Did Einstein use only two postulates to derive special relativity in 1905?

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Abstract

Einstein has been credited with deriving special relativity from two postulates, the principle of relativity and the constancy of the speed of light, in his first paper on special relativity in 1905. However, the existence of the Voigt transformation suggests that more than two postulates are needed for deriving the Lorentz transformation. In this study, Einstein's derivation of the Lorentz transformation is examined for logical consistency and implicit assumptions. It was found that Einstein's two postulates are insufficient for deriving the Lorentz transformation and essential additional assumptions were used. Because Einstein did not postulate all the necessary conditions, he arrived at the Lorentz transformation due to several logical mistakes caused by using same symbols for different quantities and variables. Therefore, Einstein did not derive special relativity from only two postulates in 1905. Einstein's most important real contribution to special relativity in his first relativity paper in 1905 is the expulsion of the medium of light waves from modern physics and the reinterpretation of the Lorentz transformation with relative velocity between reference frames instead of velocity relative to ether.

Key words: Lorentz transformation; speed of light; principle of relativity; Voigt transformation.

1. Introduction

Einstein published his first paper on relativity in 1905, in which he derived the Lorentz transformation from two postulates, the principle of relativity and the constancy of the speed of light (Einstein 1905). The physics community has generally believed that Einstein's derivation is logically valid and consistent. Einstein's feat in deriving special relativity from two simple postulates has been admired by generations of physicists and general public.

The Lorentz transformation lies in the core of special relativity, but Einstein is not the first physicist that proposed it. Larmor (1897) proposed a transformation very close to the Lorentz transformation. Lorentz (1899, 1904) and Larmor (1900) presented the transformation years before Einstein. These transformations were proposed to explain the null results of the Michelson-Morley experiment (Michelson 1881; Michelson et al. 1887). The Michelson-Morley experiment was carried out to measure the velocity of the earth relative to the medium of light surrounding the earth, because the experiments conducted by Arago (1853), Fizeau (1851), and Airy (1871) among others suggest that an object could not fully drag the medium of light around it. If the medium of light cannot be dragged by objects moving within the medium, it would be possible to measure the velocity of the object relative to the medium of light by optical or electromagnetic methods. However, all Michelson-Morley type experiments obtained null results, which have to be explained. Fitzgerald (1889) and Lorentz (1892, 1899, 1904) proposed length change hypotheses to explain the null results of the Michelson-Morley experiments, which culminates in Lorentz ether theory with the Lorentz transformation at its core.

In Lorentz ether theory, the Lorentz transformation is the primary assumptions or postulates on the space and time transformations between two reference frames. The constancy of the speed of light implied by the Michelson-Morley experiment is a consequence of the Lorentz transformation. Einstein (1905) used the constancy of the speed of light and the principle of relativity as the primary assumptions/postulates and the Lorentz transformation becomes a derived result from the two postulates. Although it is generally believed that Einstein's derivation in 1905 is logically valid, consistent and rigorous, the existence of the Voigt transformation (Voigt 1887) seems to suggest that Einstein must have implicitly used more than two postulates, or there are some logical errors in Einstein's derivation.

Why does the existence of the Voigt transformation suggest that Einstein used more than two postulates or made logical mistakes? Because Voigt transformation can ensure the constancy of the speed of light and satisfy the principle of relativity. In logic, $P \rightarrow Q$ means that P must be a sufficient condition (or a necessary and sufficient condition) of Q. If P is only a necessary condition of Q, i.e. $P \leftarrow Q$, then $(P \longleftarrow Q) \wedge P$ does not lead to Q. To prove that P is not a sufficient condition (or a necessary and sufficient condition) of Q, we only need to show that there is at least one instance where P is true, which is not Q. The Voigt transformation is just such an instance, which does not belong to the Lorentz transformation.

