the question why the universe is almost flat everywhere): due to

the exponential expansion – triggered by the release of crystallization energy – the universe is much larger than anticipated.
–horizon problem (the question why the universe looks almost the same everywhere): all parts of the universe were causally connected at the time when the hyper-crstal was born. Due to the subsequent exponential expansion they have lost their causal contact.

2.5.26 Is there an inflaton field?

Inflation was explained in section 1 and (2.5.23) as arising from the latent heat released in the crystallization of the hyper-plastics. Therefore in the tetron model, the inflaton can be interpreted as the energy/density wave that carries the crystallization energy. There is some similarity to the way dark energy is interpreted in (2.5.32).

2.5.27 Are the internal spaces compact or infinite?

They are infinite, no assumption about compactification of internal spaces needs to be made. The reason why we cannot step into the internal dimensions is because the hyper-crystal (=our universe) is restricted to a 3+1 dimensional 'surface' in R^{6+1}. Going away from the 'surface', internal space is empty, because according to (2.5.28) and (2.5.10) nothing can dissipate into the internal dimensions. Exception: other hyper-crystals may have condensed at big bang times. In general these will lie skewed with respect to the one we live in and form separate 'universes', cf. (2.3.21) and (2.5.33).

2.5.28 If internal space is infinite, how can one avoid dissipation of energy into the internal dimensions?

If internal space is infinite then matter and energy might in principle disappear into it. This can happen in the form of particles which move in the direction orthogonal to the hyper-crystal. However, in (2.1.2) and (2.1.4) I have taken the viewpoint that all observed particles including the photon are excitations of the crystal and as such cannot exist away from it. Furthermore, the tetrons, from which the crystal is made, are assumed to be so strongly bound, that they can be split off the hyper-crystal only by supply of Planck scale energies.

2.5.29 Was there a GUT era in the early universe where electroweak and strong couplings were united?

No, because there is no GUT – cf. (2.3.19). In the tetron model the strong force has a different origin than the electroweak one (cf. 2.3.25), and thus GUT unification seems unlikely.
The proper history of the universe starts with the end of the crystallization process (=the inflation era), at which point electromagnetism and weak forces are united, cf. (2.1.13). In the standard terminology this is the starting point of the radiation dominated era, with photons, effectively massless W/Z and dark matter as the dominant excitations. At the end of the electroweak era at temperatures of order Λ_F there is the alignment of isospin vectors corresponding to the electroweak phase transition which gives masses to q/l and W/Z.

2.5.30 Is there a unification of SM and gravitational forces at the Planck scale?

Not in the sense of supergravity and related models. Even at big bang temperatures gravity and SM forces have a very different nature. Although it is true that everything observed can eventually be derived from the fundamental forces among tetrons, the SM interactions trace back to interactions between isospin vectors of tetrons, whereas gravity is an elastic force between tetrahedrons which stems from remnant tetron coordinate interactions.

2.5.31 Are there dark matter candidates in the model?

Yes, there are several possibilities:
-further internal excitations of the crystal, like phinons, cf. (2.3.22) and [3]. Their interactions with mignons(=ordinary matter) is tiny, because they are not at all involved in the isomagnetic correlations giving rise to the SM.
-the pseudoscalar η arising in the 2HDM ansatz (2.1.15). Such a possibility is widely

discussed in the literature [62, 63] provided the η is inert, i.e. does not interact with quarks and leptons. This condition can be fulfilled in the present model essentially because of (43). Note that right after inflation there is the radiation dominated era where the inert scalar is copiously produced together with a soup of many other tetron-antitetron bound state excitations (photon, W/Z, Φ and Φ').