Since the existence of the Voigt transformation suggests that Einstein must have used more than two postulates or made logical mistakes in his derivation of the Lorentz transformation in 1905, it is philosophically important to understand what implicit postulates Einstein has used or what logical mistakes Einstein has made in his derivation, and to identify Einstein's real contribution to special relativity. The aim of the present study is 1) to identify such implicit postulates and possible logical mistakes by carefully examining Einstein's derivation in 1905; and 2) to establish Einstein's real contribution to special relativity if he has not rigorously derived the Lorentz transformation from the two postulates. This study does not question the correctness or validity of special relativity, nor does it question the logical consistency of special relativity per se.

The rest of the paper is organized as follows: sections 2 and 3 present the setup of Einstein's derivation and the resulted partial differential equation; section 4 examines the relationship between coordinates of the light wave front and the reflector in Einstein's derivation; section 5 looks into the role of the velocity of light perpendicular to the x-axis; section 6 analyzes Einstein's demonstration of $\phi(v) = 1$; section 7 concludes.

2. The setup of Einstein's derivation

Einstein (1905) gave his first derivation of the Lorentz transformation in his first paper on relativity. In this derivation, Einstein used the setup of two reference frames, the stationary system K (x, y, z, t) and the moving system k (ξ, η, ζ, τ), with relative velocity v along the x-axis and light signals sent from the origins of the two frames at $t = \tau = 0$ when the two origins overlap and $x = \xi = 0$. In the rest of this paper, paragraphs within quotation marks are text from Einstein's first relativity paper in 1905.

"If we place $x'=x-vt$, it is clear that a point at rest in the system k must have a system of values x', y, z, independent of time. We first define τ as a function of x', y, z, and t. To do this we have to express in equations that τ is nothing else than the summary of the data of clocks at rest in system *k*, which have been synchronized according to the rule given in §1.

From the origin of system *k* let a ray be emitted at the time τ_0 along the *X*-axis to x', and at the time τ_1 be reflected thence to the origin of the co-ordinates, arriving there at the time τ_2; we then must have $\frac{1}{2}(\tau_0+\tau_2)=\tau_1$, or, by inserting the arguments of the function τ and applying the principle of the constancy of the velocity of light in the stationary system:—

$$\frac{1}{2}\left[\tau(0,0,0,t)+\tau\left(0,0,0,t+\frac{x'}{c-v}+\frac{x'}{c+v}\right)\right]=\tau\left(x',0,0,t+\frac{x'}{c-v}\right).\text{"} \qquad (1)$$

Einstein's setup seems problematic. In this setup, x' is the distance between the origin of system *k* and the reflector as measured in *K*, which is not a variable representing the coordinate of the wave front of the light ray. Using x' instead of a usual distance notation has the risk of mixing up distance with a coordinate variable. It would be better to use symbols such *d* in place of x'. Moreover, such a setup does not relate time with the coordinates of the wave front of light beam. We may ask what τ is, and there are two possibilities: time intervals or time points. Either of the possibilities suggests modifications to equation (1).

If τ represents intervals of time ($\tau_{interval}$) needed for the two-way journey of a distance by a light pulse, equation (1) is not appropriate, because then $\tau_{interval}$ should be a function of distance x' and time interval ($t_{interval}$) in *K*.

$$\frac{1}{2}\tau\left(2x',0,0,\frac{x'}{c-v}+\frac{x'}{c+v}\right)=\tau\left(x',0,0,\frac{x'}{c-v}\right). \qquad (2)$$

In equation (2), $\frac{x'}{c-v}+\frac{x'}{c+v}$ is the time interval for the two-way journey, and $\frac{x'}{c-v}$ the time interval for the outward journey, which is half that for the two-way journey as required by the constancy of the speed of light; x' and 2 x' are distances along the *x*-axis. Changes in the distance will influence the $\tau_{interval}$ and $t_{interval}$ which are time intervals needed in *k* and *K* respectively to cover the distance x' measured in *K* and its counterpart in *k*. For the $\tau_{interval}$ and $t_{interval}$ to be meaningful, there must be a distance x' to cover. However, Einstein's equation (1) is arranged as the relationship between time points in k and space-time variables in K, indicating that τ and t are time points rather than time intervals.

If τ represents time points in the time flow, equation (1) is also inappropriate, because it indicates that x (the x-axis position of the light wave front, light source or reflector) has no influence on τ. If τ changes with t, its changes must be correlated with x, whereas x' is an arbitrary fixed distance independent of time. For the flow of time in k and K, the relationship between the $\tau_{time\ point}$ and $t_{time\ point}$ must be same no matter whether there is a reflector being placed on the x-axis or not and no matter where a reflector is placed on the x-axis. The inclusion of an arbitrary fixed distance is just to help us incorporate the constancy of the speed of light into the relation between the $\tau_{time\ point}$ and $t_{time\ point}$ in the setup. The actual value of this distance between a light source and a reflector should have no influence on the time flow in either k or K.

We know that in the Lorentz transformation τ is a function of t and x. Since τ and t are time points in the flow of time, it is more appropriate for them to link with a changing variable x than a constant distance x'. In my view, the correct representation of equation (1) should be

$$\frac{1}{2}\left[\tau(x,0,0,t)+\tau\left(x,0,0,t+\frac{x'}{c-v}+\frac{x'}{c+v}\right)\right]=\tau\left(x+x',0,0,t+\frac{x'}{c-v}\right) \qquad (3)$$

or

$$\frac{1}{2}\left[\tau(x-x',0,0,t)+\tau\left(x-x',0,0,t+\frac{x'}{c-v}+\frac{x'}{c+v}\right)\right]=\tau\left(x,0,0,t+\frac{x'}{c-v}\right). \quad (4)$$

In equations (3) and (4) τ is a function of t and x, the first argument of τ is the x-axis position of the wave front of the light ray in K, and x' is introduced to ensure the constancy of the speed of light. In equation (3), x is a variable indicating the changing position of (the light source place at) the origin of system k in coordinate system K, not one of the two fixed values 0 and x' (the origin of k and the position of the reflector). In equation (4), x is a variable indicating the changing position of the reflector (at rest in k) of the light ray in coordinate system K. When x' is chosen infinitesimally small, x becomes the position of the wave front $(x = ct)$ in both equations (3) and (4). So the two equations are equivalent.

3. The partial differential equation linking time and space variables

From equation (1) Einstein obtained a partial differential equation that relates τ with t and x', the distance between the light source and the reflector.

"Hence, if x' be chosen infinitesimally small,

$$\frac{1}{2}\left(\frac{1}{c-v}+\frac{1}{c+v}\right)\frac{\partial\tau}{\partial t}=\frac{\partial\tau}{\partial x'}+\frac{1}{c-v}\frac{\partial\tau}{\partial t}, \tag{5}$$

or

$$\frac{\partial\tau}{\partial x'}+\frac{v}{c^2-v^2}\frac{\partial\tau}{\partial t}=0 ." \tag{6}$$

It seems that Einstein took Taylor expansion for the three τ terms in equation (1) with respect to x' and t, which gives the relation between τ (the time of system k) and t as well as x', the distance from the light source to the reflector (measured in system K). But Einstein's expansion seems not to follow the Taylor expansion rules strictly. If the term on the right hand side of equation (1) is expanded with respect to x' and t, the result should be $\tau(x', 0,0, t) + \frac{x'}{c-v}\frac{\partial\tau}{\partial t}$, in which $\tau(x', 0,0, t)$ cannot be cancelled with its counterpart $\tau(0,0,0, t)$on the left hand side and there is no $x'\frac{\partial\tau}{\partial x'}$. Einstein seems to get $\tau(0,0,0, t) + x'\frac{\partial\tau}{\partial x'} + \frac{x'}{c-v}\frac{\partial\tau}{\partial t}$ from expanding the right hand side, $\tau(0,0,0, t)$ can be cancelled out with that on the left hand side and the common factor x' contained in all other terms can be cancelled out as well to arrive at equation (5).