2.5.32 Is there an explanation for dark energy?

According to the standard cosmological model the rate of expansion of the universe should decrease in time due to the gravitational attraction of the matter content of the universe. However, observations indicate that the opposite is true: the universe's expansion rate was decelerating until about 5 billion years ago, after which time the expansion began accelerating. Phenomenologically, this can be explained by including a cosmological constant in the theory (this amounts to saying that a volume in space has some intrinsic fundamental vacuum energy creating a pressure which makes the universe expand) or by a weakly fluctuating scalar 'quintessence' field (this does a similar job).
Furthermore, such a type of energy that is not matter or dark matter is also needed to explain the apparent flatness of the universe (absence of any detectable global curvature). According to that argument the contribution of dark energy should be more than twice as large as that of matter and dark matter together.
A third explanation of the dark energy effect is offered by f(R,T) models (21) by suitable accommodation of the 11 phenomenological coupling constants[57]. This is sometimes considered superior because the value of the cosmological constant needed for dark energy is of unnatural size. Unfortunately, as discussed in section 1, f(R,T) models with their many effective interactions offer only a phenomenological description of gravitation.
To understand dark energy on the microscopic level, one may use the elastic hyper-crystal of the present paper. According to that idea, the micro-elastic forces that have initially induced cosmic expansion during the big bang crystallization process are still at work today. For example, there may still be accretions to the hyper-crystal at its edges which are setting free large amounts of crystallization energy. This energy is then transferred to the other tetrahedrons of the crystal in the form of

weakly fluctuating energy/density waves traveling through the universe. This is well along the line of the quintessence idea mentioned above. It gives the accelerations still present in the universe a more substantive meaning.
Another point to notice is that the time dependence of G, h and c introduced in section 1 will modify the standard analysis of cosmic expansion, i.e. leads to a modification of the $a(t) \sim t^{2/3}$ behavior of the FLRW scale factor in a dust dominated universe. Details of this effect will be worked out in a future publication.

2.5.33 Are there other universes?

Probably yes. One may consider the original $R^{(6,1)}$ spacetime as a container of universes. When the tetron gas cooled down and temperatures reached the crystallization temperature (Planck energy), germs of 3+1-dimensional hyper-crystals came into being in various places of $R^{(6,1)}$, cf. (2.3.21). These crystals then grew in their respective 3+1 dimensionsal subspaces, each of them making up for a separate $R^{(3,1)}$ spacetime. Since they are of low dimension as compared to the whole $R^{(6,1)}$, they hardly interfere with one another. If at all, they intersect in isolated (1-dimensional) points. At those points a defect in the isomagnetic and/or coordinate crystal structure will show up, because the isospin and/or coordinate vectors of the tetrons do not know how to orient themselves.

2.5.34 What is the interpretation of a black hole? Is it a spacetime singularity?

No, from its very nature the discrete elastic hyper-crystal does not allow for real singularities. It is true that black holes correspond to solutions to the Einstein equations, and the Einstein equations according to section 1 arise from the effective action (21) for the DMESC. However, these equations are not applicable at arbitrary small distances, where the discrete structure becomes perceptible.
It is generally believed, that if enough mass M is squeezed into a roughly spherical volume of size $r = GM/c^2$, it collapses into a black hole. What happens from the standpoint of the tetrahedrons is that inside the black hole's event horizon the crystal becomes extremely compressed, i.e. the distances between the tetrahedrons

become smaller and smaller. At the same time the temperature strongly increases as more and more matter (mignons, gauge bosons and other quasi-particle excitations) is accreted. When the temperature exceeds the Fermi scale, the isospin alignment of the hyper-crystal gets lost and the accreted mignons decay to photons and weak gauge bosons. In some sense this scenario is reverse to the appearance of the 'radiation dominated' epoque of the big bang. Finally, if temperatures reach the order of the crystallization energy Λ_P, the DMESC structure completely dissolves and the tetrahedrons vaporize to form a gas which is set free into the full $R^{(6,1)}$, i.e. into the internal directions.
It is conceivable that this hot gas becomes the germ of another hyper-crystallization process making up for another universe in the sense of (2.5.33).