Einstein's equation (6) gives us an impression that the flow of time in the two systems is partially determined by where a reflector is placed. This is obviously incorrect. What the distance of a reflector from a light source can determine is only the time interval needed to reach it.

In my view, the relationship between time τ and the coordinate of the wave front represented by equation (3) or (4) is more relevant. By Taylor expansion of equation (3) or (4), we obtain

$$\frac{1}{2}\left(\frac{1}{c-v}+\frac{1}{c+v}\right)\frac{\partial \tau}{\partial t}=\frac{\partial \tau}{\partial x}+\frac{1}{c-v}\frac{\partial \tau}{\partial t}, \tag{7}$$

which can be simplified to

$$\frac{\partial \tau}{\partial x}+\frac{v}{c^2-v^2}\frac{\partial \tau}{\partial t}=0. \tag{8}$$

With Taylor expansion both equations (3) and (4) give equation (8). Therefore where a reflector is placed has no influence on the transformation of coordinates between the two systems, although the introduction of the return journey with a reflector helps us to incorporate the constancy of the speed of light into the initial equation. This is what we should have expected for the flow of time.

“It is to be noted that instead of the origin of the co-ordinates we might have chosen any other point for the point of origin of the ray, and the equation just obtained is therefore valid for all values of x', y, z.”

If Einstein was examining the relationship between the distance from the light source to the reflector and time τ needed to cover this distance, the position of light source is irrelevant. However, Einstein was deriving transformations between time points and x' was used by him as the coordinate of the reflector as well; the position of the light source will be relevant in his particular setup, but it is irrelevant in equations (3) and (4). If the light source is not at the origin of system k, let’s say x'_1 which means $x'_1 + vt$ in the system K, the distance between the light source and the reflector becomes $x' - x'_1$. The correct Taylor expansion of Einstein’s equation (1) where the light source is placed at x_1 instead of the origin should obtain

$$\frac{1}{2}\left[2\tau(x'_1,0,0,t)+\left(\frac{x'-x'_1}{c-v}+\frac{x'-x'_1}{c+v}\right)\frac{\partial \tau}{\partial t}\right]=\tau(x',0,0,t)+\frac{x'-x'_1}{c-v}\frac{\partial \tau}{\partial t}\ . \tag{9}$$

Or

$$\tau(x',0,0,t)-\tau(x'_1,0,0,t)+\frac{v(x'-x'_1)}{c^2-v^2}\frac{\partial\tau}{\partial t}=0\ . \tag{10}$$

Following Einstein's example in equation (5), we might get

$$\frac{\partial\tau}{\partial x\prime}x'-\frac{\partial\tau}{\partial x'_1}x'_1+\frac{v(x'-x'_1)}{c^2-v^2}\frac{\partial\tau}{\partial t}=0 \tag{11}$$

In such cases, Einstein's equations (5) and (6) cannot be derived from his assumptions, and another variable would be introduced. In contrast, with equations (3) and (4), where the light source or the reflector is placed has no influence on the relationship between τ and *t*.

"An analogous consideration—applied to the axes of *Y* and *Z*—it being borne in mind that light is always propagated along these axes, when viewed from the stationary system, with the velocity $\sqrt{c^2-v^2}$ gives us

$$\frac{\partial\tau}{\partial y}=0,\qquad \frac{\partial\tau}{\partial z}=0.\text{"} \tag{12}$$

The assertion that "light is always propagated along these axes, when viewed from the stationary system, with the velocity $\sqrt{c^2-v^2}$" cannot be derived from the two postulates. Without knowledge of how length in directions parallel or perpendicular to the direction of velocity, it is not certain that $c_y=c_z=\sqrt{c^2-v^2}$ when viewed from the stationary system. If Einstein uses it in his derivation, it is an additional condition, the postulate or assumption No.3. Moreover, Einstein did not say what type of light wave propagates along these axes with the velocity $\sqrt{c^2-v^2}$. Is it a ray of light sent from the origin of *k* along the η- or ζ-axis or a spherical light wave propagating from the origin of *K*?

4. Use x' to represent the position of both the reflector and the wave front

A key element in the following steps of Einstein's derivation is to use x' to represent both the distance between the origin of *k* and the wave front, $x'=x_{wave\ front}-vt=ct-vt$, and the distance between the origin of *k* and the reflector,

$x' = x_{reflector} - vt$. Einstein solved the partial differential equation to obtain τ as a function of t and x'.

"Since τ is a linear function, it follows from these equations that

$$\tau = a\left(t - \frac{v}{c^2 - v^2} x'\right) . \qquad [(13)]$$

where a is a function $\phi(v)$ at present unknown, and where for brevity it is assumed that at the origin of k, $\tau = 0$, when $t = 0$."

In Einstein's terminology, x' is originally defined as the fixed length between the origin of system k (the location of the light source) and the position of the reflector. In this context, x' should be a constant in equation (13), then the assumption 'at the origin of k, $\tau = 0$, when $t = 0$' contradicts to the equation (13). However, Einstein obviously has quietly changed the meaning of x' and used it to represent position of the wave front. As I pointed out earlier, because x' is a fixed (time independent) distance between two points at rest in system k with the expression $x' = x - vt$, it should not be a factor to influence the time transformation between two reference systems. What should be contained in equations (6) and (13) besides t is a (time dependent) variable in system K.

The correct expression for τ should be solved from equation (8)

$$\tau = a\left(t - \frac{v}{c^2 - v^2} x\right) . \qquad (14)$$

In equation (14), x is the position of the wave front, $x = ct$; when $= 0, x = 0$ and $\tau = 0$. Its simplest form is

$$\tau = t - \frac{v}{c^2 - v^2} x . \qquad (15)$$

This is the same transformation equation for time as the one proposed by Lorentz (1892). Einstein's setup seems incorrect, but there will be more logical mistakes in the rest of his derivation.

“With the help of this result we easily determine the quantities ξ , η , ζ by expressing in equations that light (as required by the principle of the constancy of the velocity of light, in combination with the principle of relativity) is also propagated with velocity *c* when measured in the moving system. For a ray of light emitted at the time $\tau = 0$ in the direction of the increasing ξ ,

$$\xi = c\tau\,,\ \xi = ac\left(t - \frac{v}{c^2 - v^2}x'\right).\text{”} \tag{16}$$

Since x' is the distance from the light source to the reflector, does this equation imply that the distance travelled by a light beam somehow depends on where a reflector is placed? Then, if a reflector was placed on Pluto, a spacecraft flying from the earth to Pluto and sending a light beam toward Pluto would find the light beam propagating backward to earth for a short while. This is obviously incorrect.

“But the ray moves relatively to the initial point of *k*, when measured in the stationary system, with the velocity *c-v*, so that

$$\frac{x'}{c - v} = t.\text{”} \tag{17}$$

Equation (17) can still be interpreted as time interval for the light ray to travel the distance x'.

$$t_{light\ to\ cover\ distance\ x'observed\ in\ K} = \frac{x'}{c-v} \tag{18}$$

“If we insert this value of *t* in the equation for ξ , we obtain

$$\xi = a\frac{c^2}{c^2 - v^2}x'.\text{”} \tag{19}$$

If x' is still what is originally defined by Einstein, now *t* in equation (13) is substituted by an expression of time interval in terms of the distance to the reflector as well. Equation (19) would be a transformation for reflector coordinates. However, if *t* in equation (13) indicates time flow rather than a time interval, equation (17) cannot be substituted into it. So Einstein has changed the meaning of x' quietly from the distance between the light source and the reflector to the position of the wave front of the light beam. This is a logic fallacy of equivocation.