2.5.35 Should gravity be quantized?

As emphasized in (2.5.6) the quantum behavior of nature is closely related to the granularity of physical space. Therefore it seems natural to believe that in circumstances where this discrete structure becomes relevant, gravitational effects should be treated in a quantum theoretical manner.
However, gravity in the tetron model point of view is an effective interaction of (internal) tetrahedrons in an elastic/plastics system. Its description by the Einstein-Hilbert action or its generalization (21) is valid only at distances $\gg L_P$, i.e. looses its validity when probed at distances where the discrete structure becomes appearant. Instead of 'quantizing' gravity one should quantize the fundamental interaction among tetrons.

2.5.36 Are there gravitons?

In section 1 general relativity has been interpreted as an effective theory for an elastic system of internal tetrahedrons. Gravitational waves exist in this theory on the classical level, as solutions to the Einstein equations. In the tetron model they can be associated to some of the density fluctuations of tetrahedrons discussed in connection with (16), and thus there is an analogy to sound waves in solids and liquids.[7]

Just as sound waves can be quantized as phonons, so can gravitational waves. However, just as for phonons this quantization procedure is sometimes necessary but does not point to any fundamental physics.

2.5.37 Why is the speed of gravitational waves equal to the speed of light?

As discussed in the last question and in section 1, in the tetron model there are elastic waves which arise from vibrations of the rigid internal tetrahedrons inside the hyper-crystal. In the Einstein theory gravitational waves are transverse metrical waves. Their velocity is forced to be equal to the speed of light by the condition of local Lorentz invariance.

Since one would like to identify some of the elastic waves with the gravitational waves, one first has to explain why elastic waves in the DMESC propagate at the same speed as the photon. From the standpoint of the microscopic model this is a rather amazing feature, because it was argued in (2.5.30) that the isomagnetic particle physics interactions of the photon and the elastic gravitational interactions do not have much in common.

In order to answer this question one should first realize that the world according to the microscopic model falls apart into 2 rather disparate pieces:

-the realm of what philosophers would call emergent or appearance phenomena, i.e. isomagnetic quasi-particles like quarks, leptons, Higgs and gauge fields. Since all these excitations fulfill Lorentz invariant wave equations, any phenomenon and signal propagation in this sphere is necessarily limited by the speed of light.

-the realm of what could be called 'true' or tetron matter, consisting of tetrons, of aligned tetrahedrons and of the DMESC with its elastic/metric structure. This may rightfully be called $'\upsilon\lambda\eta\ \pi\rho\omega\tau\eta$. However, while for Aristotle this was more an idea than a concrete material, here it can be understood in a real, *materialistic* sense. Just for joke one could call it 'tatter' to distinguish it from the ordinary mignon material which remains m-atter.

[7] In distinction the *internal* translational excitations were coined phinons in [3], while the internal rotational excitations are mignons, i.e. quarks and leptons.

Since the relevant scales $\Lambda_P \gg \Lambda_F$ are so vastly different, these 2 spheres do not have much in common. We ourselves live in the sphere of appearances and can perceive anything coming from the tatter sector only if suitable devices of mignon matter are patched in between. Gravity, for example, which originally corresponds to a shift of tetrahedron locations on the DMESC, becomes visible in our physical world only due to the reaction of suitable conglomerations of m-atter. In particular, the physical effects of a gravitational wave can only be seen by interposing appearances, i.e. ordinary matter which 'rides' the gravitational waves. The principle of relativity tells us that this ordinary matter must respect local Lorentz invariance and the principle of the speed of light. Therefore, although the fundamental interactions among tetrons may proceed at other velocities than c, the gravitational interactions between m-atter particles always appear to proceed at c.

2.5.38 How can metric velocities larger than c be interpreted in the tetron model?

This question is related to the discussion in (2.5.14),(2.5.15), (2.5.16), (2.5.24) and (2.5.37). c is the maximum speed for all the isomagnetic quasi-particles that build our known universe. However, this limit does not apply to the bound tetrahedrons which make up the hyper-crystal and are the carriers of the quasi-particles. As evident from (17), in the tetron model metrical changes are associated to displacements of tetrahedrons. The corresponding velocities of the tetrahedrons have been particularly large ($> c$) in the inflationary period shortly after the big bang (crystallization) where a lot of crystallization energy has been released.
According to (67) one can roughly identify the metric velocity in an FLRW universe with the Hubble flow Hd. In the tetron model this can be interpreted as the relative velocity of 2 tetrahedrons at distance d.