5. The role of the velocity of light perpendicular to the *x*-axis

Einstein specified earlier that "light is always propagated along these axes, when viewed from the stationary system, with the velocity $\sqrt{c^2 - v^2}$". Now this velocity plays a key role in arriving at the factor $\sqrt{1 - v^2/c^2}$.

"In an analogous manner we find, by considering rays moving along the two other axes, that

$$\eta = c\tau = ac\left(t - \frac{v}{c^2 - v^2}x'\right)\text{''} \tag{20}$$

Here τ seems to indicate time point; x' was originally defined as the distance between the light source and the reflector, and η the position of light rays moving along the η-axis. Please note here that the existence of x' does not stop light rays from propagating along the η-axis.

"When

$$\frac{y}{\sqrt{c^2 - v^2}} = t,\ x' = 0\text{.''} \tag{21}$$

Here, Einstein made three mistakes:

Firstly, let y be the position of a light ray being sent from the origin of k at $t = 0$ and propagating along y-axis, if the velocity of the light ray observed by the stationary system is $\sqrt{c^2 - v^2}$, then any rays of light perpendicular to the *x*-*z* plane will be observed by the stationary system to be $\sqrt{c^2 - v^2}$. It is wrong to assert "when $\frac{y}{\sqrt{c^2-v^2}} = t, x' = 0$". When $\frac{y}{\sqrt{c^2-v^2}} = t$, it could be $x' = 100000$, as long as the light ray perpendicular to the *x*-*z* plane.

Secondly, x' was originally defined as the distance between the light source and the reflector, so it is neither affected by time nor influenced by the propagation of light along the y-axis. Einstein seemed to forget that x' is the distance between the light source and the reflector and use x' as the x' (x)-coordinate of any point of the wave front.

Thirdly, Einstein introduced the properties of spherical light wave propagation in a model of plane wave light rays. If a spherical light wave propagates from the overlapping origins of K ($x = 0, y = 0, z = 0$) and k ($x' = 0, y' = 0, z' = 0$) at $t = 0$ and $t' = 0$, we have the velocity of light in directions parallel to the y-axis, $c_y = \sqrt{c^2 - v^2}$ at $x' = 0$, which can be obtained from the spherical light wave propagation equation

$$x^2 + y^2 + z^2 = c^2t^2 \tag{22}$$

In K, $x' = 0$ corresponds to $x_{x'=0} = vt$, so we have

$$(vt)^2 + y^2 + 0^2 = c^2t^2$$

which implies

$$c_y = \frac{y}{t} = \sqrt{c^2 - v^2} \tag{23}$$

However, a ray of plane wave light perpendicular to the x-z plane does not have such a relationship between y and x or x'.

“Thus

$$\eta = a\frac{c}{\sqrt{c^2 - v^2}}y \text{ and } \zeta = a\frac{c}{\sqrt{c^2 - v^2}}z \text{.”} \tag{24}$$

As a ray of plane wave light perpendicular to the x-z plane does not have such a relationship as “when $\frac{y}{\sqrt{c^2-v^2}} = t$, $x' = 0$”, nor does a ray of plane wave light perpendicular to the x-y plane have such a relationship as “when $\frac{z}{\sqrt{c^2-v^2}} = t$, $x' = 0$”, equation (24) is incorrect.

“Substituting for x' its value, we obtain

$$\begin{aligned}
\tau &= \phi(v)\beta(t - vx/c^2),\\
\xi &= \phi(v)\beta(x - vt),\\
\eta &= \phi(v)y,\\
\zeta &= \phi(v)z,
\end{aligned} \qquad [(25)]$$

where

$$\beta = \frac{1}{\sqrt{1 - v^2/c^2}}. \qquad [(26)]$$

and ϕ is an as yet unknown function of v. If no assumption whatever be made as to the initial position of the moving system and as to the zero point of τ, an additive constant is to be placed on the right side of each of these equations."