3 Conclusions

The present review is devoted to a model which tries to give a microscopic meaning to physical phenomena usually described by the Standard Model of elementary particles. By introducing an additional level of matter one is able to understand and to

calculate known particle properties (like the quark and lepton masses and mixings) from first principles and furthermore to make predictions for future experiments. Most prominent among the latter are:
–the existence of a fourth family of quarks and leptons with a very massive neutrino, as discussed in (2.3.22).
–the existence of a second Higgs doublet similar as in inert 2HDM models[62, 63], as discussed in (2.1.15).

The different viewpoints which have been presented in the preceding sections have supplied a set of important requirements as to the nature of tetron interaction. We know already
–that tetrons have a tendency to form excitated pairs with anti-tetrons of neighboring tetrahedrons in the crystal.
–that tetron bonds are extremely short, of the order of the Planck length.
–that they get saturated in tetrahedral configurations.
–that these configurations form 'flat' crystal structures, i.e. there is no stacking of tetrahedrons on top of one another, no growth of the hyper-crystal into internal directions.
–that isospins within the quartets of tetrons are maximally frustrated (fig. 1).
–that once tetrons are in such a saturated hyper-crystal configuration, there is a left-over elastic force among the internal tetrahedrons, which gives rise to the gravitational interactions.

After discussing particle physics properties, the implication of the tetron model on phase transitions in the early universe have been elucidated. This has led to the idea that besides the
–isomagnetic interactions among aligned isospin vectors which are relevant for particle physics
one should consider 2 other forces:
-strong rigid coordinate forces among tetrons fixing the form and extension of the (internal) tetrahedral structures and their arrangement into the hyper-crystal.
-extremely weak elastic forces between the tetrahedrons, which are the basis of the gravitational interactions.
All 3 types of forces are assumed to derive from one universal interaction among the tetrons, and it is suggested that one should use octonion multiplication and

perhaps supersysmmetry as a guideline which eventually will lead to the correct renormalizable theory in 6+1 dimensions.

References

[1] B. Lampe, arXiv:1311.6058 [hep-ph], Int. J. Mod. Phys. A30 (2015) 1550026.

[2] B. Lampe, arXiv:1405.6604 [hep-ph], Int. J. Mod. Phys. A30 (2015)1550025.

[3] B. Lampe, arXiv:1201.2281 [hep-ph], Int. J. Theor. Phys. 51 (2012) 3073.

[4] B. Lampe, arXiv:1212.0753 [hep-ph], Mod. Phys. Lett. A28 (2013) 135.

[5] B. Lampe, arXiv:0805.3762 [hep-ph], Found. Phys. 40 (2009) 573.

[6] J. Beringer et al. (Particle Data Group), Phys. Rev. D86 (2012) 010001.

[7] J.A. Wheeler, Ann. Phys. (N.Y.) 2 (1057) 604.

[8] C. Rovelli, arXiv:1102.3660 [gr-qc] (2011).

[9] A.P. Cracknell, Progr. Theor. Phys. 35 (1966) 196.

[10] A.P. Cracknell, Progr. Theor. Phys. 33 (1965) 812.

[11] A.S. Borovik-Romanov and H. Grimmer, International Tables for Crystallography D (2006) 105.

[12] R.M. White, Quantum Theory of Magnetism, Springer Verlag, ISBN-10 3-540-65116-0.

[13] D. Baumann, TASI lectures on inflation, arXiv:0907.5424 (hep-th) 2012.

[14] A. H. Guth, arXiv:astro-ph/0404546 (2004).

[15] G. Preparata and S.S. Xue, arXiv:hep-th/9503102 (1995).

[16] P. Jizba, H. Kleinert and F. Scardigli, Phys. Rev. D81 (2010) 084030.

[17] S.C. Law and K.L. McDonald, arXiv:1303.6384 [hep-ph] (2013).

[18] R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.

[19] J. Schechter and J.W.F. Valle, Phys. Rev. D 25, 2951 (1982)

[20] B.C. Allanach, S.F. King, G.K. Leontaris and S.Lola, Phys.Lett. B407 (1997) 275.

[21] H. Fritzsch and Z. Xing, Phys. Lett. B598 (2004) 237.

[22] F. Finster, J. Smoller and S. Yau, J. Math. Phys. 41 (2000) 2173.

[23] A. Ceccucci, Z. Ligeti and Y. Sakai (Particle Data Group), Phys. Rev. D86 (2012) 010001.

[24] M. Elhajal, B. Canals, R.S. Borrell and C. Lacroix, arXiv:cond-mat/0503009v1 (2005).

[25] R. Contino, The Higgs as a Composite Nambu-Goldstone Boson, TASI Lectures 2009.

[26] G. Cvetic, Rev. Mod. Phys. 71 (1999) 513.

[27] S. Weinberg, Phys. Rev. D19 (1979) 1277.

[28] S. Dimopoulos and L. Susskind, Nucl. Phys. B 155, 237 (1979).

[29] F. Sannino, Acta Phys. Polon. B 40, 3533 (2009), arXiv:0911.0931 [hep-ph].

[30] T.W. Appelquist, D. Karabali and L.C.R. Wijewardhana, Phys. Rev. Lett. 57, 957 (1986).

[31] E. Eichten et al., Rev. Mod. Phys. 56 (1984) 579.

[32] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345.

[33] J.O. Andersen and L.T. Kyllingstad, J. Phys. G37 (2010) 19.

[34] D. Ebert and M.K. Volkov, Z. Phys. C16 (1983) 205.

[35] A. Maiezza, M. Nemevsek, F. Nesti and G. Senjanovic, Phys. Rev. D 82 (2010) 055022.

[36] R. Jora, S. Nasrib and J. Schechter, arXiv:1302.6344v3 [hep-ph] (2013).

[37] S. Oneda and K. Terasaki, Progr. Theor. Phys. Suppl. 82 (1985) 1.

[38] R. Slansky, Phys. Rep. 79 (1981) 1.

[39] J. Schnack, Dalton Trans. 39 (2010) 4677, arXiv:0912.0411v1 [cond-mat] (2009).

[40] G.M. Dixon, Division Algebras, Kluwer Books, 2009.

[41] J. Conway and D. Smith, On Octonions and Quaternions, Peters Publishing, Natick, MA (2003).

[42] I. L. Kantor and A. S. Solodovnikov, Hypercomplex Numbers, Berlin, 1989.

[43] R.D. Schafer, An Introduction to Nonassociative Algebras,

[44] R. Arnowitt, S. Deser and C. Misner, Gen. Rel. Grav. 40 (2008) 1997.

[45] B. Carter and H. Quintana, Proc. Roy. Soc. 331 (1972) 57.

[46] arXiv:1403.1213v2 [astro-ph.CO] (2014).

[47] J.M. Cline, Les Houches Lectures, arXiv:hep-ph/0609145 (2006).

[48] K. Kajantie, M. Laine, K. Rummukainen and M.E. Shaposhnikov, Phys. Rev. Lett. 77 (1996) 2887.

[49] J.M. Cline and P.A. Lemieux, arXiv:hep-ph/9609240 (2007).

[50] C.P. Bean and D.S. Rodbell, Phys. Rev. 126 (1962) 104.

[51] M.J. Duff, Contemporary Physics 12 (2014) 56.

[52] J.W. Moffat, arXiv:0208109 [hep-th] (2012).

[53] E. and F. Cosserat, Thorie des Corps deformables, Paris 1909, Hermann et Fils.

[54] D. Röder, J. Ruppert and D.H. Rischke, Phys. Rev. D68 (2003) 016003.