Einstein did not say which x' value is substituted, $x' = ct - vt$ or $x' = x - vt$? The former has been used to arrive at equation (19), $\xi = a\frac{c^2}{c^2 - v^2}x'$, and the latter was defined at the very beginning of this derivation. From the results, it appears that Einstein used the latter in obtaining $\xi = \phi(v)\beta(x - vt)$ from equation (19), another logical fallacy of equivocation. From the following four equations,

$$\tau = a\left(t - \frac{v}{c^2 - v^2}x'\right),$$

$$\xi = a\frac{c^2}{c^2 - v^2}x',$$

$$\eta = a\frac{c}{\sqrt{c^2 - v^2}}y,$$

$$\zeta = a\frac{c}{\sqrt{c^2 - v^2}}z. \qquad (27)$$

Einstein appears to substitute first $x' = x - vt$, and then let $\phi(v) = a/\sqrt{1 - v^2/c^2}$ to arrive at the equations (25). This step shows how important it is to postulate the velocity of light along the η- and ζ-axes to be $\sqrt{c^2 - v^2}$ for deriving the Lorentz transformation. Without this assumption, Einstein would not be able to derive equation (25). Once these conditions are accepted, there is no specific logical error in obtaining equation (25) from these conditions.

6. Einstein's demonstration of $\boldsymbol{\phi(v) = 1}$

So far, Einstein has used three assumptions or postulates, the principle of relativity, the constancy of the speed of light, and the velocity of light in the moving system along the axes perpendicular to the velocity between the two reference frames being $\sqrt{c^2 - v^2}$. There are also several mistakes or inadequacies in Einstein's derivation so far. The final step of Einstein's derivation is a demonstration of $\phi(v) = 1$, which will complete the derivation of the Lorentz transformation.

With equation (25), Einstein needs to solve $\phi(v)$ to complete his derivation of the Lorentz transformation. He used double application of the transformation of the same reference frame via another reference frame and also asserted that the length of a moving rod perpendicular to the direction of the velocity measured in the stationary frame does not change.

"In the equations of transformation which have been developed there enters an unknown function ϕ of *v*, which we will now determine.

For this purpose we introduce a third system of co-ordinates *K'*, which relatively to the system *k* is in a state of parallel translatory motion parallel to the axis of *Ξ*, such that the origin of co-ordinates of system *K'*, moves with velocity -*v* on the axis of *Ξ*. At the time *t=0* let all three origins coincide, and when *t=x=y=z=0* let the time *t'* of the system *K'* be zero. We call the co-ordinates, measured in the system *K'*, *x'*, *y'*,*z'*, and by a twofold application of our equations of transformation we obtain

$$\begin{aligned} t' &= \phi(-v)\beta(-v)(\tau + v\xi / c^2) = \phi(v)\phi(-v)t, \\ x' &= \phi(-v)\beta(-v)(\xi + v\tau) = \phi(v)\phi(-v)x, \\ y' &= \phi(-v)\eta = \phi(v)\phi(-v)y, \\ z' &= \phi(-v)\zeta = \phi(v)\phi(-v)z. \end{aligned} \qquad [(28)]$$

Since the relations between x', y', z' and x, y, z do not contain the time t, the systems K and K' at rest with respect to one another, and it is clear that the transformation from K to K' must be the identical transformation. Thus

$$\phi(v)\phi(-v) = 1.\text{”} \qquad (29)$$

Einstein used two applications of the transformations which goes back to the same stationary (observing) system K (now named K' as a third coordinate system) to arrive at $\phi(v)\phi(-v) = 1$ by the fact t (now named t' in the identical coordinate system K') must be equal to t. He then obtained $\phi(v) = 1$ by assuming that the length of a rod at directions perpendicular to the velocity does not change.