[55] R.J. Adler, Am. J. Phys. 78 (2010) 925.

[56] T. Hambyea, F.S. Linga, L. Lopez Honoreza and J. Rochera, arXiv:0903.4010v1 [hep-ph] (2009).

[57] W. Lu, W. Lee and K.W. Ng, arXiv:1401.3155v1 [gr-qc] (2014).

[58] K. Hayashi and T. Shirafuji, Prog. Theor. Phys. 64 (1980) 866, 883, 1435, 2222.

[59] L.N. Mihaila, J. Salomon and M. Steinhauser, Phys. Rev. D86 (2012) 096008.

[60] V.A. Kosteleck. Phys. Rev. D69 (2004) 105009.

[61] G.C. Branco et al., Phys. Rep. 516 (2012) 1.

[62] I.F. Ginzburg, K.A. Kanishev, M. Krawczyk and D. Sokolowska, Phys. Rev. D82 (2010) 123533.

[63] M. Gustafson, arxiv:1106.1719 [hep-ph] (2011).

[64] A.L. Chernyshev and M.E. Zhitomirsky, Phys. Rev. B79 (2009) 144416.

[65] R. Yanes, J. Jackson, L. Udvardi, L. Szunyogh and U. Nowak, Phys. Rev. Lett. 111 (2013) 217202.

[66] L. Know and M.S. Turner, Phys. Rev. Lett. 73 (1994) 3347.

[67] D. Spolyar, arXiv:1111.3629v1 [astro-ph.CO] (2011).

[68] B. Tent, J. Smit and A. Tranberg, arXiv:hep-ph/0404128v2 (2004).

[69] G. German, G. Ross and S. Sarkar, Nucl. Phys. B608 (2001) 423.

[70] H. Georgi and C. Jarlskog, Phys. Lett. B86 (1979) 297.

[71] K. Bora, arXiv:1206.5909v1 [hep-ph] (2012).

[72] Y. Zhang, H. An, X. Ji and R.N. Mohapatra, Nucl. Phys. B802 (2008) 247.

[73] G. Altarelli, arXiv:hep-ph/0508053 [hep-ph] (2005).

[74] G. Altarelli, F. Feruglio and L. Merlo, arXiv:1205.5133 [hep-ph] (2012).

[75] P.F. Harrison, D.H. Perkins, and W.G. Scott, Phys. Lett. B530 (2002) 167.

[76] C. Jarlskog, Z. Phys. C29 (1985) 491.

[77] A. D. Sakharov, Doklady Akad. Nauk S. S. R. 177, 70-71 (1987).

[78] G. Senjanovic, arXiv:1012.4104 [hep-ph] (2011).

[79] M. Blagojevic, Kopaonik lectures (2002).

[80] T.P. Sotiriou and V. Faraoni arXiv:0805.1726 [gr-qc] (2008).

[81] G.E. Volovik, arXiv:gr-qc/0005091 (2000).

[82] S. Capozziello and M. Laurentis, Phys. Rep. 509 (2011) 167.

[83] S. L. Adler, Rev. Mod. Phys. 54, 729 (1982).

[84] F. Gronwald and F.W. Hehl, in Advances in Modern Continuum Dynamics, Isola Elba, 1991.

[85] F.W. Hehl and Y.N. Obukhov, Ann. Fond. L. de Broglie 32 (2007) 157.

[86] C.G. Bohmer and N. Tamanini, arXiv:1301.5471v2 [gr-qc] (2013)

[87] A. Einstein, Preuss. Akad. d. Wiss., Sitzungsberichte (1928) 217.

[88] V.C. Andrade, L.C.T. Guillen and J.G. Perreira, arXiv:gr-qc/0011087v1 (2000).

[89] G. G. Ross, Grand Unified Theories, Oxford University Press, 1984.

[90] C. Barcelo and G. Jannes, arXiv:0705.4652v2 [gr-qc] (2007).

[91] T.M. Davis and C.H. Lineweaver, arXiv:astro-ph/0310808v2 (2003).