“We now inquire into the signification of $\phi(v)$. We give our attention to that part of the axis of Y of system k which lies between $\xi = 0$, $\eta = 0$, $\zeta = 0$ and $\xi = 0$, $\eta = l$, $\zeta = 0$. This part of the axis of Y is a rod moving perpendicularly to its axis with velocity v relatively to system K. Its ends possess in K the co-ordinates

$$x_1 = vt, y_1 = \frac{l}{\phi(v)}, z_1 = 0$$

and

$$x_2 = vt, y_2 = 0, z_2 = 0.$$

The length of the rod measured in K is therefore $l/\phi(v)$; and this gives us the meaning of the function $\phi(v)$. From reasons of symmetry it is now evident that the length of a given rod moving perpendicularly to its axis, measured in the stationary system, must depend only on the velocity and not on the direction and the sense of the motion. The length of the moving rod measured in the stationary system does not change, therefore, if v and $-v$ are interchanged. Hence follows that $l/\phi(v) = l/\phi(-v)$, or $\phi(v) = \phi(-v)$.

It follows from this relation and the one previously found that $\phi(v) = 1$, so that the transformation equations which have been found become

$$
\begin{aligned}
\tau &= \beta(t - vx/c^2),\\
\xi &= \beta(x - vt),\\
\eta &= y,\\
\zeta &= z,
\end{aligned}
\qquad [(30)]
$$

where

$$\beta = 1/\sqrt{1 - v^2/c^2}\ ."$$

In the above reasoning, the constant length at the directions perpendicular to the velocity of the moving frame is essential. However, the principle of relativity does not exclude the possibility that a rod of length L in k, perpendicular to the direction of the velocity, is observed by K to be $L\sqrt{1 - v^2/c^2}$ and a rod of length L in K is observed by k to be $L\sqrt{1 - v^2/c^2}$ (as in the Voigt transformation). The equations $\eta = y$ and $\zeta = z$ cannot be obtained from the principle of relativity, as the relativistic length contraction in any direction in another reference frame cannot tell the absolute velocity of your own reference frame and the law of physics can still be the same in all reference frames. Therefore, the statement "$\eta = y$ and $\zeta = z$ when the velocity between K and k is along the x-axis" is another postulate needed for deriving the Lorentz transformation, the postulate or assumption No.4.

7. Conclusions

Einstein's reputation is partly built on the belief in the mainstream physicists, many historians and philosophers of science, and the general public that Einstein derived special relativity from only two postulates in his first paper on special relativity. The present study has demonstrated that Einstein not only implicitly used more than two postulates or assumptions, but also made logical mistakes because he did not assume all the necessary conditions.

In addition to the two postulates, Einstein also stipulated 1) the velocity of light along other axis is $\sqrt{c^2 - v^2}$; and 2) the length of a rod perpendicular to the direction of the velocity of the moving frame does not change, as observed by the stationary frame. Since Einstein did not postulate a proper function form for the space transformation, even with those additional assumptions he still had to mix up two different concepts, the position of wave fronts and the position of the reflector. With this logical mistake and additional assumptions, Einstein arrived at the Lorentz transformation.

The Lorentz transformation and consequently special relativity cannot be derived from only these two postulates, the principle of relativity and constancy of the speed of light without additional postulates on the function form of the transformation and length/distance of vertical directions. To obtain the Lorentz transformation through deduction, two additional postulates have to be made:

1. The length/distance in the directions orthogonal to the direction of the velocity does not change so that Voigt transformation and similar ones are excluded.
2. Basic function forms of the transformation (Ma 2014)

$$x' = ax - avt$$
$$t' = mt - nx \tag{31}$$

Since the Lorentz transformation cannot be derived uniquely from the constancy of the speed of light and the principle of relativity, it is incorrect to assert that Einstein derived special relativity from only two postulates. Therefore, Einstein's most important real contribution in his first relativity paper in 1905 is not the derivation of special relativity from only two postulates; instead it is to reinterpret the Lorentz transformation by expelling ether from Lorentz ether theory and designating the velocity in the Lorentz transformation as the relative velocity between two reference frames, which ushers in Einstein's new relativistic space-time views.

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